**DEVELOPMENT OF SOME RIDGE ESTIMATORS FOR CLASSICAL AND GENERALIZED LINEAR REGRESSION MODELS**

**BY**

**ALADEITAN, BENEDICTA BOLUWAJI**

**(19PGCF000076)**

**A Thesis submitted to the Department of Physical Sciences, Mathematics Programme, College of Pure and Applied Sciences, Landmark University, Omu-Aran. Nigeria.**

**In Partial Fulfilment of the Requirements for the Award of the Degree of Doctor of Philosophy (Ph.D.) in Mathematics.**

**SEPTEMBER, 2022.**

DECLARATION

I, **BENEDICTA, BOLUWAJI ALADEITAN,** a Ph.D. student in the Department of Physical Sciences, (Mathematics Programme), Landmark University, Omu-Aran, hereby declare that this thesis entitled “Development of some ridge estimators for the classical and generalized linear regression models”, submitted by me is based on my original work. Any material(s) obtained from other sources or work done by any other persons or institutions have been duly acknowledged.

ALADEITAN, BENEDICTA BOLUWAJI (19PGCF000076)

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Signature & Date

CERTIFICATION

This is to certify that this thesis has been read and approved as meeting the requirements of the Department of Physical Sciences, Landmark University, Omu-Aran, Nigeria, for the Award of Ph.D. Degree.

Prof. O. Adebimpe Date

Supervisor

Prof. K. Ayinde Date

Co- Supervisor

Dr. S.O. Ikubanni Date

Head of Department

External Examiner Date

ABSTRACT

Regression analysis is a statistical tool usually used to study the relationship between a dependent variable and independent variables. The classical linear regression model (CLRM) and Generalized Linear Regression Model (GLRM) are examples of regression analysis techniques. The Ordinary Least Squares (OLS) and the Maximum Likelihood Estimator (MLE) are used in estimating parameters in CLRM and GLRM respectively when there is no violation of any assumptions made on the models. Recent researches have shown that time series variables and economic variables grow together which results into the problem of multicollinearity. The aim of this study was to develop some estimators to address multicollinearity problem in CLRM and GLRMs more efficiently. The objectives were to propose some ridge estimators and their parameters by modifying the KL estimator; compare performances of the proposed ridge estimators and their ridge parameters with some existing ones; identify the ridge parameters that are more efficient; and apply the estimators to real life data sets.

The Kibria Lukman (KL) estimator recently developed was modified by replacing the with and a new Modified KL (MKL) estimator was obtained. New generalized versions of the shrinkage parameters were developed from the KL and MKL parameters. These generalized ridge parameters were further considered in forms of minimum, maximum, median, mid-range, arithmetic mean, geometric mean and harmonic mean of the eigen values of the design matrix to obtain other generalized and ordinary ridge parameters. The new estimator and different versions of the shrinkage parameters were introduced to the Linear, Poisson and Logistic regression models and their performance examined through Monte Carlo simulation study and real life data sets.

The MRKLHM, GMKL1AM and the MAMKL2 with frequency of 142, 57 and 132 were selected for KL, MKL1 and MKL2 respectively as the best performing parameters in the linear regression model. The MAMKL2 consistently performed well in a simulation and real life study. The GMKLHM, MNMKL1 and GMMKL2GM with frequency of 34, 58 and 39 were selected for the KL, MKL1 and MKL2 estimators respectively for the Poisson regression model. The GMKLHM parameter showed more consistency and efficiency in the simulation and real life study. The MAKLMN, GMKL1MN and AMMKL2HM were selected with frequency of 20, 20 and 19 for the KL, MKL1 and MKL2 estimators respectively for the logistic regression model. The AMMKL2HM parameter performed consistently and efficiently in the simulation and real life study.

The parameter(s) with highest frequency after being ranked between 1 and 10 for the KL and MKL estimators were considered the most efficient parameters for the KL and MKL estimators respectively. The different forms and types of these ridge parameters for KL, MKL1 and MKL2 were extended to the Poisson and logistic regression models. Versions of MKL2 ridge parameter that performed well for the Linear and Logistic regression models could be adopted for estimating parameters in the presence of multicollinearity while the versions of KL ridge parameter that performed well for the Poisson regression model could be used for parameter estimation in the presence of multicollinearity.

**Word Count: 500 words.**

DEDICATION

This thesis is dedicated to the great God, the Lord Almighty, the author and the finisher of faith.

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LIST OF ABBREVATION

OLS Ordinary Least Sqaure

MLE Maximum Likelihood Estimator

CLRM Classical Linear Regression Model

GLRM Generalized Linear Regression Model

PRM Poisson Regression Model

SE Standard Error

PCR Principal Component Regression

MSE Mean Square Error

MRT Modified Ridge Type

IGRM Inverse Gaussian Regression Model

KL Kibria Lukman

MKL Modified Kibria Lukman

GKL Generalized Kibria Lukman parameter

GKLAM Generalized Kibria Lukman with Arithmetic Mean of .

GKLMN Generalized Kibria Lukman with Minimum of .

GKLMA Generalized Kibria Lukman with Maximum of

GKLMR Generalized Kibria Lukman with Midrange of .

GKLMD Generalized Kibria Lukman with Median of .

GKLGM Generalized Kibria Lukman with Geometric Mean of

GKLHM Generalized Kibria Lukman with Geometric Mean of .

AMKL Arithmetic Mean of Kibria Lukman parameter.

AMKLAM Arithmetic Mean of Kibria Lukman with Arithmetic Mean of .

AMKLMN Arithmetic Mean of Kibria Lukman with Minimum of .

AMKLMA Arithmetic Mean of Kibria Lukman with Maximum of .

AMKLMR Arithmetic Mean of Kibria Lukman with Midrange of .

AMKLMD Arithmetic Mean of Kibria Lukman with Median of .

AMKLGM Arithmetic Mean of Kibria Lukman with Geometric Mean of .

AMKLHM Arithmetic Mean of Kibria Lukman with Arithmetic Mean of .

MNKL Minimum Kibria Lukman parameter.

MNKLAM Minimum Kibria Lukman with Arithmetic Mean of

MNKLMN Minimum Kibria Lukman with Minimum of

MNKLMA Minimum Kibria Lukman with Maximum of

MNKLMR Minimum Kibria Lukman with Midrange of

MNKLMD Minimum Kibria Lukman with Median of

MNKLGM Minimum Kibria Lukman with Geometric Mean of

MNKLHM Minimum Kibria Lukman with Harmonic Mean of

MAKL Maximum Kibria Lukman parameter

MAKLAM Maximum Kibria Lukman with Arithmetic Mean of .

MAKLMN Maximum Kibria Lukman with Minimum of .

MAKLMA Maximum Kibria Lukman with Maximum of .

MAKLMR Maximum Kibria Lukman with Midrange of .

MAKLMD Maximum Kibria Lukman with Median of .

MAKLGM Maximum Kibria Lukman with Geometric Mean of .

MAKLHM Maximum Kibria Lukman with Harmonic Mean of .

MRKL Midrange Kibria Lukman parameter.

MRKLAM Midrange Kibria Lukman with Arithmetic Mean of .

MRKLMN Midrange Kibria Lukman with Minimum of .

MRKLMA Midrange Kibria Lukman with Maximum of .

MRKLMR Midrange Kibria Lukman with Midrange of .

MRKLMD Midrange Kibria Lukman with Median of .

MRKLGM Midrange Kibria Lukman with Geometric Mean of .

MRKLHM Midrange Kibria Lukman with Mean of .

MDKL Median Kibria Lukman parameter.

MDKLAM Median Kibria Lukman with Arithmetic Mean of .

MDKLMN Median Kibria Lukman with Minimum of .

MDKLMA Median Kibria Lukman with Maximum of .

MDKLMR Median Kibria Lukman with Midrange of .

MDKLMD Median Kibria Lukman with Median of .

MDKLGM Median Kibria Lukman with Geometric Mean of .

MDKLHM Median Kibria Lukman with Harmonic Mean of .

GMKL Geometric Mean Kibria Lukman parameter.

GMKLAM Geometric Mean Kibria Lukman with Arithmetic Mean of .

GMKLMN Geometric Mean Kibria Lukman with Minimum of .

GMKLMA Geometric Mean Kibria Lukman with Maximum of .

GMKLMR Geometric Mean Kibria Lukman with Midrange of .

GMKLMD Geometric Mean Kibria Lukman with Median of .

GMKLGM Geometric Mean Kibria Lukman with Geometric Mean of .

GMKLHM Geometric Mean Kibria Lukman with Harmonic Mean of .

HMKL Harmonic Mean Kibria Lukman parameter.

HMKLAM Harmonic Mean Kibria Lukman with Arithmetic Mean of .

HMKLMN Harmonic Mean Kibria Lukman with Minimum of .

HMKLMA Harmonic Mean Kibria Lukman with Maximum of .

HMKLMR Harmonic Mean Kibria Lukman with Midrange of .

HMKLMD Harmonic Mean Kibria Lukman with Median of .

HMKLGM Harmonic Mean Kibria Lukman with Geometric Mean of .

HMKLHM Harmonic Mean Kibria Lukman with Harmonic Mean of .

GMKL1 Generalized Modified Kibria Lukman 1 parameter.

MKL1AM Modified Kibria Lukman 1 with Arithmetic Mean of .

MKL1MN Modified Kibria Lukman 1 with Minimum of .

MKL1MA Modified Kibria Lukman 1 with Maximum of .

MKL1MR Modified Kibria Lukman 1 with Midrange of .

MKL1MD Modified Kibria Lukman 1 with Median of .

MKL1GM Modified Kibria Lukman 1 with Geometric Mean of .

MKL1HM Modified Kibria Lukman 1 with Harmonic Mean of .

AMMKL1 Arithmetic Mean of Modified Kibria Lukman 1.

AMMKL1AM Arithmetic Mean of Modified Kibria Lukman 1 with Arithmetic Mean of .

AMMKL1MN Arithmetic Mean of Modified Kibria Lukman 1 with Minimum of .

AMMKL1MA Arithmetic Mean of Modified Kibria Lukman 1 with Maximum of .

AMMKL1MR Arithmetic Mean of Modified Kibria Lukman 1 with Midrange of .

AMMKL1MD Arithmetic Mean of Modified Kibria Lukman 1 with Median of .

AMMKL1GM Arithmetic Mean of Modified Kibria Lukman 1 with Geometric Mean of .

AMMKL1HM Arithmetic Mean of Modified Kibria Lukman 1 with Harmonic Mean of .

MNMKL1 Minimum of Modified Kibria Lukman 1 parameter.

MNMKL1AM Minimum of Modified Kibria Lukman 1 parameter with Arithmetic Mean of .

MNMKL1MN Minimum of Modified Kibria Lukman 1 with Minimum of .

MNMKL1MA Minimum of Modified Kibria Lukman 1 with Maximum of .

MNMKL1MR Minimum of Modified Kibria Lukman 1 with Midrange of .

MNMKL1MD Minimum of Modified Kibria Lukman 1 with Median of .

MNMKL1GM Minimum of Modified Kibria Lukman 1 with Geometric Mean of .

MNMKL1HM Minimum of Modified Kibria Lukman 1 with Harmonic Mean of .

MAMKL1 Maximum of Modified Kibria Lukman 1 parameter.

MAMKL1AM Maximum of Modified Kibria Lukman 1 with Arithmetic Mean of .

MAMKL1MN Maximum of Modified Kibria Lukman 1 with Minimum of .

MAMKL1MA Maximum of Modified Kibria Lukman 1 with Maximum of .

MAMKL1MR Maximum of Modified Kibria Lukman 1 with Midrange of .

MAMKL1MD Maximum of Modified Kibria Lukman 1 with Median of .

MAMKL1GM Maximum of Modified Kibria Lukman 1 with Geometric Mean of .

MAMKL1HM Maximum of Modified Kibria Lukman 1 with Harmonic Mean of .

MRMKL1 Midrange of Modified Kibria Lukman 1 parameter.

MRMKL1AM Midrange of Modified Kibria Lukman 1 with Arithmetic Mean of .

MRMKL1MN Midrange of Modified Kibria Lukman 1 with Minimum of .

MRMKL1MA Midrange of Modified Kibria Lukman 1 with Maximum of .

MRMKL1MR Midrange of Modified Kibria Lukman 1 with Midrange of .

MRMKL1MD Midrange of Modified Kibria Lukman 1 with Median of .

MRMKL1GM Midrange of Modified Kibria Lukman 1 with Geometric Mean of .

MRMKL1HM Midrange of Modified Kibria Lukman 1 with Harmonic Mean of .

MDMKL1 Median of Modified Kibria Lukman 1 parameter.

MDMKL1AM Median of Modified Kibria Lukman 1 with Arithmetic Mean of .

MDMKL1MN Median of Modified Kibria Lukman 1 with Minimum of .

MDMKL1MA Median of Modified Kibria Lukman 1 with Maximum of .

MDMKL1MR Median of Modified Kibria Lukman 1 with Midrange of .

MDMKL1MD Median of Modified Kibria Lukman 1 with Median of .

MDMKL1GM Median of Modified Kibria Lukman 1 with Geometric Mean of .

MDMKL1HM Median of Modified Kibria Lukman 1 with Harmonic Mean of .

GMMKL1 Geometric Mean of Modified Kibria Lukman 1 parameter.

GMMKL1AM Geometric Mean of Modified Kibria Lukman 1 with Arithmetic Mean of .

GMMKL1MN Geometric Mean of Modified Kibria Lukman 1 with Minimum of .

GMMKL1MA Geometric Mean of Modified Kibria Lukman 1 with Maximum of .

GMMKL1MR Geometric Mean of Modified Kibria Lukman 1 with Midrange of .

GMMKL1MD Geometric Mean of Modified Kibria Lukman 1 with Median of .

GMMKL1GM Geometric Mean of Modified Kibria Lukman 1 with Geometric Mean of .

GMMKL1HM Geometric Mean of Modified Kibria Lukman 1 with Harmonic Mean of .

HMMKL1 Harmonic Mean of Modified Kibria Lukman 1 with Arithmetic Mean of .

HMMKL1AM Harmonic Mean of Modified Kibria Lukman 1 with Arithmetic Mean of .

HMMKL1MA Harmonic Mean of Modified Kibria Lukman 1 with Maximum of .

HMMKL1MN Harmonic Mean of Modified Kibria Lukman 1 with Minimum of .

HMMKL1MR Harmonic Mean of Modified Kibria Lukman 1 with Midrange of .

HMMKL1MD Harmonic Mean of Modified Kibria Lukman 1 with Median of .

HMMKL1GM Harmonic Mean of Modified Kibria Lukman 1 with Geometric Mean of .

HMMKL1HM Harmonic Mean of Modified Kibria Lukman 1 with Harmonic Mean of .

GMKL2 Generalized Modified Kibria Lukman 2 parameter.

MKL2AM Modified Kibria Lukman 2 with Arithmetic Mean of .

MKL2MN Modified Kibria Lukman 2 with Minimum of .

MKL2MA Modified Kibria Lukman 2 with Maximum of .

MKL2MR Modified Kibria Lukman 2 with Midrange of .

MKL2MD Modified Kibria Lukman 2 with Median of .

MKL2GM Modified Kibria Lukman 2 with Geometric Mean of .

MKL2HM Modified Kibria Lukman 2 with Harmonic Mean of .

AMMKL2 Arithmetic Mean of Modified Kibria Lukman 2.

AMMKL2AM Arithmetic Mean of Modified Kibria Lukman 2 with Arithmetic Mean of .

AMMKL2MN Arithmetic Mean of Modified Kibria Lukman 2 with Minimum of .

AMMKL2MA Arithmetic Mean of Modified Kibria Lukman 2 with Maximum of .

AMMKL2MR Arithmetic Mean of Modified Kibria Lukman 2 with Midrange of .

AMMKL2MD Arithmetic Mean of Modified Kibria Lukman 2 with Median of .

AMMKL2GM Arithmetic Mean of Modified Kibria Lukman 2 with Geometric Mean of .

AMMKL2HM Arithmetic Mean of Modified Kibria Lukman 2 with Harmonic Mean of .

MNMKL2 Minimum of Modified Kibria Lukman 2 parameter.

MNMKL2AM Minimum of Modified Kibria Lukman 2 parameter with Arithmetic Mean of .

MNMKL2MN Minimum of Modified Kibria Lukman 2 with Minimum of .

MNMKL2MA Minimum of Modified Kibria Lukman 2 with Maximum of .

MNMKL2MR Minimum of Modified Kibria Lukman 2 with Midrange of .

MNMKL2MD Minimum of Modified Kibria Lukman 2 with Median of .

MNMKL2GM Minimum of Modified Kibria Lukman 2 with Geometric Mean of .

MNMKL2HM Minimum of Modified Kibria Lukman 2 with Harmonic Mean of .

MAMKL2 Maximum of Modified Kibria Lukman 2 parameter.

MAMKL2AM Maximum of Modified Kibria Lukman 2 with Arithmetic Mean of .

MAMKL2MN Maximum of Modified Kibria Lukman 2 with Minimum of .

MAMKL2MA Maximum of Modified Kibria Lukman 2 with Maximum of .

MAMKL2MR Maximum of Modified Kibria Lukman 2 with Midrange of .

MAMKL2MD Maximum of Modified Kibria Lukman 2 with Median of .

MAMKL2GM Maximum of Modified Kibria Lukman 2 with Geometric Mean of .

MAMKL2HM Maximum of Modified Kibria Lukman 2 with Harmonic Mean of .

MRMKL2 Midrange of Modified Kibria Lukman 2 parameter.

MRMKL2AM Midrange of Modified Kibria Lukman 2 with Arithmetic Mean of .

MRMKL2MN Midrange of Modified Kibria Lukman 2 with Minimum of .

MRMKL2MA Midrange of Modified Kibria Lukman 2 with Maximum of .

MRMKL2MR Midrange of Modified Kibria Lukman 2 with Midrange of .

MRMKL2MD Midrange of Modified Kibria Lukman 2 with Median of .

MRMKL2GM Midrange of Modified Kibria Lukman 2 with Geometric Mean of .

MRMKL2HM Midrange of Modified Kibria Lukman 2 with Harmonic Mean of .

MDMKL2 Median of Modified Kibria Lukman 2 parameter.

MDMKL2AM Median of Modified Kibria Lukman 2 with Arithmetic Mean of .

MDMKL2MN Median of Modified Kibria Lukman 2 with Minimum of .

MDMKL2MA Median of Modified Kibria Lukman 2 with Maximum of .

MDMKL2MR Median of Modified Kibria Lukman 2 with Midrange of .

MDMKL2MD Median of Modified Kibria Lukman 2 with Median of .

MDMKL2GM Median of Modified Kibria Lukman 2 with Geometric Mean of .

MDMKL2HM Median of Modified Kibria Lukman 2 with Harmonic Mean of .

GMMKL2 Geometric Mean of Modified Kibria Lukman 2 parameter.

GMMKL2AM Geometric Mean of Modified Kibria Lukman 2 with Arithmetic Mean of .

GMMKL2MN Geometric Mean of Modified Kibria Lukman 2 with Minimum of .

GMMKL2MA Geometric Mean of Modified Kibria Lukman 2 with Maximum of .

GMMKL2MR Geometric Mean of Modified Kibria Lukman 2 with Midrange of .

GMMKL2MD Geometric Mean of Modified Kibria Lukman 2 with Median of .

GMMKL2GM Geometric Mean of Modified Kibria Lukman 2 with Geometric Mean of .

GMMKL2HM Geometric Mean of Modified Kibria Lukman 2 with Harmonic Mean of .

HMMKL2 Harmonic Mean of Modified Kibria Lukman 2 with Arithmetic Mean of .

HMMKL2AM Harmonic Mean of Modified Kibria Lukman 2 with Arithmetic Mean of .

HMMKL2MA Harmonic Mean of Modified Kibria Lukman 2 with Maximum of .

HMMKL2MN Harmonic Mean of Modified Kibria Lukman 2 with Minimum of .

HMMKL2MR Harmonic Mean of Modified Kibria Lukman 2 with Midrange of .

HMMKL2MD Harmonic Mean of Modified Kibria Lukman 2 with Median of .

HMMKL2GM Harmonic Mean of Modified Kibria Lukman 2 with Geometric Mean of .

HMMKL2HM Harmonic Mean of Modified Kibria Lukman 2 with Harmonic Mean of .



GENERAL INTRODUCTION

# Introduction

This chapter introduces the classical linear regression model and the generalized linear regression model and the assumptions that govern their use.

## Background to the Study

Regression analysis is a statistical method mostly adopted for estimating a dependent variable from one or more independent variables. Regression analysis is used mainly for forecasting (trend forecasting) and for drawing inferences about relationship among interrelated variables (Ayinde *et al.* 2012). Various forms of regression analysis such as simple linear regression, multiple linear regression and the non-linear regression exist. The most common types of regression models are the simple and multiple linear regression models. The non-linear regression models are used mostly in cases where there is no linear relationship between the dependent and the independent variables. Examples of non-linear models include the exponential models, sigmoidal functions and power models.

A linear regression model follows after the linear mathematical model and it is governed by some assumptions (section 1.1.3). The linear regression model has been in use for a long time. It is widely accepted by almost all the fields of study especially in science and technology. The Classical Linear Regression Model (CLRM) is generally expressed as:

(1.1)

where

=,

where y is a *n×1* response vector*, X* is a *n* x *p* matrix of independent variables,  *β* is a *p×1* vector of parameters and ε is a *n×1* vector of errors. The ordinary least square (OLS) estimator is commonly used for the estimation of *β*. It is given as:

(1.2)

where is the OLS estimator and is the transpose of the matrix *X.*

The CLRM is formulated under some basic assumptions and when the assumptions are satisfied, then the OLS estimator is best to estimate the parameters of the model. According to Gilmour *et al.* (1995), the OLS estimator is the best linear unbiased estimator (BLUE) for *β* when there is no violation of the assumptions in CLRM. However, in LRM the OLS estimator does not perform well and causes high instability when multicollinearity is present (Hoerl and Kennard, 1970; Liu, 1993; Liu, 2003). Multicollinearity according to Gilmour *et al.* (1995) is defined as near or strong linear relationship among the independent variables. The effect of multicollinearity includes unstable OLS estimates, wrong signs of the regression estimates, increase in the estimated variance, imprecise confidence interval and incorrect t-ratio (Kibria and Lukman, 2020; Lukman *et al*. 2020a). Several estimators have been suggested in the linear regression model to tackle the problem of multicollinearity. These include but not limited to the principal component regression estimator, ridge estimator, Liu estimator, two parameter estimators and KL estimator.

### The Simple Linear Regression Model

The simple linear regression model attempts to model the relationship between one dependent variable and another independent variable e.g age and years of education, length and breadth of a table, dose and response of a drug e.t.c. The simple linear regression model can be expressed in the form:

(1.3)

where *y* is the dependent variable, *x* is the independent variable, is the random error term, is the intercept and is the regression coefficient. These parameters are estimated using the observed values of x and y from which inferences can be made. Predictions of the dependent variable can also be made from independent variable.

### The Multiple Linear Regression Model

The dependent variable y is frequently influenced by multiple independent variables. Regression models are referred to as multiple linear regression models when they include more than one explanatory variable; for example, crop yield can be influenced by the amount of rainfall, quantity of nitrogen and potash in the soil. The multiple linear regression model is expressed in the form

(1.4)

The is the intercept and are the regression coefficients to be estimated. There is an assumption of linearity in the regression coefficients which makes the regression model linear. The regression model can be expressed in matrix form as shown in equation (1.1).

Regression models are employed for various purposes including: forecasting, parameter estimation, data description, variable selection and output control.

### Assumptions of the Linear Regression Model

The Classical Linear Regression model is formulated under some assumptions. These includes:

1. The regression model should be linear in parameters.
2. The residual is distributed with mean 0 and variance i.e and
3. The residual has a constant variance i.e Homoscedasticity.
4. The residuals must be uncorrelated i.e covariance of the residuals must be 0.
5. The explanatory variables should be independent i.e uncorrelated.
6. The explanatory variables and the residuals are uncorrelated.
7. The X values in repeated sampling are fixed. In repeated samples, the values used by the regressors X are regarded as fixed. Technically, X is thought to be non-stochastic.
8. Zero covariance between and i.e E( i.e non-stochastic explanatory variables and stochastic error terms are independent.
9. The number of expected estimated parameters *p* should be less than the sample size *n*.
10. The residuals are normally distributed.

### Non-Linear Regression Models

Non-linear models are models in which the response variables cannot be expressed as a linear function of the unknown parameters. It is sometimes referred to as a mechanistic model. The non-linear equation also shows that the prediction equation does not linearly depend on one or more unknown parameters. Examples of non-linear models include the exponential models, power models, Fourier models and Weibull growth (Smyth, 2002).

### Generalized Linear Models

A generalized linear model (GLM) is a variation of classical linear regression that takes into account response variables with error distribution models that do not conform to the normal distribution (Gill, 2000).

A GLM consists of three major components which includes:

1. A random component: This component specifies the conditional distribution of the outcome variable *y*i given the value of the independent variable in the model.
2. A linear predictor that consists of a linear function of the independent variables.
3. A link function that helps transform the expectation of the dependent variable.

The procedure generally adopted for fitting generalized linear models based on likelihood is called the Maximum Likelihood Estimator (MLE).

The GLM includes the logistic regression model, gamma regression model, Poisson regression model and Inverse Gaussian regression model among others (Nelder and Wedderburn, 1972; Algamal, 2018a; Algamal, 2018b; Shamany *et al*. 2019; Amin *et al*. 2020; Lukman *et al*. 2020b; Lukman *et al*. 2020c).s

#### The Logistic Regression Model

When describing the relationship between independent variable(s) and a dichotomous dependent variable, the binary logistic regression model is usually adopted. The binary logistic regression model which follows a Bernoulli distribution is given as:

(1.5)

where is the ith row of an matrix and is a vector of unknown regression coefficient. The parameters of the model are usually estimated using the method of maximum likelihood estimator (MLE). The log likelihood function is given as:

(1.6)

and equating the derivative to zero we have:

(1.7)

The iterative weighted least squares algorithm is used in solving equation (1.6) which results in the maximum likelihood for the logistic regression model as follows:

(1.8)

where and *z* is a vector where the *i*th element equals which is asymptotically unbiased estimate of *β* (Lukman *et al*., 2020b)*.*

#### The Poisson Regression Model

The Poisson Regression Model (PRM), which is typically used for count or frequency data modeling, is another special case of the GLM. When a response variable represents a rare event or count data, PRM is used to model the relationship between the response variable and one or more independent variables. The response variable also takes the shape of a non-negative variable, and it can be used in the social and physical sciences, including economics and health. It is also a common practice to estimate the regression coefficient in a PRM using the MLE.

Given that the response variable, is a count variable, it is assumed to follow a Poisson distribution such as P(*µi*) where *µi*= *exp* ()*,* such that *xi* is the *i*th row of *X* which is a *n*×*p* data matrix with *p* independent variables and is a *p*×*1* vector of coefficients (Hamad and Algamal 2022). The model's log likelihood is provided as:

(1.9)

The iterated weighted least squares (IWLS) algorithm is the most popular technique for maximizing the likelihood function and it is given as:

(1.10)

where and is a vector with *i*th element equals

#### The Gamma Regression Model

When simulating real-world data, the gamma regression model is preferred when the response variable is positively skewed and follows that distribution (Dunder *et al*., 2016; Algamal and Asar, 2018; Lukman *et al*. 2020c). The gamma regression model is widely accepted in fields such as the medical sciences, healthcare, economics, automobile insurance claims and so on. When the probability density function is given as:

(1.11)

then a distribution is said to follow a gamma distribution where k is a positive shape parameter, is the scale parameter such that:

;

and

; and

such that *n* is the sample size and *p* is the number of independent variables. The log-likelihood of the function in equation (1.10) is defined as:

(1.12)

And when solved using the iterative weighted least squares procedure, it gives the estimate of the regression parameter as:

(1.13)

where is the matrix and is a vector while the ith element (Lukman *et al*. 2020c).

### Assumptions of the Generalized Linear Model

Observations are taken from independent individuals.

1. Although the dependent variable may not have a normal distribution, it usually takes on one from the exponential family, such as binomial, Poisson, logistic, etc.
2. A GLM does assume a linear relationship between the transformed expected response in terms of the link function and the explanatory variables but does not assume a linear relationship between the response variable and the explanatory variables, for example, the logistic regression,  .
3. Independent variables can be nonlinear transformations of some original variables and should not be correlated.
4. Although they should not be normally distributed, errors must be independent.
5. Instead of using ordinary least squares (OLS), parameters are estimated using maximum likelihood estimation (MLE) (McCullagh and Nelder, 1989).

### Properties of the OLS and GLM

Some useful properties of are that it is an unbiased estimator

(1.14)

Gauss-Markov theorem guarantees that among all unbiased estimators of β, the Ordinary Least Squares (OLS) estimator has the minimum variance (Gujarati, 1995).

. (1.15)

A common estimator of  is the Mean Square Error defined as:

MSE = /(n-p) (1.16)

Under the premise that the error terms are independent and identically distributed normal variates with mean 0 and variance-covariance matrix , the least squares estimator of β is also the maximum likelihood estimator. The usual notation for this assumption is ~N() and if it holds, then the OLS estimator is also the Uniform Minimum Variance Unbiased Estimator (UMVUE). Model inferences such as confidence intervals and hypothesis tests are also powerful if the error terms are distributed normally and independently. From the statistical point of view, the OLS estimator is the optimal under a non-violation of any of the model’s assumptions.

### Violations of the Assumptions of the Linear Regression Models

The linear regression model leads to a number of issues when its assumptions are not met. It is therefore important to know the causes of these violations and the remedial measures to put in place.

#### Multicollinearity

This problem arises as a result of the violation of the assumption of independence between the explanatory variables. In the case of a simple regression model, the assumption is upheld. However, when there is more than one explanatory variable as in the case of a multiple regression model, it is possible that the explanatory variables are related. Multicollinearity is therefore defined as the correlation of the explanatory variable with one or more of the other explanatory variables in a multiple regression model and varies from low correlation to very high or perfect correlation. Multicollinearity poses as a serious threat as it reduces the significance of an explanatory variable that produces large standard error. As multicollinearity increases in severity, the determinant approaches zero and the inverse increases which in turn causes serious round off errors in the regression calculations. As a result, the regression coefficients may not be computationally accurate. More importantly, the regression coefficients tend to be imprecise because of large sampling variances.

The presence of multicollinearity can be mainly detected by the following:

**High with low t-Statistic Values**

Individual regression coefficients may be statistically insignificant, but the overall fit of the equation may be high.

**High Variance Inflation Factor (VIF)**

The VIF evaluates how much multicollinearity raises an estimated coefficient's variance. It describes the degree to which all of the other explanatory variables in the equation can account for a given explanatory variable. Values of VIF ranges between 1 and . VIF of 1 indicates no multicollinearity present. VIF values of greater than 1 show the independent variables are moderately correlated while VIF values between 5 and 10 Indicates that there is a high correlation between the independent variables. When VIF exceeds 10, then the correlation is severe and should be combated accordingly (Akinwande *et al*. 2015).

**High correlation coefficient**

There may be high pairwise correlations among the explanatory factors (in absolute value). The general rule is that severe multicollinearity may be present if the correlation is greater than 0.8.

When the OLS estimator and its standard errors are sensitive to minute changes in the data, multicollinearity can also be found. Additionally, confidence intervals sometimes have a tendency to be much wider, which makes it easier to accept the null hypothesis (Gujarati, 1995).

**NOTE:** This violation also exists in the GLMs.

#### Autocorrelation

Numerous parametric statistical techniques presuppose that the errors of the models used for the analysis are unrelated to one another (i.e. the errors are not correlated). The errors are referred to as autocorrelated or dependent when this assumption is not met. The assumption of independent errors present in many parametric statistical analyses may not be satisfied in time-series designs because they involve collecting data from a single participant at numerous points in time rather than from many participants at one point in time. When this happens, the results of these analyses and the conclusions reached as a result are probably going to be inaccurate unless some corrective measures are taken (Huitema and Laraway, 2015). Autocorrelation can occur due to the omission of important explanatory variables, incorrect functional form, manipulation of data, interpolation in the statistical observations among others (Gujarati, 1995).

#### Normality

The error term is assumed to follow a normal distribution. It is assumed to have zero mean and constant variance . This is necessary to conduct statistical tests of significance of the regression coefficients and to construct the confidence intervals. If this assumption is violated, the OLS regression estimates are still best and unbiased but the hypothesis testing (t test, F test) breaks down (Greene, 2003). The efficiency of hypothesis testing with the OLS estimates rapidly decrease with non-normal error term. One of the most common violations of a normal distribution for the error terms is the presence of one or more outliers in the sample

#### Heteroscedasticity

Heteroscedasticity can be simply defined as a condition where the variance of the error term in a regression model varies. This condition arises mainly as a result of outliers being present in a data set, incorrect specification of the model, incorrect transformation of data for regression analysis, skewness in the distribution of a regressor, mixed observation with different scales of measurement and some other sources. Due to the inconsistency in the covariance matrix of the estimated regression coefficients and the high variance of the predictions, the presence of heteroscedasticity can lead to estimators that are no longer efficient and the invalidation of hypothesis tests (like the t-test and F-test). The consequences of the problem of heteroscedasticity observed by Gujarati (1995) are summarized below:

1. The regression estimates are still unbiased

ii. The variances of the regression estimates do not have minimum variance.

iii. We are unable to perform tests of significance and create confidence intervals using the formulas for the variance of the coefficients.

iv. Predictions would have high variance.

### Parameter Estimation of Linear Regression Models

The CLRM in matrix form defined in (1.1) and the estimated regression model is , where is the vector of predicted response values and is the estimator of regression coefficient. OLS computes the parameter estimates of by minimizing the sum of the squared residuals. Therefore, the objective is to find those values of that lead to the minimum value of . Gujarati (1995) reported the fundamental derivation of OLS parameter estimates as follows:

(1.17)

Differentiating equation (1.18) with respect to and equate the result to zero in order to minimize S(β).

=0 (1.18)

Simplifying (1.19) gives the least squares equation as:

(1.19)

This can be solved since is of full rank to have the OLS estimator as:

(1.20)

### Parameter Estimation for the Generalized Linear Regression Models

Given a set of observations , its Maximum Likelihood Estimate (MLE) is a function such that

(1.21)

The function over the sample space of observations is called the MLE.

#### Properties of a Good Estimator

**Unbiasedness**

An estimator is said to be an unbiased estimator of β if E () =β. The difference between an estimator's expected value and true parameter is referred to as bias. Symbolically,

Bias ( E () – β (1.22)

**Minimum variance**

is said to be a minimum-variance estimator ofβif the variance of is smaller than the variance of where is another estimator of β. Symbolically, has minimum variance if

V(< V( (1.23)

**Efficiency**

If an estimator is unbiased and has the lowest variance when compared to all other unbiased estimators, it is said to be efficient. The relative efficiency (RE) of two estimators is the ratio of their variances,

RE= (1.24)

**Minimum Mean Squared Error**

If the mean squared error of an estimator , MSE (), is not higher than the mean squared error of any other estimator in the class, then that estimator is said to have a minimum mean squared error. The biasedness and variance properties are combined to form an estimator's mean squared error. Itis defined as:

MSE () =E

= E

= ,

+

(1.25)

**Best Linear Unbiased Estimator (BLUE)**

If an estimator is the best linear, unbiased estimator among all other linear, unbiased estimators of β, it is said to be BLUE (Ayinde and Lukman, 2014).

## Statement of the Problem

The most popular techniques for estimating the parameters of CLRMs and GLRMs, respectively, are the ordinary least squares estimator (OLS estimator) and the maximum likelihood estimator (MLE). These techniques are only effective when none of the assumptions are violated. One of the assumptions is that the explanatory variables should not be related to one another. Recent researches have shown that time series variable and economic variables grow together which results into linear relationship among the explanatory variables called multicollinearity (Ayinde *et al*. 2012; Ayinde *et al*. 2018; Lukman *et al*. 2020a). This problem poses serious threat to the efficiency of the OLS estimator and MLE. Different estimators have been developed in literature as alternatives to both estimators. However, this research seeks to develop some new estimators to efficiently handle multicollinearity challenge in CLRMs and GLRMs.

## Justification of the Study

Multicollinearity affects the efficiencies of the OLS estimator and the MLE. In literature, some alternative estimators have been developed to respectively replace the OLS estimator and the MLE in CLRM and GLRM and these include the ridge estimators. Different ridge estimators exist with their ridge parameter(s) and the commonest ones include the Hoerl ans Kennard ridge and Kibria-Lukman ridge estimators (Hoerl and Kennard, 1970; Kibria and Lukman, 2020). The performances of each of these ridge estimators depends largely on the choice of the shrinkage (ridge) parameter (Dorugade, 2014; Kibria and Shipra, 2016; Asar and Genç, 2017b; Owolabi *et al*. 2022). Therefore, there is a need to develop some new ridge estimators and their ridge parameters in different forms and types so as to identify the most efficient type of ridge estimator(s) for CLRMs and GLRMs.

## Aim and Objectives

The aim of this study is to develop some estimators to address multicollinearity problem in CLRMs and GLRMs more efficiently. The objectives are to:

1. propose some ridge estimators and their parameters by modifying the KL estimator;
2. compare the performances of the proposed ridge estimators and their ridge parameters with some existing ones;
3. identify the ridge parameters that are more efficient; and
4. apply the estimators to real life data sets.

## Scope of the study

This study suggests some new ridge estimators to deal with the multicollinearity issue in multiple linear regression, binary logistic regression and poison regression models. Both simulated and real-life data sets were used in the study.

## Research questions

1. Is it possible to propose some new ridge estimators by modifying the KL estimator?
2. How well do these proposed estimators perform when compared with some existing ones?
3. Which of these estimators would produce most efficient estimates of the model parameters?
4. How well do these estimators perform with real life data sets?

## Significance of the study

In both linear regression model and generalized linear models with the problem of multicollinearity, the generally acceptable method of parameter estimation is no longer efficient (Hoerl and Kennard, 1970; Dorugade, 2014; Kibria and Lukman, 2020). Therefore, this study proposes some new methods of parameter estimation that can handle the problem of multicollinearity.



LITERATURE REVIEW

# Introduction

In this chapter, we have the review of literature on multicollinearity and different estimators to mitigate the challenge. Different forms and types of existing ridge parameters were also reviewed.

## Multicollinearity Problems and its Detection

The linear relationships between the independent variables in multiple regression analysis are referred to as multicollinearity. It denotes relationship between two or more independent variables that exhibit linear combination with one another (Shrestha, 2020). As a result of multicollinearity, the standard errors and variances of the regression coefficient estimates increases which translate to lower t-statistics. Multicollinearity can also result in larger confidence intervals and less solid likelihood esteems for the predictors leading to unreliable results. Due to the effect of multicollinearity, some of the significant variables under investigation will also become statistically insignificant. Several methods of detecting the presence of multicollinearity have been suggested in literatures and the most common methods are discussed as follows:

### Correlation Coefficient and Pairwise Scattered Plots

While the correlation coefficient can also show a relationship between two independent variables, the scatter plot is a graphic representation that shows the linear relationship between pairs of independent variables. If the correlation coefficient value with the pairwise variables is greater than or equal to 0.8, then it indicates the possibility of multicollinearity.

### Variance Inflation Factor

The variance inflation factor (VIF) is a way to quantify how much multicollinearity has inflated the estimated regression coefficient's variance. The VIF is estimated as:

(2.1)

where are the coefficient of determination obtained by regressing *X*i on other independent variables and *i=1,…,p* (Daoud, 2017).

A VIF of < 1 implies that there is no linear relationship between the independent variables (no multicollinearity). VIF values between 1 and 5 indicates moderate level of multicollinearity. It becomes an issue of concern when VIF ranges between 5 and 10 (high multicollinearity). A VIF value that is greater than 10 (severe multicollinearity) shows that the regression coefficient are not dependable due to the implicating presence of multicollinearity (Shrestha, 2020).

### The method of eigenvalue

The variance of a linear combination of variables is indicated by the term eigenvalue. Very small eigenvalues (close to 0.05) indicate multicollinearity because the sum of eigenvalues must equal the number of independent variables. Even small changes in the data result in large changes in the regression coefficient estimates. Alternatively, the condition number or condition index can be adopted when the eigenvalues are small as they become difficult to interpret. Condition number (CN) is defined as:

CN= (2.2)

and condition index (CI) is defined as:

CI= = (2.3)

Severe multicollinearity results from CN values greater than 1000 and moderate to strong multicollinearity from CN values between 100 and 1000. Alternatively, there is a moderate to strong multicollinearity if the CI = is between 10 and 30, and there is severe multicollinearity if it is greater than 30 (Gujarati, 1995).

## Remedial Measures for Multicollinearity

Several measures have been proposed in literature as means to combat multicollinearity in regression analysis.

### Re-specification of the model

One of the common causes of multicollinearity is the selection of the model such as using two independent variables that are highly correlated in the regression equation. It is therefore important to re-specify the regression equation by redefining the independent variables so as to minimize the effect of multicollinearity. Another method for model re-specification is by eliminating one of the related explanatory variables. This, often times, leads to a specification bias or specification error especially when the variable is relevant thereby reducing the predictive power of the model.

### Incorporating new data

It is possible to have a new set of samples with the same variable but without the multicollinearity issue that existed in the first set of samples because multicollinearity is a sample feature. Since the standard errors are based on both the correlation between variables and the sample size, an increase in sample size can occasionally reduce the impact of multicollinearity. The Standard Error (SE) decreases as sample size increases. However, it is often difficult to obtain additional data, most especially when using secondary data. Even though this suggestion is often not practicable, the possibility should not be overlooked (Gujarati, 1995). The multicollinearity issue cannot be solved by collecting more data if the population or model constraints are the cause of the multicollinearity.

### Disaggregation of Data

Brown *et al*. (1973) stated that when the data are in aggregated form, returning to the individual observations will reduce the level of multicollinearity. Of course, the individual observations may not always be available. This is usually the case when the data are subject to confidentiality restrictions or are secondary data (e.g. time series, census data).

### Pooling data

Combining cross sectional and time-series data is a variation on the extraneous or a priori information technique and is referred to as pooling data. Although it is an appealing technique, combining cross-sectional and time series data in the manner just mentioned could lead to interpretation issues. Despite this, the method has a wide range of applications and should be taken into account when the cross-sectional estimates do not significantly differ from one cross section to another (Gujarati, 1995).

### Use of Biased Estimator

Poor estimates of the regression coefficients can be found when the least-squares method is used on nonorthogonal data. The requirement that be an unbiased estimator of is the issue with the least squares method. The least-squares estimator has the lowest variance in the group of unbiased linear estimators, according to the Gauss-Markov property. Dropping the demand that the estimator of be unbiased is one way to solve this issue. For the purpose of obtaining biased estimators of regression coefficients, various techniques have been developed.

#### Biased Estimator in Linear Regression Models

Numerous researches (Liu, 1993; Yang and Chang, 2010; Kibria and Lukman, 2020; Ahmad and Aslam, 2020; Dawoud, 2021) have been carried out in linear regression in order to examine the effect of multicollinearity on the estimated regression coefficient. Quite a number of estimators have been proposed in the linear regression model to combat the problem of multicollinearity. These are:

**The Principal Component Analysis**

The regression coefficient in a regression model is frequently estimated using the Principal Component Regression (PCR) model. The principal component analysis is typically used in PCR. The dependent variables are regressed on the independent variable's principal components rather than the dependent variable directly on the independent variable(s). The standard error is decreased and it is anticipated that the regression estimates will provide more accurate results by adding a certain amount of bias.

The principal component regression estimator of a linear regression model which uses r principal components and is defined as:

(2.4)

such that when *y* is regressed on the *r* principal components, is the predicted variable (Lukman *et al*. 2020d).

**The Stein Estimator**

Stein (1956) defined a linear function of the Ordinary Least Square (OLS) estimator as Stein estimator. It is given as:

(2.5)

where 0<k<1.

This can be expressed in the form of the generalized least square estimator as:

(2.6)

where *T*=*kI* and 0<k<1. P is the orthogonal matrix such that

The OLS coefficients are shrunk by a factor of k toward the origin using the Stein estimator. Each OLS component's shrinkage factor is the same, which leads to instability.

**The Ridge Estimator**

The ridge estimator was developed by Hoerl and Kennard (1970). The estimator was proposed by adding a small positive constant ‘k’ to the diagonals of the *X’X* matrix. The estimator, also known as the ridge estimator, provides a mean square error that is lower than that of the ordinary least square (OLS) estimator. The ridge estimator by Hoerl and Kennard (1970) is defined as:

(2.7)

where *I* is the identity matrix and *k* is a diagonal matrix with non-negative diagonal elements () such that,

**Ridge Estimator with Prior Information**

By re-examining the work of Hoerl and Kennard (1970), another ridge estimator based on prior information was proposed by Swindel (1976), where b is an arbitrary point in the parameter space that was chosen to reflect the prior information on *β*. The estimator outperformed the OLS estimator and is known as the modified ridge regression estimator as well as a good ridge estimator based on prior information. The ridge estimator is defined as:

, 0 ≤ k (2.8)

**Jackknifed Ridge Estimator**

Singh *et al*. (1986) proposed an almost unbiased ridge estimator as a method of jackknifing the ridge parameter which helps to further reduce the bias of an estimator when multicollinearity is involved. By jackknifing the ridge estimator, the resulting estimator was able to reduce the bias in the ridge estimator uniformly in all components. The jackknifed ridge estimator is defined as:

(2.9)

where , and

**Liu Estimator**

Liu (1993) came up with another biased estimator called the Liu estimator. The advantages of the ridge estimator and the contraction estimator are combined in this estimator. It is assumed that the Liu estimator can address the issue of multicollinearity more effectively than Ridge because it is a linear function of the parameter *d*, making the shrinkage parameter selection smaller than that of Ridge. The Liu estimator is defined as:

( + I) ( (2.10)

**Jackknifed Liu Estimator**

The almost-unbiased generalized Liu estimator, which was introduced by Akdeniz and Kaçiranlar (1995), was superior to both the generalized Liu estimator and the OLS estimator in comparison tests. The almost unbiased Liu estimator is defined as:

(2.11)

**The r-k estimator**

Other forms of estimators proposed have been combined with the ridge estimator such as the ‘r-k’ class estimator by (Baye and Parker, 1984). They combined the principal component technique with the ridge estimator. The estimator harnesses the gains of the principal component estimator and the ridge regression estimator and it was demonstrated that there are advantages from jointly utilizing the PCR and the Ridge estimators rather than individually.

**The r-d Estimator**

The r-d class estimator, which Kaçiranlar and Sakalliolu (2007) proposed, combined the Liu estimate and the PCR, which followed the same process as (Baye and Parker, 1984). Using the Mean Square Error (MSE) as a criterion, the estimator outperforms the Liu estimator, OLS estimator and PCR estimator. The k-d class estimator was proposed by Sakalliolu and Kaçiranlar (2008) and is a biased estimator. The OLS, the Ridge regression estimator, and the Liu estimator are special cases of this general estimator. The OLS, Ridge, and Liu estimators were found to be inferior to the estimator.

**The KL Estimator**

Kibria and Lukman (2020) proposed the KL estimator as an additional single parameter estimator. Theoretical comparison and simulation results demonstrated that, when using the MSE as a criterion, the estimator outperformed the Liu and the Ridge estimators in some situations. The KL estimator is defined below as:

(2.12)

**Combined Estimators**

Sarkar (1992) proposed the restricted ridge estimator which was later improved on by (GroGroß, 2003). He proposed another ridge-like estimator which was referred to as a restricted ridge estimator and the superiority of this estimator to the restricted least square estimator was demonstrated and confirmed.

Kaçiranlar *et al*. (1999) combined the ridge estimator by Hoerl and Kennard (1970) and the Liu estimator by Liu (1993) and proposed the restricted Liu estimator. The restricted Liu estimator was observed to perform better than the OLS and the Liu estimator using the MSE as a criterion. The estimator is defined as:

(2.13)

where and the *R* matrix is a of full row rank . The vector *r* is where *r* and *R* are known.

Liu (2003) developed the Liu-type estimator to combat multicollinearity in the linear regression model. The estimator is referred to as a new two-parameter estimator due to the inclusion of two parameters *k* and *d.* The Liu estimator was able to address the ill conditioning problem identified in the ridge estimator and also had a smaller MSE when compared with the ridge estimator. The Liu-type estimator is defined as:

(2.14)

where k>0 and

Another two-parameter was introduced by Özkale and Kaçiranlar (2007) which was referred to as the restricted and the unrestricted two-parameter estimator. The estimator was obtained by incorporating the contraction estimator to the work done by (Swindel, 1976). The two-parameter estimator combines the OLS, Ridge regression estimator, Liu estimator and the contraction estimator as unique cases. The superiority of the new two-parameter over the OLS estimator was established and comparison was made using the MSE.

Following after the work of Kaçiranlar *et al*. (1999), another two-parameter estimator was proposed by (Yang and Chang, 2010). Using the MSE as criterion, the new estimator was found superior to the OLS, Ridge, Liu estimator and the two-parameter estimator proposed by (Ozkale and Kaciranlar, 2007).

Chang and Yang (2012) proposed the two-parameter estimator in conjunction with PCR and named it principal component two-parameter (PCTP) estimator as an alternative to solving multicollinearity. The superiority of the estimator over OLS, Ridge, Liu, Two-parameter and the PCR estimators were established using the MSE as criterion. Wu and Yang (2013) introduced the almost unbiased two-parameter (AUTP) estimator which is a jackknifed form of the two-parameter estimator. The estimator makes use of the unbiased shrinkage estimators and therefore performs better than the OLS and the two-parameter estimator (restricted and unrestricted two-parameter) adopting the MSE as criterion.

By combining the ridge estimator with other forms of estimators, a ridge type estimator was also introduced by Dorugade (2014) as a means of combating the problem of multicollinearity. The estimator combined the ridge and the Liu parameters. The estimator is defined as:

(2.15)

where *k* and *d* are the biasing parameters and *I* is the identity matrix.

Inan (2015) proposed another two-parameter estimator and its efficiency was compared with other two-parameter estimators. It was found to be superior to the PCR, r-k class estimator and the Liu-type estimator using MSE as criterion.

The Modified Ridge Type (MRT) estimator by Lukman *et al*. (2019c) was also introduced as an alternative to the OLS in the presence of multicollinearity. The MRT estimator is defined as:

, *k* > 0 and 0 < *d* <1 (2.16)

In addition, the modified almost unbiased two-parameter estimator was also proposed by (Lukman *et al*., 2019a). The two-parameter estimator is defined as:

(2.17)

Dawoud and Kibria (2020) also developed the two-parameter estimator referred to as Dawoud-Kibria estimator and it is denoted as

(2.18)

where and

Another class of the biased two-parameter estimator was proposed by Ahmad and Aslam (2020) when regressors are correlated. The two-parameter estimator is defined as:

(2.19)

The unbiased modified ridge type (UMRT) estimator was also developed by Lukman *et al*. (2020a) as a means of estimating parameters in the presence of multicollinearity. The estimator is defined as:

(2.20)

Such that and

*J* is estimated by

Consequently, for

Lattef and Alheety (2021) introduced the modified unbiased two-parameter (MUTP) estimator which was developed as an extension of the UTPE earlier developed by Wu (2014). The MUTPE is defined as:

(2.21)

and *k* > 0, 0 < *d* < 1,

Dawoud (2021) proposed an improved estimator to reduce the effect of multicollinearity in linear regression models. The estimator is defined as:

(2.22)

#### Biased Estimators in Generalized Linear Models

Some of the proposed estimators have been extended to various forms of generalized linear models such as the logistic regression model, multinomial logistic regression model, the Poisson regression model, Gamma regression model, Beta regression model, Bell regression model, inverse Gaussian regression model, Probit model and so on.

**Ridge Estimator**

Schaeffer *et al*. (1984) proposed the logistic ridge regression estimator as an alternative to the Maximum Likelihood estimator (MLE) when the independent variables are correlated. The Poisson Ridge estimator for the Poisson regression model was introduced by Månsson and Shukur (2011) while Algamal (2018a) developed a ridge estimator for the gamma regression model. Algamal (2018b) introduced the Ridge estimator into the inverse gaussian regression model and a new ridge type estimator was introduced by Shamany *et al*. (2019).

**Modified Ridge type Estimator**

The Modified Ridge Type (MRT) was introduced to the logistic regression model by (Lukman *et al.*, 2020b). The estimator was compared with the MLE, Ridge and the Liu estimators in the logistic regression model and was found to perform the best through a simulation study and real life application. The MRT was introduced to the gamma regression model by (Lukman *et al*., 2020c).

**Liu Estimator**

The Liu estimator was introduced into the logistic regression model by Månsson *et al*. (2012a) while the improve Liu estimator was introduced to the Poisson regression model by (Mansson *et al*., 2012b). Qasim *et al*. (2019) proposed a new Poisson Liu regression estimator for the Poisson regression model.

**Combined Estimators**

Inan and Erdogan (2013) introduced the Liu-type estimator into the logistic regression model. Asar and Genç (2017b) introduced the two parameter ridge estimator to the binary logistic regression model and was also observed to perform well. A new two parameter estimator for the Poisson regression was also introduced by (Asar and Genç, 2017a). Wu and Asar (2017) proposed another estimator that combined the PCR and the Liu-type estimator in logistic regression model. A new stochastic restricted Liu estimator for the logistic regression model was proposed by Zuo and Li (2018) while the Liu-type estimator for the Gamma regression model was introduced by Algamal and Asar (2018). A new two parameter estimator was introduced to the Inverse Gaussian Regression Model (IGRM) by (Shamany *et al*., 2019). A new Liu-type estimator for IGRM was introduced by (Akram *et al*., 2020). and a modified ridge type logistic estimator was introduced by (Lukman *et al*. 2020b). The modified ridge type estimator for the gamma regression model was introduced by Lukman *et al*. (2020c) while a new Liu-type for the inverse Gaussian model was introduced by (Akram *et al*., 2020). A new adjusted Liu estimator for the Poisson regression model was proposed by (Amin *et al.,* 2021).

**Jackknifed Estimator**

Wu and Asar (2016) introduced the almost unbiased ridge estimator to the logistic regression model and it outperformed the Ridge, Liu and the MLE estimators. The modified jackknifed estimator for the Poisson regression model was introduced by (Türkan and Özel, 2016). Amin *et al*. (2020) developed the almost unbiased ridge estimator for the gamma regression model.

**KL Estimator**

Recently, Poisson KL estimator was developed by Lukman *et al*. (2021a) for combating multicollinearity in the PRM. Lukman *et al*. (2021b) proposed the KL estimator for the Inverse Gaussian regression model while the jackknifed KL estimator for the Poisson regression model was proposed by (Hamad and Algamal, 2022).

### Estimation of the Ridge parameter k

Different methods of estimating the k parameter have been proposed in different studies and a number of them are being considered.

Hoerl and Kennard (1970) proposed . They suggested estimating ridge parameter by taking the maximum of such that the estimator of k is:

(2.23)

where *i=1,…,p.*

Hoerl *et al.* (1975) proposed a different estimator of *k* by taking the Harmonic Mean of the ridge parameter . This estimator is given as:

(2.24)

where *p* is the number of independent variables and *i=1,…,p*.

Kibria (2003) proposed some new estimators of k by taking the Geometric Mean, Arithmetic Mean and Median (p ≥ 3) of the ridge parameter . These estimators are respectively defined as:

(2.25) (2.26)

Median (2.27)

Lawless and Wang (1976) proposed a different estimator of k resulting from taking the Harmonic Mean of the ridge parameter . The estimator is defined as:

(2.28)

where is the eigenvalue of the matrix .

Alkhamisi *et al*. (2006) proposed another ridge parameter . They proposed estimators of k as the Arithmetic Mean and Median of the ridge parameter . These estimators are respectively defined as:

(2.29)

(2.30)

Other forms of the generalized ridge parameter have been proposed by other researchers such as Nomura (1988) who proposed a new ridge parameter defined as

. (2.31)

Troskie and Chalton (1996) also proposed another ridge parameter which is defined as:

(2.32)

Firinguetti (1999) proposed another ridge parameter which is defined as:

(2.33)

Batach *et al*., (2008) proposed the ridge parameter defined as:

(2.34)

Dorugade (2016) proposed a ridge parameter defined as:

(2.35)

Lukman and Ayinde (2017) proposed another ridge parameter which is in line with Lawless and Wang (1976) and it is defined as:

(2.36)

Fayose and Ayinde (2019) examined various forms of these parameters which includes the median (MD), arithmetic mean (AM), midrange (MR), maximum (MA), minimum (MN), geometric mean (GM) and the harmonic mean (HM) of the eigen values of the matrix. The varying forms were found to perform better than their original forms.

## Monte-Carlo Simulation

Monte-Carlo simulation technique is a mathematical procedure that is used to enhance decision making by professionals in various fields of study such as finance, science, economics, engineering, insurance. The procedure helps to make informed choices about a wide range of possible outcome which can be used to model future outcomes. This computation algorithm helps to obtain quantitative results based on repeated random sampling where the entire system is simulated a large number of times (usually from 1000) with each simulation being equally likely.

A statistical method used to simulate data can be defined as a Monte Carlo method. A simulation is a technique that uses random number sequences as its input. The Monte Carlo method uses computerized statistical sampling experiments to approximate solutions to a range of mathematical issues. The approach can be used to solve both issues with built-in probabilistic structure and issues without any probabilistic content. In order to evaluate statistical estimators for structural equation models, Monte Carlo simulations are now frequently used. However, finite sample properties of estimators in structural equation models are frequently outside the scope of the established asymptotic theory, despite the fact that analytical statistical theory can address some research questions. Other times, even asymptotically, the distributions are unknown. When this occurs, Monte Carlo simulations offer a great way to assess estimators and goodness-of-fit statistics in a variety of circumstances, such as sample size, non-normality, dichotomous or ordinal variables, model complexity, and model misspecification.

Using simulated random numbers, the Monte Carlo method allows for the investigation of the characteristics of random variable distributions (Gentle, 1985). Most estimators' asymptotic properties are typically known, but it's possible to know their finite sampling properties as well. By establishing controlled conditions from which sampling distributions of parameter estimates are generated, Monte Carlo simulations enable researchers to evaluate the finite sampling performance of estimators. The key to assessing a statistic's behavior is understanding the sampling distribution (Dorugade, 2014). For instance, a researcher can learn a statistic's bias, effectiveness, and other desirable characteristics from the sampling distribution. Even when a researcher artificially creates a sampling distribution using the Monte Carlo method, sampling distributions are theoretical and unobserved. The analyst then draws repeated samples of size n from that population and estimates the parameters of interest for each sample. The researcher starts by developing a model with known population parameters (i.e., the values are set by the researcher). The parameter estimates from each sample are then combined to estimate a sampling distribution for each population parameter (Wichern and Churchill, 1978). This estimated sampling distribution provides the sampling distribution's characteristics, such as mean and variance. These nine steps are crucial for organizing and carrying out a Monte Carlo analysis.:

i. developing a theoretically derived research question

ii. creating a valid model

iii. designing specific experimental conditions

iv. choosing values of population parameters

v. choosing an appropriate software package

vi. executing the simulations

vii. file storage

viii. troubleshooting and verification

ix. summarizing results.



# METHODOLOGY

# Introduction

In this chapter, properties of the ridge and KL estimators were shown and the newly modified KL estimators were developed for the CLRM and the GLMs. Existing ridge parameters were also expressed and new ones developed.

## The Ridge, KL and the Proposed Estimator in Multiple Linear Regression Model

The properties of the Ridge, KL and the MKL estimators are as shown:

### The Ridge Estimator

Hoerl and Kenard (1970) proposed the ridge estimator which is defined as:

(3.1)

where k is the ridge parameter. The following properties are also obtained from the ridge estimator:

**MEAN:**

where

= ` (3.2)

**BIAS:**

= (3.3)

**VARIANCE:**

=

**=** (3.4)

**MEAN SQUARED ERROR**:

The mean square error (MSE) becomes:

= (3.5)

Moreover, since is a positive definite matrix, the regression model can be re-written in canonical form such that:

(3.6)

where and , and *P* is an orthogonal matrix whose columns consists of the eigenvectors of such that and are the eigenvalues of. The MSE of ridge estimator in canonical form becomes:

(3.7)

where is the ith element of the vector.

For this transformed model, MSE of the OLS estimator is the trace of the variance covariance matrix, therefore the MSE for the OLS is obtained as:

(3.8)

### The KL Estimator

A one parameter estimator proposed by Kibria and Lukman (2020) is defined as:

, (3.9)

The properties of the KL estimator become:

**MEAN**

(3.10)

**BIAS:**

(3.11)

(3.12)

**MEAN SQUARED ERROR:**

The MSE therefore becomes:

(3.13)

Similarly, the MSE of the estimator in canonical form becomes:

(3.14)

### The Modified KL Estimator (proposed estimator)

In this study, a new one parameter ridge estimator is obtained by modifying the KL estimator following the report from authors including ( Liu, 1993; Kaçiranlar *et al*., 1999; Chang and Yang, 2012; Ahmad and Aslam, 2020).

From the KL estimator of (3.9), is replaced with the to obtain the proposed estimator as:

(3.15)

where k is the shrinkage parameter.

The properties of the proposed estimator are as follows:

**MEAN:**

(3.16)

**BIAS:**

(3.17)

**VARIANCE:**

The variance of the proposed estimator is expressed as follows:

(3.18)

**MEAN SQUARED ERROR:**

(3.19)

Consequently, the Mean square error in canonical form is:

(3.20)

## The Ridge, KL and MKL Estimators in Poisson Regression Model

The Poisson regression method is typically used as the standard statistical technique for analyzing regression models with count data as the dependent variable. Poisson regression model is sometimes referred to as a log-linear model which is a linear combination of the model’s parameters enabling linear regression to be implemented. The distribution is assumed to follow a Poisson distribution as P(*µi*) where *µi*= *exp*(*xiβ*)*,* *xi* is the *i*th row of *X* which is a *n*×*p* data matrix with *p* independent variables and *β* is a *p*×*1* vector of coefficients. The log likelihood of the model is obtained as:

(3.21)

The most common method of parameter estimation for a Poisson regression model is the method of maximum likelihood. By solving the equation (3.21) the parameters are estimated as follows:

(3.22)

Using the iterated weighted least squares (IWLS) algorithm in solving the above equation, the maximum likelihood estimator is obtained as:

(3.23)

where and is a vector while the *i*th element equals

The MLE is normally distributed with a covariance matrix that is equivalent to the inverse of the second derivative as:

(3.24)

and the mean square error is given as:

(3.25)

Where is the ith eigenvalue of the matrix. With multicollinearity problem in the explanatory variables, high instability in the variance of the MLE is inevitable. This makes the interpretation of the estimated parameters cumbersome due to the size of the vector of the estimated coefficients.

### Poisson Ridge Estimator

The Poisson ridge estimator proposed by Månsson and Shukur (2011) to combat multicollinearity for the Poisson regression model is defined as:

(3.26)

with the model in canonical form expressed as:

(3.27)

where and ;

given that and , *P* is an orthogonal matrix whose columns consists of the eigenvectors o such that and are the eigenvalues of.

is the ith component of *P* is the matrix which the columns are the eigenvectors of .

The Mean squared error of the Poisson ridge regression is defined as:

(3.28)

### Poisson KL Estimator

Lukman *et al*., (2021a) proposed the Poisson KL estimator and it defined as:

(3.29)

Similarly, the Poisson KL estimator in canonical form is expressed as:

(2.30)

The mean squared error for the Poisson KL estimator is defined as:

(3.31)

### Poisson Modified KL Estimator (Proposed Estimator)

From the MKL estimator proposed above for the linear regression model and by extension to the Poisson regression model, the Poisson MKL estimator is defined as:

(3.32)

The Poisson modified KL estimator in canonical form is expressed as:

The Mean Squared Error of the MKL estimators is defined as:

(3.33)

## The Ridge, KL and the Modified KL Estimators in Logistic Regression Model

The outcome variable in a logistic regression model follows a Bernoulli distribution y~Be such that:

(3.34)

where *x*i is the ith row of an *n*×*p* matrix of *X*, *β* is a p×1 vector of unknown regression coefficients.

The MLE of *β* can be obtained using the iterative weighted least squares algorithm for the differentiated function and the estimate obtained as:

(3.35)

where and z is a vector where the *i*th element equals which is asymptotically unbiased estimate of *β.*

### Logistic Ridge Estimator

The logistic ridge regression estimator (LRR) by Schaefer *et al*. (1984) is defined as:

(3.36)

The estimator is expressed in canonical form as:

where the MSE expressed as:

### Logistic KL Estimator

The KL estimator introduced into the Logistic regression model is defined as:

(3.37)

Consequently, the logistic KL estimator can be expressed in canonical form as:

(3.38)

and the mean squared error expressed as:

### Logistic Modified KL Estimator (Proposed Estimator)

The Modified KL (MKL) estimator defined earlier is introduced as the logistic version of the modified KL estimator. The Logistic Modified KL estimator is therefore defined as:

(3.39)

The estimator is expressed in canonical form as:

(3.40)

The mean squared error is expressed as

## Theoretical Comparison

The new estimator (MKL) is theoretically compared with the ridge estimator and the KL estimator in the LRM. The comparison is extended to the Poisson and the logistic regression models. Given that is a positive definite matrix and can be expressed as shown in equation (3.6) where and , and *P* is an orthogonal matrix whose columns consists of the eigenvectors of such that and are the eigenvalues of, the Ridge estimator of in the canonical regression model is given as:

(3.41)

The KL estimator of is given as:

(3.42)

The MKL estimator of is given as:

(3.43)

where *k* is the shrinkage parameter.

The following lemmas are considered when comparing estimators.

**Lemma 3.1:** Let M be an n×n positive definite matrix, that is *M* >0, and be some vector, then if and only if Farebrother (1976).

**Lemma 3.2:** Let be two linear estimators of . Suppose that where *i*=1,2 denotes the covariance matrix of and. i=1,2.Consequently, if and only if where (Trenkler and Toutenburg, 1990).

### Comparison in Linear Regression Model

The proposed estimator is theoretically compared to other one parameter ridge estimator in order to determine superiority.

#### Comparison Between and .

**Theorem 3.4.1.1**

If *k>0*, then the estimator is superior to the estimator using the MSE matrix criterion if and only if .

**Proof**

We observed that is positive definite such that . Lemma 3.2 completed.

#### Comparison Between and .

**Theorem 3.4.1.2**

If *k > 0*, then the estimator is superior to the estimator using the MSE matrix criterion if and only if .

**Proof**

We observed that is non-negative such that . Therefore, Lemma 3.2 completed.

#### Comparison Between and .

**Theorem 3.4.1.3**

If *k > 0*, then the estimator is superior to the estimator using the MSE matrix criterion if and only if: .

**Proof**

Simplifying further, we have such that for *k >*0. Lemma completed

### Comparison in Poisson Regression Model

Theoretical comparisons are carried out between the estimators for the Poisson regression model.

#### Comparison Between and

**Theorem 3.4.2.1:** is preferred to iff, provided *k*>0.

**Proof**

It is observed that such that the expression above is non-negative for k>0

#### Comparison Between and

**Theorem 3.4.2.2:** is preferred to iff, provided *k*>0.

**Proof**

We can observe that the difference of the variance of the estimator is non-negative since for *k>*0.

#### Comparison Between and

**Theorem 3.4.2.3:**  is preferred to iff, provided *k*>0.

**Proof**

The difference of the MSE is non-negative since for k > 0. Lemma completed.

### Comparison in Logistic Regression Model

Theoretical comparisons are carried out between the estimators for the logistic regression model.

#### Comparison Between and

**Theorem 3.4.3.1:**  is preferred to iff, provided the shrinkage parameter *k*>0.

**Proof**

It is observed that is positive definite such that . Lemma completed.

#### Comparison Between and .

**Theorem 3.4.3.2:**  is preferred to iff, provided the shrinkage parameter *k*>0.

**Proof**

We observed that since is non-negative such that . Lemma completed.

#### Comparison Between and

**Theorem 3.4.3.3:**  is preferred to iff, provided the shrinkage parameter *k*>0.

**Proof**

The difference of the variance is non-negative since such that for *k >*0. Proof completed.

## Selection of the Shrinkage Parameter k

How well an estimator perform is dependent on the selection of its shrinkage parameter (Lukman and Ayinde, 2017). Various forms and types of shrinkage parameters that exists such as the arithmetic mean, minimum, maximum, midrange, median, geometric mean and harmonic mean were considered. New forms and types of shrinkage parameters for the KL and MKL estimators based on the different available methods are then developed.

### Ridge Parameters Based on Hoerl and Kennard

Hoerl and Kennard (1970) proposed a biasing parameter which is of the form:

(3.44)

and also suggested estimating ridge parameter by taking the maximum of such that the estimator of k is consequently defined as:

(3.45)

where is the ith element of the vector , *i=1,2,…,p* ; *p* is the number of regressors and

(3.46)

Hoerl *et al.* (1975) proposed a different estimator of *k* by taking the Harmonic Mean of the ridge parameter . This estimator is given as:

Kibria (2003) proposed new estimators of *k* by taking the geometric mean, arithmetic mean and median of the ridge parameter . These estimators are respectively defined as:

(3.47) (3.48)

### Ridge Parameters Based on Alkhamisi *et al.* (2006)

Alkhamisi *et al*. (2006) proposed another ridge parameter . They proposed estimators of *k* as the Arithmetic Mean and Median of the ridge parameter . These estimators are respectively defined as:

(3.49)

(3.50)

### Ridge Parameters Based on Muniz *et al.* (2012)

Muniz *et al.* (2012) proposed the estimator of the ridge parameter K as the Varying Maximum and Arithmetic Mean of the ridge parameter . These estimators are respectively defined as:

(3.51)

(3.52)

### Other Forms of Shrinkage Parameters

Khalaf and Shukur (2005) suggested a new method of selecting the parameter *k*. This can be seen in the form of Fixed Maximum of the ridge parameter . The estimator is defined as:

(3.53)

Muniz and Kibria (2009) proposed the estimator of the ridge parameter *k* as the Geometric Mean of the ridge parameter . The estimator is given as:

(3.54)

Fayose and Ayinde (2019) proposed ridge parameter defined as:

(3.55)

Three different versions of the ridge parameter also proposed were examined in this study. These are defined as follow:

(3.56)

(3.57)

(3.58)

### Proposed Ridge Parameters

Lukman and Ayinde (2017) introduced the classification of existing generalized ridge parameters using the ideas of forms and types. By adopting the same idea of Lukman and Ayinde (2017), different forms and types of the ridge parameters were developed.

#### Proposed Ridge Parameters Based on the KL Estimator

Kibria and Lukman (2020) proposed a biasing parameter of the generalized form as:

(3.59)

Other proposed forms of the KL parameter are shown from equation (3.60) to equation (3.66) while the different forms and types of the KL ridge parameter are then further summarized in Table 3.1.

(3.60)

where

(3.61)

where

(3.62)

where

(3.63)

where

(3.64)

where

(3.65)

where

(3.66)

where

Table 3.1: Different forms and types of KL Ridge Parameter.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| FORMS | TYPES | | | | | | |
| AR | MN | MA | MR | MD | GM | HM |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

### Ridge Parameter Based on Modified KL Estimator

Following previous studies such as Kibria (2003), Kibria and Lukman (2020), Ahmad and Aslam (2020), the mean squared error of the Modified Kibria Lukman estimator is differentiated with respect to k (the shrinkage parameter) and the expression equated to zero.

Differentiate eqn. (3.15) with respect to *k*,

(3.67)

(3.68)

Equating (3.68) to zero and solving for *k*, the resulting *k* values are:

(3.69)

(3.70)

and

(3.71)

The performance of this estimator depends largely on the use of the biasing parameter *k.* Thus, the and the were used as the biasing parameter for the modified Kibria-Lukman estimator given that and such that . The is not used because i.e .

Different forms of the ridge parameter proposed in this study include:

(3.72)

where

(3.73)

where

(3.74)

(3.75)

where

(3.76)

where

(3.77)

where

(3.78)

where

Different types of the ridge parameter is not considered due to the peculiarity of the parameter.

Different forms of the ridge parameter proposed in this study include:

(3.79)

(3.80)

where

(3.81)

where

(3.82)

where

(3.83)

where

(3.84)

where

(3.85)

where

(3.86)

where

Table 3.2: Proposed forms and types of the MKL2 Ridge Parameter

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| FORMS | TYPES | | | | | | |
| AR | MN | MA | MR | MD | GM | HM |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |

## Simulation Study

In this section, simulation study was used to provide information on the performance of the estimators by using the R Studio programming language. Simulations were carried out for the CLRM and the GLMs.

### Procedure for Generating the Explanatory Variable

The simulation procedure used by Kibria (2003) was adopted to generate the explanatory variables in this study and is given as:

(3.87)

Where are independent standard normal pseudo-random numbers and is the correlation between the variables. Different levels of correlation were considered for the simulation in this study. The values of were taken to be 0.8, 0.9, 0.95, 0.99 and 0.999 respectively (Liu, 2003; Ozkale and Karciranlar, 2007; Kibria and Lukman, 2020; Ahmad and Aslam, 2020).

### Procedure for Generating the Dependent Variable

The response variable was determined using

(3.88)

where is assumed to be normally distributed with mean of 0 and variance such that p is the number of independent variables, p in this study was taken to be 4 and 8.The parameter values were chosen such that =1. This is a common restriction in simulation studies of this type (Muniz and Kibria, 2009). Simulation studies was repeated 5000 times for the following sample sizes: n = 10, 20, 30, 50, 100 and 200 for the CLRM. Sample sizes of n=50, 75, 100 and 200 were used for the GLMs. Error variance = 1, 9, 25, 49 and 100 were also used. For the CLRM, at a specified value of n, p, σ and , the fixed X’s were first generated; followed by the , and the values of Y were then obtained using the regression model. For the logistic regression model, the n dependent variables were generated using the Bernoulli distribution where and xi is the ith row of the data matrix X. The dependent variables for the Poisson regression model were done using pseudo-random numbers from Po(µi) where and Xi is the ith row of the design matrix with being the coefficient vector. This process was replicated 5000 times for each case.

### Criterion for Investigating the Performance of the Ridge Parameters

Several authors have used the Mean Square Error (MSE) to compare the performance of ridge regression estimator with the Ordinary Least Square estimator and other proposed estimators when there is multicollinearity (Hoerl and Kennard, 1970; Lawless and Wang, 1976; Kibria, 2003; Kibria and Lukman, 2020; Ahmad and Aslam, 2020). To investigate the performance of the ridge estimators, the MSE is calculated and generated using different ridge parameters as follows:

(3.89)

where would be any of the estimators. The estimator with the smallest MSE was considered the best.

## Real Life Data Application

For the purpose of real-life analysis some of the datasets that have been adopted in other studies of this type were employed for the CLRM and GLMs.

### Data for Linear Regression Model

The famous Portland cement data initially applied by Woods *et al*. (1932) have been adopted by other researchers such as Li and Yang (2012), Kibria and Lukman (2020), Aslam and Ahmad (2020) was used in this study. The data set has its outcome and independent variables listed as follow:

*Y*= Heat evolved after 180 days of curing/calorie per gram of cement

*X*1 = Tricalcium Aluminate

*X*2 = Tricalcium Silicate

*X*3 = Tetracalcium Aluminoferrite

*X*4 = *β*-Dicalcium Silicate

The regression model for the data is written as:

(3.90)

The variance inflation factors for the data set were VIF1=38.50, VIF2=254.42, VIF3=46.87 and VIF4=282.51. The eigenvalues of the X’X matrix were , , and and while the condition number was also obtained as 424. The sample size n is 13. The three parameters for determining the presence of multicollinearity all indicated the presence of multicollinearity.

### Real-Life Application for Logistic Regression Model

The data set adopted for the logistic regression model was initially adopted by Pena *et al*. (2011) and subsequently by (Ozkale, 2016; Lukman *et al*. 2020b). The growth limit of *Alicyclobacillus acidoterrestris* CRA7152 in apple juice was modelled. The logistic regression model was used to investigate the effect of pH, concentration of Nisin (Ni), temperature (T), soluble solids concentration (Brix) on the growth probability of *Alicyclobacillus acidoterrestris* CRA7152 in apple juice (*g*). The model is expressed as:

(3.91)

The data set was studied by Ozkale (2016) and the following eigenvalues of the   
matrix at the final iteration were obtained as = 4.2143, = 0.1774,  
 = 0.1145, = 0.0718 and = 0.0303. The condition number was then obtained as 138.8583 which indicates the presence of multicollinearity.

### Real-Life Application for Poisson Regression Model

Myers et al. (2012) were the first researchers to use the Poisson regression model on an aircraft damage dataset, and other researchers have since done the same (Asar and Genc, 2017a; Amin et al., 2020). The McDonnell Douglas A-4 Skyhawk and the A-6 Grumman Itruder are two distinct aircraft that are covered in some detail by the dataset. The number of damaged areas on the aircraft is the dependent variable, and it has a Poisson distribution (Asar and Genc, 2017a; Amin et al., 2020). Three explanatory variables are included in the data set: X1 (aircraft type) determines the binary nature of the outcome (A-4 is coded as 0 and A-6 is coded as 1), X2 (bomb load in tons), and X3 (number of months of aircrew experience). Multicollinearity has a significant impact on the data set, as Myers et al. (2012) were able to determine. The eigenvalues used to interpret the data were 4.333, 374.8961, and 2085.2251, while the condition number is 219.365.



# RESULTS AND DISCUSSION

# Introduction

In this chapter, we have the results and discussion from simulation and real life data application.

## Summary of Results of the Estimators with Linear Regression Model

The summary of the three ridge parameters KL, MKL1 and MKL2 for the linear regression model is provided as follows:

### KL Result with Linear Regression Model

The frequency (number of times) of each ridge parameter for the KL estimator in linear regression model whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity and error variance is presented in Table 4.1. See also appendix 1-30 for the MSE of the different KL ridge parameters. The performance of the KL ridge parameter shows that when the number of explanatory variables is 4, the MRKLHM had the highest frequency of 142 and the performance was consistent as the sample size increased. Its performance was closely marked by MAKLHM with frequency of 137. Other forms of the ridge parameter that performed well and had frequency of 132,125 and 120 include the MAKLMN, AMKLHM and MRKLMD respectively. With an increase in the number of explanatory variables, the MRKLHM also had the highest frequency of 117 while the MRKLMD and MAKLMN also performed well with a frequency of 110 and 100 respectively. In general, for the KL ridge parameter in linear regression, the MRKLHM had the best performance at all sample size and number of explanatory variables. Figure 4.1 illustrates the 5 best KL ridge parameter at different levels of explanatory variables and sample sizes. The Figure shows the consistency of the MRKLHM ridge parameter which was selected as best performing ridge parameter for the KL estimator.

Table 4.1:Frequency of KL Parameters that ranked between one and ten across all Sample Sizes for the KL Ridge Estimator in Linear Regression Model

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | Estimators | n | | | | | | Total | Rank |
| 10 | 20 | 30 | 50 | 100 | 200 |
| 4 | GKL | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 31 |
| GKLAM | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 34 |
| GKLMN | 0 | 0 | 0 | 0 | 2 | 2 | 4 | 28 |
| GKLMA | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 35 |
| GKLMR | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 35 |
| GKLMD | 0 | 0 | 0 | 0 | 2 | 2 | 4 | 28 |
| GKLGM | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 31 |
| GKLHM | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 31 |
| AMKL | 3 | 4 | 6 | 6 | 6 | 9 | 34 | 15 |
| AMKLAM | 0 | 0 | 0 | 3 | 6 | 3 | 12 | 20 |
| AMKLMN | 0 | 0 | 0 | 2 | 1 | 3 | 6 | 26 |
| AMKLMR | 0 | 0 | 0 | 1 | 2 | 1 | 4 | 28 |
| AMKLMD | 25 | 25 | 25 | 17 | 6 | 7 | 105 | 6 |
| AMKLGM | 7 | 13 | 12 | 17 | 18 | 18 | 85 | 7 |
| **AMKLHM** | **25** | **23** | **25** | **22** | **16** | **14** | **125** | **4** |
| MAKL | 0 | 0 | 0 | 1 | 6 | 8 | 15 | 19 |
| **MAKLMN** | **25** | **25** | **15** | **23** | **22** | **22** | **132** | **3** |
| MAKLMD | 0 | 9 | 1 | 22 | 21 | 21 | 74 | 9 |
| MAKLGM | 0 | 0 | 0 | 2 | 5 | 4 | 11 | 21 |
| **MAKLHM** | **25** | **25** | **24** | **21** | **21** | **21** | **137** | **2** |
| MRKL | 2 | 3 | 4 | 5 | 8 | 12 | 34 | 15 |
| MRKLAM | 0 | 0 | 0 | 3 | 3 | 2 | 8 | 25 |
| MRKLMN | 0 | 0 | 1 | 2 | 4 | 3 | 10 | 23 |
| **MRKLMD** | **25** | **25** | **25** | **22** | **12** | **11** | **120** | **5** |
| MRKLGM | 7 | 12 | 12 | 14 | 16 | 14 | 75 | 8 |
| **MRKLHM** | **25** | **25** | **25** | **23** | **22** | **22** | **142** | **1** |
| MDKLMD | 11 | 14 | 9 | 2 | 0 | 0 | 36 | 14 |
| MDKLGM | 0 | 0 | 0 | 0 | 5 | 4 | 9 | 24 |
| MDKLHM | 23 | 11 | 12 | 5 | 4 | 4 | 59 | 12 |
| GMKL | 12 | 10 | 10 | 11 | 9 | 10 | 62 | 11 |
| GMKLAM | 0 | 3 | 6 | 4 | 5 | 7 | 25 | 17 |
| GMKLMN | 0 | 0 | 0 | 2 | 2 | 2 | 6 | 26 |
| GMKLMR | 0 | 0 | 5 | 0 | 3 | 3 | 11 | 21 |
| GMKLMD | 17 | 8 | 14 | 1 | 3 | 2 | 45 | 13 |
| GMKLGM | 14 | 11 | 16 | 10 | 10 | 11 | 72 | 10 |
| GMKLHM | 4 | 4 | 3 | 1 | 3 | 2 | 17 | 18 |
| HMKL | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLAM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLMN | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLMA | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLMR | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLMD | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLGM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| HMKLHM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 35 |
| Total | 250 | 250 | 250 | 250 | 250 | 250 | 1500 |  |
|  | Estimators | n | | | | | | Total |  |
| 10 | 20 | 30 | 50 | 100 | 200 |  |  |
| 8 | GKL | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| GKLAM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| GKLMN | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 34 |
| GKLMR | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| GKLMD | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 34 |
| GKLGM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| GKLHM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| AMKL | 0 | 1 | 4 | 12 | 10 | 11 | 38 | 14 |
| AMKLAM | 0 | 0 | 0 | 5 | 3 | 3 | 11 | 28 |
| AMKLMN | 25 | 0 | 0 | 2 | 2 | 1 | 30 | 18 |
| AMKLMD | 0 | 25 | 25 | 16 | 2 | 4 | 72 | 9 |
| **AMKLGM** | **0** | **8** | **9** | **24** | **23** | **22** | **86** | **5** |
| **AMKLHM** | **0** | **25** | **25** | **9** | **15** | **14** | **88** | **4** |
| MAKL | 0 | 0 | 0 | 1 | 3 | 4 | 8 | 32 |
| **MAKLMN** | **25** | **3** | **4** | **23** | **23** | **22** | **100** | **3** |
| MAKLMD | 0 | 0 | 1 | 22 | 22 | 20 | 65 | 10 |
| MAKLGM | 0 | 0 | 0 | 4 | 4 | 3 | 11 | 28 |
| MAKLHM | 0 | 13 | 8 | 22 | 22 | 19 | 84 | 6 |
| MRKL | 0 | 0 | 3 | 6 | 8 | 9 | 26 | 20 |
| MRKLAM | 0 | 0 | 0 | 3 | 0 | 0 | 3 | 33 |
| MRKLMN | 25 | 0 | 1 | 2 | 3 | 2 | 33 | 16 |
| **MRKLMD** | **0** | **25** | **25** | **24** | **20** | **16** | **110** | **2** |
| MRKLGM | 0 | 8 | 9 | 23 | 22 | 21 | 83 | 7 |
| **MRKLHM** | **0** | **25** | **25** | **22** | **23** | **22** | **117** | **1** |
| MDKL | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 36 |
| MDKLMN | 25 | 0 | 0 | 0 | 1 | 0 | 26 | 20 |
| MDKLMD | 0 | 7 | 14 | 0 | 0 | 0 | 21 | 27 |
| MDKLGM | 0 | 0 | 0 | 0 | 4 | 6 | 10 | 31 |
| MDKLHM | 0 | 23 | 21 | 0 | 0 | 0 | 44 | 13 |
| GMKL | 0 | 19 | 18 | 8 | 8 | 9 | 62 | 11 |
| GMKLAM | 0 | 1 | 1 | 10 | 12 | 9 | 33 | 16 |
| GMKLMN | 25 | 0 | 0 | 0 | 1 | 2 | 28 | 19 |
| GMKLMR | 0 | 0 | 1 | 4 | 3 | 3 | 11 | 28 |
| GMKLMD | 0 | 25 | 21 | 1 | 1 | 3 | 51 | 12 |
| GMKLGM | 0 | 25 | 21 | 6 | 12 | 11 | 75 | 8 |
| GMKLHM | 0 | 17 | 14 | 1 | 2 | 2 | 36 | 15 |
| HMKL | 25 | 0 | 0 | 0 | 0 | 0 | 25 | 22 |
| HMKLAM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| HMKLMN | 25 | 0 | 0 | 0 | 0 | 0 | 25 | 22 |
| HMKLMA | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| HMKLMR | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 36 |
| HMKLMD | 22 | 0 | 0 | 0 | 0 | 0 | 22 | 26 |
| HMKLGM | 23 | 0 | 0 | 0 | 0 | 0 | 23 | 25 |
| HMKLHM | 25 | 0 | 0 | 0 | 0 | 0 | 25 | 22 |
|  | Total | 245 | 250 | 250 | 250 | 250 | 250 | 1495 |  |

Figure 4.1: Best KL Ridge Parameters in Linear Regression Model.

### MKL1 Results with Linear Regression Model

Table 4.2 shows the frequency of each ridge parameter for the MKL estimator whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity and error variance for the MKL1 ridge parameter (see appendix 1-30 for MSE of MKL 1 ridg parameters). The result for the MKL1 ridge parameter when the number of explanatory variables is 4 shows that the GMKL1MN, AMMKL1MN, MNMKL1, MNMKL1MN, MAMKL1MN, MRMKL1MN, MDMKL1MN and the GMMKL1MN had the highest frequency of 57 which all had minimum form of . These were followed in performance by HMMKL1GM with a frequency of 49 and GMKL1AM, AMMKL1AM, AMMKL1, MNMKL1AM, MAMKL1AM, MRMKL1AM, MDMKL1AM, GMMKL1AM with frequency of 45 which all have arithmetic mean form of . The performance of the estimators generally improved as the sample size increased. When the number of explanatory variables is increased to 8, the GMKL1AM, AMMKL1, AMMKL1AM, MNMKL1AM, MAMKL1AM, MRMKL1AM, MDMKL1AM and GMMKL1AM had the highest frequency of 89. The parameters all had the arithmetic mean form of . The performance of the best MKL1 ridge parameter is shown in Figure 4.2. The MKL1 ridge parameters that has the Arithmetic mean form of λ had better performance at low sample size while the MKL1 ridge parameters that has the minimum form of λ improved as the sample size increased.

Table 4.2: Frequency of MKL1 Parameters that ranked between one and ten across all Sample Size for the MKL Ridge Estimator in Linear Regression Model

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | Estimators | n | | | | | | Total | Rank |
| 10 | 20 | 30 | 50 | 100 | 200 |
| 4 | GMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **GMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| GMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| GMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| GMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| GMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| AMMKL1 | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| AMMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **AMMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| AMMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| AMMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| AMMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| AMMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| **MNMKL1** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| MNMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **MNMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| MNMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| MNMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MNMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| MNMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MAMKL1 | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| MAMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **MAMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| MAMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| MAMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MAMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| MAMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MRMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **MRMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| MRMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| MRMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MRMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| MRMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MDMKL1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MDMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **MDMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| MDMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| MDMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| MDMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| MDMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| GMMKL1 | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| GMMKL1AM | 13 | 12 | 7 | 7 | 4 | 2 | 45 | 10 |
| **GMMKL1MN** | **4** | **6** | **9** | **11** | **14** | **13** | **57** | **1** |
| GMMKL1MA | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| GMMKL1MD | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| GMMKL1GM | 5 | 7 | 7 | 7 | 7 | 10 | 43 | 18 |
| GMMKL1HM | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| HMMKL1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 40 |
| HMMKL1AM | 3 | 0 | 0 | 0 | 0 | 0 | 3 | 31 |
| HMMKL1MN | 1 | 1 | 3 | 4 | 5 | 2 | 16 | 29 |
| HMMKL1MA | 14 | 9 | 4 | 6 | 2 | 1 | 36 | 27 |
| HMMKL1MR | 6 | 1 | 0 | 0 | 0 | 0 | 7 | 30 |
| HMMKL1MD | 5 | 8 | 6 | 8 | 5 | 6 | 38 | 26 |
| HMMKL1GM | 8 | 10 | 9 | 7 | 8 | 7 | 49 | 9 |
| HMMKL1HM | 3 | 6 | 6 | 7 | 6 | 6 | 34 | 28 |
| Total | 240 | 235 | 228 | 232 | 226 | 222 | 1383 |  |
|  | Estimators | n | | | | | |  |  |
| 10 | 20 | 30 | 50 | 100 | 200 |  |  |
| 8 | **GMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| GMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| GMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| GMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| GMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| GMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| GMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| **AMMKL1** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| **AMMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| AMMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| AMMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| AMMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| AMMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| AMMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| AMMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| MNMKL1 | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| **MNMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| MNMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| MNMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| MNMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| MNMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| MNMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| MNMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| MAMKL1 | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| **MAMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| MAMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| MAMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| MAMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| MAMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| MAMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| MAMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| MRMKL1 | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| **MRMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| MRMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| MRMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| MRMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| MRMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| MRMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| MRMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| MDMKL1 | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| **MDMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| MDMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| MDMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| MDMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| MDMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| MDMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| MDMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| GMMKL1 | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| **GMMKL1AM** | **18** | **16** | **15** | **13** | **14** | **13** | **89** | **1** |
| GMMKL1MN | 0 | 2 | 4 | 12 | 10 | 12 | 40 | 12 |
| GMMKL1MA | 2 | 1 | 1 | 0 | 0 | 0 | 4 | 32 |
| GMMKL1MR | 2 | 0 | 0 | 0 | 0 | 0 | 2 | 48 |
| GMMKL1MD | 1 | 1 | 0 | 0 | 0 | 0 | 2 | 48 |
| GMMKL1GM | 2 | 4 | 3 | 0 | 1 | 0 | 10 | 23 |
| GMMKL1HM | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| HMMKL1 | 0 | 1 | 2 | 0 | 0 | 0 | 3 | 40 |
| HMMKL1AM | 2 | 1 | 1 | 1 | 0 | 0 | 5 | 31 |
| HMMKL1MN | 0 | 5 | 4 | 6 | 8 | 9 | 32 | 22 |
| HMMKL1MA | 18 | 9 | 9 | 7 | 3 | 3 | 49 | 9 |
| HMMKL1MR | 16 | 6 | 6 | 4 | 1 | 0 | 33 | 21 |
| HMMKL1MD | 6 | 10 | 9 | 7 | 8 | 9 | 49 | 9 |
| HMMKL1GM | 5 | 9 | 8 | 8 | 9 | 10 | 49 | 9 |
| HMMKL1HM | 1 | 6 | 7 | 6 | 10 | 10 | 40 | 12 |
|  | Total | 248 | 246 | 244 | 239 | 239 | 241 | 1457 |  |

Figure 4.2: Best MKL1 Ridge Parameter in Linear Regression Mode

### MKL2 Result with Linear Regression Model

The results on Table 4.3 shows the frequency of each ridge parameter for the MKL estimator whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity and error variance for MKL2 ridge parameter (see appendix 1-30 for MSE of MKL2 ridge parameters).The results when the number of explanatory variables are 4 shows that MAMKL2 had the highest frequency of 132 and was consistent at all the sample sizes. Other maximum versions of MKL2 also performed well although the performance of the parameters were better at lower sample sizes except for MAMKL2MD, MAMKL2GM and MAMKL2HM which had higher frequencies as the sample size increased. As the number of explanatory variables is increased to 8, all maximum versions of MKL2 also performed well with MAMKL2 still having the highest frequency of 123 although the estimator performed better at lower sample sizes except for MAMKL2MD, MAMKL2GM and MAMKL2HM that had higher frequencies at higher sample size. Figure 4.3 shows the performance of the five best MKL2 ridge parameters across different sample size and number of explanatory variables. The MAMKL2 consistently performed well across the sample size although its performance decreased as the sample size increased.

Table 4.3: Frequency of MKL2 parameters that ranks between one and ten across all sample size for the MKL estimator in linear regression model

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | Estimators | n | | | | | | Total | Rank |
| 10 | 20 | 30 | 50 | 100 | 200 |
| 4 | GMKL2 | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| GMKL2AM | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| GMKL2MN | 0 | 0 | 0 | 0 | 2 | 2 | 4 | 29 |
| GMKL2MA | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| GMKL2MR | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| GMKL2MD | 0 | 0 | 0 | 0 | 2 | 2 | 4 | 29 |
| GMKL2GM | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| GMKL2HM | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 33 |
| AMMKL2 | 0 | 0 | 1 | 4 | 6 | 5 | 16 | 21 |
| AMMKL2AM | 1 | 3 | 3 | 5 | 9 | 9 | 30 | 15 |
| AMMKL2MN | 0 | 0 | 1 | 2 | 2 | 2 | 7 | 26 |
| AMMKL2MA | 6 | 5 | 5 | 2 | 2 | 1 | 21 | 18 |
| AMMKL2MR | 0 | 2 | 4 | 4 | 6 | 8 | 24 | 17 |
| AMMKL2MD | 1 | 1 | 2 | 3 | 2 | 2 | 11 | 23 |
| AMMKL2GM | 2 | 2 | 3 | 4 | 7 | 10 | 28 | 16 |
| AMMKL2HM | 0 | 1 | 1 | 2 | 3 | 5 | 12 | 22 |
| **MAMKL2** | **25** | **25** | **24** | **20** | **19** | **19** | **132** | **1** |
| **MAMKL2AM** | **24** | **23** | **22** | **20** | **14** | **15** | **118** | **2** |
| MAMKL2MN | 23 | 21 | 19 | 13 | 6 | 6 | 88 | 6 |
| MAMKL2MA | 23 | 21 | 19 | 13 | 6 | 6 | 88 | 6 |
| **MAMKL2MR** | **23** | **23** | **21** | **19** | **9** | **11** | **106** | **3** |
| MAMKL2MD | 7 | 9 | 11 | 18 | 20 | 20 | 85 | 8 |
| MAMKL2GM | 11 | 11 | 14 | 12 | 11 | 12 | 71 | 13 |
| MAMKL2HM | 4 | 7 | 8 | 17 | 18 | 19 | 73 | 12 |
| MRMKL2 | 17 | 14 | 10 | 9 | 12 | 12 | 74 | 11 |
| MRMKL2AM | 13 | 15 | 17 | 12 | 13 | 12 | 82 | 9 |
| MRMKL2MN | 21 | 19 | 15 | 11 | 11 | 5 | 82 | 9 |
| **MRMKL2MA** | **21** | **20** | **18** | **13** | **10** | **9** | **91** | **5** |
| **MRMKL2MR** | **23** | **21** | **21** | **14** | **14** | **13** | **106** | **3** |
| MRMKL2MD | 2 | 2 | 4 | 2 | 4 | 4 | 18 | 20 |
| MRMKL2GM | 2 | 3 | 4 | 8 | 15 | 12 | 44 | 14 |
| MRMKL2HM | 1 | 2 | 2 | 3 | 5 | 6 | 19 | 19 |
| MDMKL2MN | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| GMMKL2 | 0 | 0 | 0 | 2 | 3 | 3 | 8 | 24 |
| GMMKL2AM | 0 | 0 | 0 | 2 | 1 | 2 | 5 | 27 |
| GMMKL2MN | 0 | 0 | 0 | 2 | 3 | 3 | 8 | 24 |
| GMMKL2MR | 0 | 0 | 1 | 1 | 0 | 1 | 3 | 33 |
| GMMKL2MD | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 29 |
| GMMKL2GM | 0 | 0 | 0 | 2 | 1 | 2 | 5 | 27 |
| GMMKL2HM | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 29 |
| HMMKL2 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| HMMKL2AM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| HMMKL2MN | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| HMMKL2MA | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 40 |
| HMMKL2MR | 0 | 0 | 0 | 1 | 0 | 1 | 2 | 40 |
| HMMKL2MD | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| HMMKL2GM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| HMMKL2HM | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 42 |
| Total | 250 | 250 | 250 | 250 | 250 | 250 | 1500 |  |
|  |  | n | | | | | |  |  |
| Estimators | 10 | 20 | 30 | 50 | 100 | 200 |  |  |
| 8 | GMKL2 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| GMKL2AM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| GMKL2MN | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 33 |
| GMKL2MA | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| GMKL2MR | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| GMKL2MD | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 33 |
| GMKL2GM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| GMKL2HM | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 33 |
| AMMKL2 | 0 | 0 | 1 | 3 | 2 | 5 | 11 | 21 |
| AMMKL2AM | 0 | 3 | 2 | 7 | 8 | 7 | 27 | 16 |
| AMMKL2MN | 0 | 0 | 1 | 2 | 3 | 2 | 8 | 24 |
| AMMKL2MA | 0 | 16 | 12 | 3 | 6 | 5 | 42 | 15 |
| AMMKL2MR | 0 | 5 | 5 | 5 | 6 | 3 | 24 | 17 |
| AMMKL2MD | 0 | 1 | 2 | 2 | 2 | 4 | 11 | 21 |
| AMMKL2GM | 0 | 1 | 3 | 4 | 5 | 6 | 19 | 18 |
| AMMKL2HM | 0 | 0 | 1 | 2 | 2 | 5 | 10 | 23 |
| **MAMKL2** | **25** | **23** | **24** | **20** | **17** | **14** | **123** | **1** |
| **MAMKL2AM** | **25** | **24** | **22** | **16** | **15** | **9** | **111** | **3** |
| MAMKL2MN | 25 | 20 | 19 | 13 | 11 | 8 | 96 | 7 |
| MAMKL2MA | 25 | 20 | 19 | 13 | 11 | 8 | 96 | 7 |
| **MAMKL2MR** | **25** | **22** | **21** | **14** | **14** | **10** | **106** | **4** |
| MAMKL2MD | 5 | 4 | 6 | 17 | 18 | 17 | 67 | 11 |
| MAMKL2GM | 2 | 6 | 8 | 12 | 10 | 8 | 46 | 13 |
| MAMKL2HM | 0 | 3 | 4 | 17 | 18 | 16 | 58 | 12 |
| MRMKL2 | 21 | 13 | 12 | 10 | 9 | 17 | 82 | 10 |
| MRMKL2AM | 25 | 18 | 18 | 15 | 11 | 8 | 95 | 9 |
| MRMKL2MN | 23 | 21 | 19 | 14 | 14 | 12 | 103 | 6 |
| **MRMKL2MA** | **24** | **22** | **20** | **14** | **13** | **11** | **104** | **5** |
| **MRMKL2MR** | **25** | **23** | **22** | **14** | **16** | **12** | **112** | **2** |
| MRMKL2MD | 0 | 1 | 3 | 4 | 3 | 7 | 18 | 20 |
| MRMKL2GM | 0 | 2 | 3 | 9 | 13 | 17 | 44 | 14 |
| MRMKL2LHM | 0 | 1 | 2 | 4 | 5 | 7 | 19 | 18 |
| MDMKL2 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 37 |
| MDMKL2MN | 0 | 0 | 0 | 1 | 1 | 1 | 3 | 32 |
| MDMKL2MD | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 37 |
| GMMKL2 | 0 | 0 | 0 | 2 | 2 | 3 | 7 | 26 |
| GMMKL2AM | 0 | 0 | 0 | 3 | 3 | 2 | 8 | 24 |
| GMMKL2MN | 0 | 0 | 0 | 1 | 1 | 3 | 5 | 30 |
| GMMKL2MA | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 33 |
| GMMKL2MR | 0 | 0 | 1 | 2 | 1 | 3 | 7 | 26 |
| GMMKL2MD | 0 | 0 | 0 | 2 | 2 | 2 | 6 | 28 |
| GMMKL2GM | 0 | 0 | 0 | 2 | 2 | 1 | 5 | 30 |
| GMMKL2HM | 0 | 0 | 0 | 2 | 2 | 2 | 6 | 28 |
| HMMKL2AM | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| HMMKL2MA | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
| HMMKL2MR | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 37 |
|  | Total | 250 | 249 | 250 | 249 | 249 | 250 | 1497 |  |

Figure 4.3: Best MKL2 Ridge Parameters in Linear Regression Model

### Combined Result of KL, MKL1 and MKL2

Table 4.4 shows the selected KL, MKL1 and MKL2 ridge parameter for the linear regression model. The MRKLHM, GMKL1AM and the MAMKL2 were selected for KL, MKL1 and MKL2 respectively. The performance of the three parameters were examined using their MSE’s as criterion. Figure 4.4 and Figure 4.5 shows that at low levels of multicollinearity and variance when the number of explanatory variables is 4, the MRKLHM and MAMKL2 had low and consistent MSE in comparison to the GMKL1AM. When the number of explanatory variables is 8, The MRKLHM had really high MSE at low sample sizes. In general, the MAMKL2 had lower and consistent MSE at all levels of variance, level of multicollinearity and number of explanatory variables.

Table 4.4: Best Ridge Parameter of KL, MKL1 and MKL2 in Linear Regression Model.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | KL | | | MKL1 | | | MKL2 | | |
| S/No | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | |
| P=4 | P=8 | P=4 | P=8 | P=4 | P=8 |
| 1 | MRKLHM | 142 | 117 | GMKL1AM | 45 | 89 | MAMKL2 | 132 | 123 |
| 2 |  |  | | AMMKL1 | 45 | 89 |  |  |  |
| 3 |  |  | | AMMKL1AM | 45 | 89 |  |  |  |
| 4 |  |  | | MNMKL1AM | 45 | 89 |  |  |  |
| 5 |  |  | | MAMKL1AM | 45 | 89 |  |  |  |
| 6 |  |  | | MRMKL1AM | 45 | 89 |  |  |  |
| 7 |  |  | | MDMKL1AM | 45 | 89 |  |  |  |
| 8 |  |  | | GMMKL1AM | 45 | 89 |  |  |  |

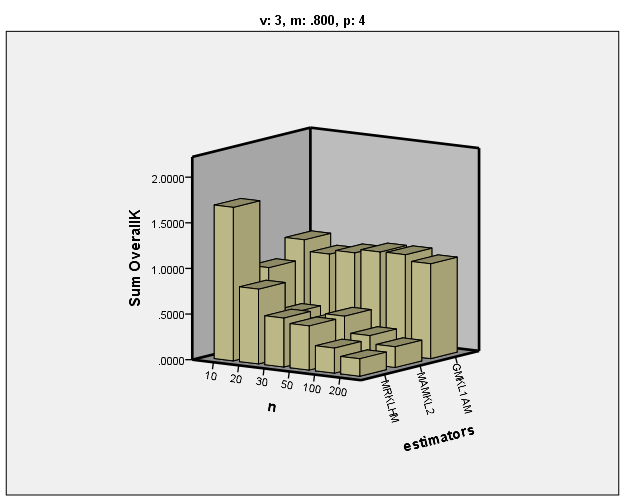


Figure 4.4: MSE of three Parameters with highest frequency in Linear Regression when v=9, and P=4

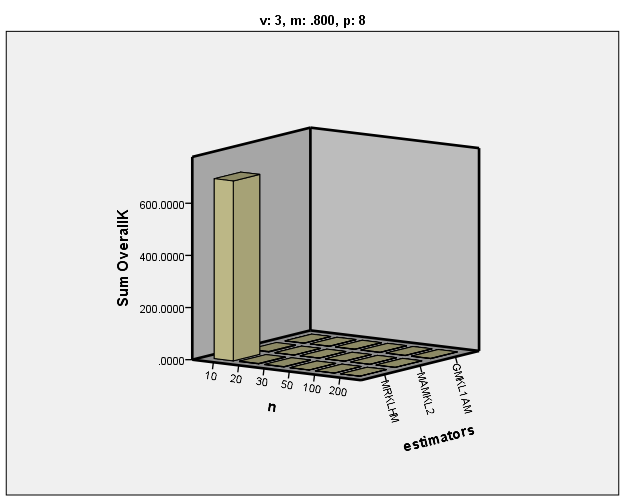


Figure 4.5: MSE of three Parameters with highest frequency in Linear Regression when v=9, and P=8

### Comparison of Selected Best Parameters in Linear Regression Model with Existing Parameters

The selected best performing ridge parameters for the KL, MKL1 and MKL2 were compared with existing ridge parameters and the results are shown in Table 4.5*.* The result from the overall comparison of all ridge parameters and estimators for the linear regression model shows that the MAMKL2 had the highest frequency of 49 when the number of explanatory variables is 4. It’s performance increased as the sample size increased. When the number of explanatory variables is increased to 8, the MKL1AM, AMMKL1, AMMKL1AM, MNMKL1AM, MAMKL1AM, MRMKL1AM, MDMKL1AM and GMMKL1AM all had equal and highest frequency of 77. The parameters at this level all had the arithmetic mean version of and had higher frequencies at small sample sizes.

Table 4.5: Overall Performance of the Ridge Parameters for Linear Regression Model

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | Estimators | n | | | | | | Total |
| 10 | 20 | 30 | 50 | 100 | 200 |
| 4 | RIDGE | 0 | 0 | 0 | 0 | 1 | 1 | 2 |
| ASIMOTA | 9 | 7 | 5 | 4 | 2 | 1 | 28 |
| FAYINDE | 0 | 0 | 0 | 8 | 16 | 14 | 38 |
| FAYINDE1 | 1 | 1 | 0 | 0 | 0 | 0 | 2 |
| FAYINDE2 | 2 | 1 | 0 | 0 | 0 | 0 | 3 |
| FAYINDE3 | 3 | 2 | 1 | 0 | 0 | 0 | 6 |
| MRKLHM | 0 | 1 | 1 | 1 | 3 | 4 | 10 |
| MKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| AMMKL1 | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| AMMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| MNMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| MAMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| MRMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| MDMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| GMMKL1AM | 12 | 9 | 4 | 4 | 1 | 0 | 30 |
| MAMKL2 | 5 | 3 | 3 | 9 | 14 | 15 | 49 |
| Total | 116 | 87 | 42 | 54 | 44 | 35 | 378 |
|  |  | n | | | | | |  |
| Estimators | 10 | 20 | 30 | 50 | 100 | 200 |  |
| 8 | ASIMOTA | 9 | 6 | 5 | 3 | 4 | 1 | 28 |
| FAYINDE1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| FAYINDE2 | 3 | 1 | 0 | 0 | 0 | 0 | 4 |
| FAYINDE3 | 7 | 1 | 1 | 0 | 0 | 0 | 9 |
| MRKLHM | 0 | 0 | 1 | 2 | 2 | 4 | 9 |
| MKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| AMMKL1 | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| AMMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| MNMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| MAMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| MRMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| MDMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| GMMKL1AM | 18 | 16 | 15 | 13 | 8 | 7 | 77 |
| MAMKL2 | 0 | 0 | 0 | 5 | 12 | 9 | 26 |
| Total | 164 | 136 | 127 | 114 | 82 | 70 | 693 |

### Real Life Application for Linear Regression

The selected best ridge parameters for the KL, MKL1 and MKL2 were compared with the ridge estimator and the ordinary least squares estimator with a real life application in linear regression. The MSE and the ranks of the parameter are shown in Table 4.6. The MAMKL2 version of MKL2 ridge parameter had the lowest MSE in the real life application.

Table 4.6: MSE of Selected Parameter for Real Life Application in Linear Regression Model

|  |  |  |
| --- | --- | --- |
| Estimators | MSE | Rank of MSE |
| OLS | 4910.000 | 5.000 |
| RIDGE2 | 96.500 | 2.000 |
| MRKLHM | 1090.000 | 3.000 |
| MKL1AM | 3110.000 | 4.000 |
| MAMKL2 | 0.1190 | 1.000 |

## Summary of Results of the Estimators with Poisson Regression Model

The summary of the three ridge parameters KL, MKL1 and MKL2 for the Poisson regression model is provided as follows:

### KL Result with Poison Regression Model

The number of times each ridge parameter whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity is presented in Table 4.7*.* The results show that the AMKLHM had the highest frequency of 38 and MAKLMN with frequency of 37. The two parameters had high frequencies at low sample sizes even though they both performed well at all sample sizes when the number of explanatory variables are 4. With the number of explanatory variables at 8, the GMKLMD and the GMKLHM had the highest frequency of 35 and 34 respectively. It was closely followed by GMKL with a frequency of 33. The parameters had higher frequencies at lower sample sizes. In general, the AMKLHM and GMKLMD had the best performance when the number of explanatory variables are 4 and 8 respectively. Figure 4.6 shows the performance of the five best KL ridge parameters selected across sample size and the number of explanatory variables. The AMKLHM had the highest frequency across the sample size. Its frequency decreased as the sample size increased.

Table 4.7: Frequency of KL Parameters that ranks between one and ten across all Sample Size for of the KL Ridge Estimator in Poisson Regression Model

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total | Rank |
| 50 | 75 | 100 | 200 |
| 4 | GKL | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLAM | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLMN | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLMA | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLMR | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLMD | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLGM | 0 | 1 | 0 | 0 | 1 | 43 |
| GKLHM | 0 | 3 | 0 | 0 | 3 | 34 |
| AMKL | 6 | 7 | 7 | 5 | 25 | 8 |
| AMKLMN | 8 | 6 | 6 | 6 | 26 | 6 |
| AMKLMD | 4 | 4 | 5 | 4 | 17 | 14 |
| AMKLGM | 5 | 3 | 4 | 4 | 16 | 15 |
| **AMKLHM** | **12** | **10** | **9** | **7** | **38** | **1** |
| MNKL | 0 | 0 | 0 | 1 | 1 | 43 |
| MNKLAM | 0 | 0 | 0 | 1 | 1 | 43 |
| MNKLMN | 0 | 0 | 0 | 1 | 1 | 43 |
| MNKLMA | 0 | 0 | 1 | 2 | 3 | 34 |
| MNKLMR | 0 | 0 | 1 | 2 | 3 | 34 |
| MNKLMD | 0 | 0 | 0 | 1 | 1 | 43 |
| MNKLGM | 0 | 0 | 0 | 1 | 1 | 43 |
| MNKLHM | 0 | 0 | 0 | 2 | 2 | 40 |
| MAKL | 2 | 2 | 3 | 2 | 9 | 21 |
| MAKLAM | 0 | 0 | 0 | 1 | 1 | 43 |
| **MAKLMN** | **12** | **9** | **9** | **7** | **37** | **2** |
| MAKLMR | 0 | 0 | 0 | 1 | 1 | 43 |
| MAKLMD | 2 | 2 | 3 | 2 | 9 | 21 |
| MAKLGM | 0 | 0 | 0 | 1 | 1 | 43 |
| **MAKLHM** | **6** | **8** | **8** | **6** | **28** | **4** |
| MRKL | 3 | 5 | 7 | 4 | 19 | 12 |
| MRKLMN | 7 | 7 | 7 | 4 | 25 | 8 |
| MRKLMD | 2 | 2 | 3 | 2 | 9 | 21 |
| MRKLGM | 1 | 2 | 2 | 1 | 6 | 32 |
| **MRKLHM** | **10** | **9** | **9** | **7** | **35** | **3** |
| MDKL | 4 | 2 | 2 | 2 | 10 | 20 |
| MDKLAM | 1 | 1 | 1 | 0 | 3 | 34 |
| MDKLMN | 4 | 4 | 4 | 3 | 15 | 18 |
| MDKLMA | 0 | 1 | 1 | 0 | 2 | 40 |
| MDKLMR | 0 | 1 | 1 | 0 | 2 | 40 |
| MDKLMD | 1 | 1 | 1 | 0 | 3 | 34 |
| MDKLGM | 1 | 1 | 1 | 0 | 3 | 34 |
| MDKLHM | 6 | 4 | 2 | 2 | 14 | 19 |
| **GMKL** | **8** | **6** | **6** | **8** | **28** | **4** |
| GMKLAM | 4 | 5 | 3 | 4 | 16 | 15 |
| GMKLMN | 5 | 3 | 4 | 6 | 18 | 13 |
| GMKLMA | 1 | 0 | 1 | 3 | 5 | 33 |
| GMKLMR | 4 | 4 | 3 | 5 | 16 | 15 |
| GMKLMD | 7 | 7 | 5 | 5 | 24 | 11 |
| GMKLGM | 7 | 8 | 6 | 5 | 26 | 6 |
| GMKLHM | 10 | 6 | 4 | 5 | 25 | 8 |
| HMKL | 1 | 1 | 2 | 3 | 7 | 29 |
| HMKLAM | 1 | 1 | 3 | 3 | 8 | 26 |
| HMKLMN | 0 | 1 | 2 | 4 | 7 | 29 |
| HMKLMA | 1 | 1 | 3 | 3 | 8 | 26 |
| HMKLMR | 1 | 1 | 3 | 3 | 8 | 26 |
| HMKLMD | 1 | 1 | 3 | 4 | 9 | 21 |
| HMKLGM | 1 | 2 | 3 | 3 | 9 | 21 |
| HMKLHM | 1 | 1 | 2 | 3 | 7 | 29 |
| Total |  | 149 | 150 | 149 | 598 |  |
|  |  | n | | | | |  |
| Estimators |  | 50 | 75 | 100 | 200 |  |
| 8 | GKLMN | 3 | 2 | 1 | 0 | 6 | 27 |
| AMKL | 0 | 1 | 2 | 4 | 7 | 25 |
| AMKLMN | 7 | 5 | 4 | 2 | 18 | 14 |
| AMKLMD | 0 | 1 | 3 | 4 | 8 | 24 |
| **AMKLHM** | **8** | **10** | **7** | **5** | **30** | **4** |
| MNKL | 0 | 1 | 1 | 4 | 6 | 27 |
| MNKLAM | 0 | 0 | 1 | 3 | 4 | 35 |
| MNKLMN | 0 | 0 | 1 | 4 | 5 | 33 |
| MNKLMA | 0 | 1 | 2 | 4 | 7 | 25 |
| MNKLMR | 0 | 1 | 1 | 4 | 6 | 27 |
| MNKLMD | 0 | 0 | 2 | 4 | 6 | 27 |
| MNKLGM | 0 | 0 | 2 | 3 | 5 | 33 |
| MNKLHM | 0 | 0 | 1 | 3 | 4 | 35 |
| MAKLMN | 5 | 10 | 6 | 5 | 26 | 8 |
| MAKLMA | 1 | 0 | 0 | 0 | 1 | 40 |
| MAKLHM | 0 | 0 | 0 | 2 | 2 | 38 |
| MRKLMN | 6 | 5 | 2 | 2 | 15 | 19 |
| MRKLMD | 0 | 0 | 0 | 4 | 4 | 35 |
| MRKLHM | 7 | 8 | 6 | 4 | 25 | 10 |
| MDKL | 6 | 5 | 4 | 2 | 17 | 15 |
| MDKLAM | 0 | 0 | 1 | 0 | 1 | 40 |
| MDKLMN | 7 | 4 | 3 | 3 | 17 | 15 |
| MDKLMA | 0 | 0 | 1 | 0 | 1 | 40 |
| MDKLMR | 0 | 0 | 1 | 0 | 1 | 40 |
| MDKLMD | 0 | 1 | 3 | 2 | 6 | 27 |
| MDKLGM | 0 | 0 | 1 | 1 | 2 | 38 |
| MDKLHM | 8 | 8 | 6 | 3 | 25 | 10 |
| **GMKL** | **11** | **9** | **9** | **4** | **33** | **3** |
| GMKLAM | 3 | 2 | 4 | 2 | 11 | 22 |
| GMKLMN | 4 | 4 | 5 | 4 | 17 | 15 |
| GMKLMA | 0 | 2 | 3 | 1 | 6 | 27 |
| GMKLMR | 3 | 2 | 3 | 2 | 10 | 23 |
| **GMKLMD** | **12** | **10** | **10** | **3** | **35** | **1** |
| **GMKLGM** | **8** | **9** | **7** | **5** | **29** | **5** |
| **GMKLHM** | **13** | **11** | **8** | **2** | **34** | **2** |
| HMKL | 2 | 2 | 3 | 6 | 13 | 20 |
| HMKLAM | 8 | 6 | 6 | 6 | 26 | 8 |
| HMKLMN | 1 | 2 | 4 | 9 | 16 | 18 |
| HMKLMA | 8 | 7 | 6 | 6 | 27 | 6 |
| HMKLMR | 8 | 7 | 6 | 6 | 27 | 6 |
| HMKLMD | 4 | 6 | 4 | 6 | 20 | 13 |
| HMKLGM | 5 | 6 | 5 | 6 | 22 | 12 |
| HMKLHM | 2 | 2 | 3 | 6 | 13 | 20 |
|  | Total | 150 | 150 | 148 | 146 | 594 |  |

Figure 4.6: Best KL ridge Parameter in Poisson Regression

### MKL1 Result with Poisson Regression Model

The frequency of each ridge parameter for MKL1 whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity in Poisson regression model is presented in Table 4.8. The result of simulation for the Poisson regression model for the MKL1 ridge parameter when the number of explanatory variables are 4 show that GMKL1MN, AMMKL1MN, MNMKL1, MNMKL1MN, MNMKL1HM, MAMKL1MN, MRMKL1MN, MDMKL1MN, GMMKL1MN were the best performing estimators with frequency of 58. When the number of explanatory variables is increased to 8, GMKL1MN, AMMKL1MN, MNMKL1, MNMKL1MN, MNMKL1HM, MAMKL1MN, MRMKL1MN, MDMKL1MN, GMMKL1MN were the best performing parameters also with frequency of 58. The parameters all had the minimum form of except for MNMKL1HM which had a minimum version of the MKL1 ridge parameter when the number of explanatory variables is 4 and 8. The overall best performing parameters were AMMKL1MN, MNMKL1, MNMKL1MN, MNMKL1HM, MAMKL1MN, MRMKL1MN, MDMKL1MN, GMMKL1MN for when the number of explanatory variables are 4 and 8. The performance of the MKL1 ridge parameter for the Poisson regression model is shown in Figure 4.7. The selected parameters all performed consistently well across the sample size and number of explanatory variables.

Table 4.8: Frequency of MKL1 Parameters that rank between one and ten across all Sample Size of the MKL Ridge Estimator in Poisson Regression Model.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total | Rank |
| 50 | 75 | 100 | 200 |
| 4 | GMKL1 | 2 | 0 | 1 | 1 | 4 | 10 |
| GMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **GMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| AMMKL1 | 0 | 0 | 0 | 1 | 1 | 18 |
| AMMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **AMMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| AMMKL1HM | 0 | 0 | 0 | 1 | 1 | 18 |
| **MNMKL1** | **15** | **15** | **15** | **13** | **58** | **1** |
| MNMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **MNMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| MNMKL1GM | 0 | 0 | 0 | 1 | 1 | 18 |
| **MNMKL1HM** | **15** | **15** | **15** | **13** | **58** | **1** |
| MAMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **MAMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| MRMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **MRMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| MDMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **MDMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| GMMKL1AM | 0 | 0 | 0 | 2 | 2 | 11 |
| **GMMKL1MN** | **15** | **15** | **15** | **13** | **58** | **1** |
| HMMKL1AM | 1 | 0 | 0 | 0 | 1 | 18 |
| HMMKL1MA | 1 | 0 | 0 | 0 | 1 | 18 |
| HMMKL1MR | 1 | 0 | 0 | 0 | 1 | 18 |
| HMMKL1HM | 0 | 0 | 0 | 1 | 1 | 18 |
| Total | 140 | 135 | 136 | 136 | 547 |  |
|  |  | n | | | | |  |
| Estimators | 50 | 75 | 100 | 200 | Total |  |
| 8 | GMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **GMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| GMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| AMMKL1 | 1 | 0 | 0 | 0 | 1 | 10 |
| AMMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **AMMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| AMMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| **MNMKL1** | **14** | **14** | **15** | **15** | **58** | **1** |
| MNMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **MNMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| MNMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| **MNMKL1HM** | **14** | **14** | **15** | **15** | **58** | **1** |
| MAMKL1 | 0 | 1 | 0 | 0 | 1 | 10 |
| MAMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **MAMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| MAMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| MRMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **MRMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| MRMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| MDMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **MDMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| MDMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| GMMKL1AM | 1 | 0 | 0 | 0 | 1 | 10 |
| **GMMKL1MN** | **14** | **14** | **15** | **15** | **58** | **1** |
| GMMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| HMMKL1AM | 0 | 1 | 0 | 0 | 1 | 10 |
| HMMKL1MA | 0 | 1 | 0 | 0 | 1 | 10 |
| HMMKL1MR | 0 | 1 | 0 | 0 | 1 | 10 |
| HMMKL1GM | 1 | 0 | 0 | 0 | 1 | 10 |
|  | Total | 135 | 137 | 135 | 135 | 542 |  |

Figure 4.7: Best MKL1 Ridge Parameter in Poisson Regression Model

### MKL2 Result with Poisson Regression Model

The frequency of each ridge parameter for MKL2 whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity in Poisson regression model is presented in Table 4.9. The results reveal that AMMKL2HM had the highest frequency of 38 when the number of explanatory variables is 4. When the number of explanatory variables is increased to 8, the GMMKL2GM had the highest frequency of 39 while AMMKL2HM and the GMMKL2 closely followed with a frequency of 36. The parameters both had higher frequencies at lower sample size at when the number of explanatory variables is 4 and 8. In general, the AMMKL2HM performs well for the Poisson regression model. Figure 4.8 show the performance of the MKL2 ridge parameter for the Poisson regression model. The AMMKL2HM had the highest and consistent result across the sample size and number of explanatory variables.

Table 4.9: Frequency of MKL2 Parameters that rank between one and ten across all Sample Size of the MKL Ridge Estimator in Poisson Regression Model.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total | Rank |
| 50 | 75 | 100 | 200 |
| 4 | GMKL2 | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2AM | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2MN | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2MA | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2MR | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2MD | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2GM | 0 | 1 | 0 | 0 | 1 | 47 |
| GMKL2HM | 1 | 1 | 0 | 0 | 2 | 46 |
| **AMMKL2** | **6** | **8** | **6** | **6** | **26** | **4** |
| AMMKL2MN | 7 | 5 | 5 | 3 | 20 | 10 |
| AMMKL2MD | 5 | 5 | 5 | 7 | 22 | 8 |
| AMMKL2GM | 5 | 6 | 6 | 5 | 22 | 8 |
| **AMMKL2HM** | **11** | **10** | **9** | **8** | **38** | **1** |
| MNMKL2 | 1 | 1 | 2 | 3 | 7 | 30 |
| MNMKL2AM | 1 | 1 | 2 | 3 | 7 | 30 |
| MNMKL2MN | 1 | 1 | 2 | 3 | 7 | 30 |
| MNMKL2MA | 1 | 2 | 3 | 3 | 9 | 23 |
| MNMKL2MR | 1 | 1 | 3 | 3 | 8 | 25 |
| MNMKL2MD | 1 | 1 | 2 | 3 | 7 | 30 |
| MNMKL2GM | 1 | 1 | 2 | 3 | 7 | 30 |
| MNMKL2HM | 1 | 1 | 2 | 4 | 8 | 25 |
| MAMKL2 | 3 | 2 | 5 | 4 | 14 | 18 |
| MAMKL2MD | 0 | 2 | 2 | 2 | 6 | 36 |
| MAMKL2GM | 1 | 0 | 1 | 1 | 3 | 45 |
| **MAMKL2HM** | **8** | **8** | **8** | **6** | **30** | **3** |
| **MRMKL2** | **7** | **7** | **7** | **5** | **26** | **4** |
| MRMKL2MD | 5 | 4 | 4 | 6 | 19 | 12 |
| MRMKL2GM | 3 | 3 | 4 | 6 | 16 | 15 |
| **MRMKL2HM** | **9** | **9** | **10** | **7** | **35** | **2** |
| MDMKL2 | 5 | 4 | 4 | 2 | 15 | 17 |
| MDMKL2AM | 2 | 1 | 1 | 0 | 4 | 39 |
| MDMKL2MN | 4 | 2 | 3 | 0 | 9 | 23 |
| MDMKL2MA | 2 | 1 | 1 | 0 | 4 | 39 |
| MDMKL2MR | 2 | 1 | 1 | 0 | 4 | 39 |
| MDMKL2MD | 2 | 1 | 1 | 0 | 4 | 39 |
| MDMKL2GM | 2 | 1 | 1 | 0 | 4 | 39 |
| MDMKL2HM | 5 | 3 | 2 | 2 | 12 | 19 |
| GMMKL2 | 7 | 6 | 5 | 7 | 25 | 6 |
| GMMKL2AM | 4 | 5 | 4 | 5 | 18 | 13 |
| GMMKL2MN | 3 | 2 | 1 | 4 | 10 | 21 |
| GMMKL2MA | 3 | 3 | 2 | 3 | 11 | 20 |
| GMMKL2MR | 3 | 5 | 3 | 5 | 16 | 15 |
| GMMKL2MD | 6 | 5 | 5 | 4 | 20 | 10 |
| GMMKL2GM | 7 | 7 | 6 | 4 | 24 | 7 |
| GMMKL2HM | 6 | 4 | 4 | 3 | 17 | 14 |
| HMMKL2 | 0 | 1 | 1 | 2 | 4 | 39 |
| HMMKL2AM | 2 | 1 | 2 | 2 | 7 | 30 |
| HMMKL2MN | 1 | 2 | 3 | 4 | 10 | 21 |
| HMMKL2MA | 2 | 2 | 2 | 2 | 8 | 25 |
| HMMKL2MR | 2 | 2 | 2 | 2 | 8 | 25 |
| HMMKL2MD | 0 | 1 | 3 | 4 | 8 | 25 |
| HMMKL2GM | 0 | 1 | 2 | 2 | 5 | 38 |
| HMMKL2HM | 1 | 2 | 1 | 2 | 6 | 36 |
| Total | 150 | 149 | 150 | 150 | 599 |  |
|  |  | n | | | | |  |
| Estimators | 50 | 75 | 100 | 200 | Total |  |
| 8 | GMKL2MN | 3 | 2 | 1 | 1 | 7 | 26 |
| GMKL2HM | 0 | 0 | 0 | 1 | 1 | 42 |
| AMMKL2 | 0 | 2 | 3 | 5 | 10 | 21 |
| AMMKL2MN | 8 | 6 | 4 | 2 | 20 | 10 |
| AMMKL2MD | 5 | 3 | 6 | 5 | 19 | 11 |
| AMMKL2GM | 0 | 1 | 2 | 2 | 5 | 36 |
| **AMMKL2HM** | **12** | **11** | **7** | **6** | **36** | **2** |
| MNMKL2 | 0 | 0 | 2 | 4 | 6 | 30 |
| MNMKL2AM | 0 | 0 | 2 | 4 | 6 | 30 |
| MNMKL2MN | 0 | 0 | 2 | 5 | 7 | 26 |
| MNMKL2MA | 0 | 0 | 2 | 4 | 6 | 30 |
| MNMKL2MR | 0 | 0 | 2 | 4 | 6 | 30 |
| MNMKL2MD | 0 | 0 | 2 | 5 | 7 | 26 |
| MNMKL2GM | 0 | 0 | 2 | 4 | 6 | 30 |
| MNMKL2HM | 0 | 0 | 2 | 4 | 6 | 30 |
| MAMKL2MD | 0 | 0 | 0 | 1 | 1 | 42 |
| MAMKL2HM | 0 | 1 | 0 | 4 | 5 | 36 |
| MRMKL2 | 0 | 1 | 1 | 2 | 4 | 38 |
| MRMKL2MD | 0 | 0 | 0 | 4 | 4 | 38 |
| MRMKL2HM | 8 | 8 | 7 | 4 | 27 | 6 |
| MDMKL2 | 9 | 7 | 6 | 2 | 24 | 9 |
| MDMKL2AM | 0 | 0 | 1 | 0 | 1 | 42 |
| MDMKL2MN | 3 | 4 | 3 | 1 | 11 | 20 |
| MDMKL2MA | 0 | 1 | 2 | 0 | 3 | 40 |
| MDMKL2MR | 0 | 0 | 2 | 0 | 2 | 41 |
| MDMKL2MD | 6 | 4 | 6 | 1 | 17 | 13 |
| MDMKL2GM | 2 | 1 | 3 | 1 | 7 | 26 |
| MDMKL2HM | 11 | 8 | 6 | 2 | 27 | 6 |
| **GMMKL2** | **13** | **11** | **8** | **4** | **36** | **2** |
| GMMKL2AM | 4 | 7 | 8 | 6 | 25 | 8 |
| GMMKL2MN | 4 | 4 | 5 | 4 | 17 | 13 |
| GMMKL2MA | 4 | 2 | 5 | 3 | 14 | 17 |
| GMMKL2MR | 4 | 4 | 5 | 5 | 18 | 12 |
| **GMMKL2MD** | **13** | **11** | **8** | **3** | **35** | **4** |
| **GMMKL2GM** | **13** | **11** | **11** | **4** | **39** | **1** |
| **GMMKL2HM** | **13** | **12** | **7** | **3** | **35** | **4** |
| HMMKL2 | 1 | 2 | 2 | 3 | 8 | 25 |
| HMMKL2AM | 3 | 3 | 3 | 5 | 14 | 17 |
| HMMKL2MN | 1 | 2 | 3 | 6 | 12 | 19 |
| HMMKL2MA | 3 | 6 | 3 | 5 | 17 | 13 |
| HMMKL2MR | 3 | 6 | 3 | 5 | 17 | 13 |
| HMMKL2MD | 1 | 3 | 1 | 4 | 9 | 24 |
| HMMKL2GM | 2 | 3 | 1 | 4 | 10 | 21 |
| HMMKL2HM | 1 | 2 | 1 | 6 | 10 | 21 |
| Total | 150 | 149 | 150 | 148 | 597 |  |

Figure 4.8: Best Performing MKL2 for the Poisson Regression Model

### Combined Result of KL, MKL1 and MKL2 for Poisson Regression Model

Table 4.10 shows the ridge parameter with the highest frequency for the KL and MKL1 and MKL2 in the Poisson regression model. The GMKLHM, MNMKL1 and GMMKL2GM were selected for the KL, MKL1 and MKL2 estimators respectively due to their consistency across all levels of variance, beta and multicollinearity. Figure 4.9, Figure 4.10 and Figure 4.11 show that the GMKLHM had lower and consistent MSE with low levels of multicollinearity and when beta is negative one. As the number of explanatory variables increased, the MNMKL1 and GMKLHM had low and consistent MSE but as the level of multicollinearity increased, the GMKLHM had the best performance across all levels of multicollinearity, beta and sample size.

Table 4.10: Best Ridge Parameter of KL, MKL1 and MKL2 in Poisson Regression Model.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | KL | | | MKL1 | | | MKL2 | | | |
| S/No | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | |
| P=4 | P=8 | P=4 | P=8 | P=4 | P=8 |
| 1 | AMKLHM | 38 | 30 | GMKL1MN | 58 | 58 | AMMKL2HM | 38 | 36 |
| 2 | MAKLMN | 37 | 26 | AMMKL1MN | 58 | 58 | GMMKL2 | 25 | 36 |
| 3 | GMKLMD | 24 | 35 | MNMKL1 | 58 | 58 | GMMKL2GM | 24 | 39 |
| 4 | GMKLHM | 25 | 34 | MNMKL1MN | 58 | 58 |  |  |  |
| 5 |  |  | | MNMKL1HM | 58 | 58 |  |  |  | |
| 6 |  |  | | MAMKL1MN | 58 | 58 |  |  |  | |
| 7 |  |  | | MRMKL1MN | 58 | 58 |  |  |  | |
| 8 |  |  | | MDMKL1MN | 58 | 58 |  |  |  | |
|  |  |  | | GMMKL1MN | 58 | 58 |  |  |  | |

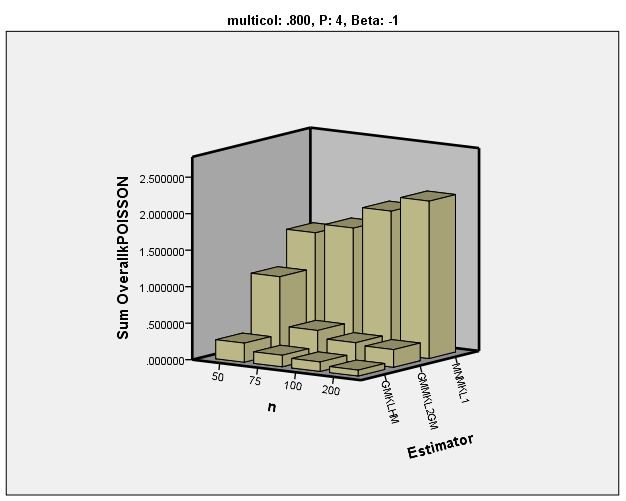


Figure 4.9: MSE of Best Performing Parameters when beta=-1, and P=4 for Poisson Regression.

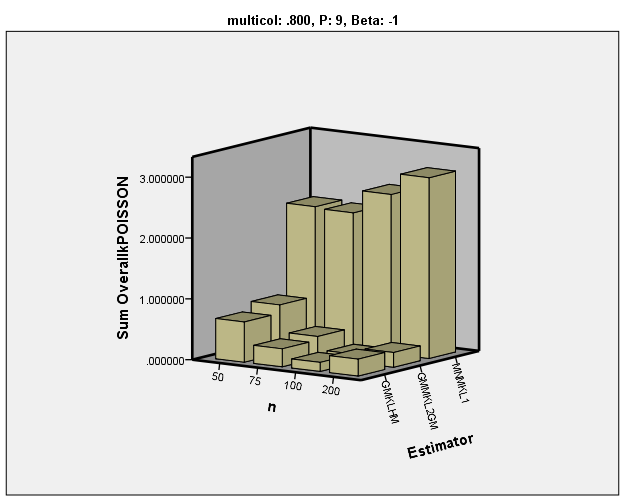


Figure 4.10: MSE of Best Performing Parameters when beta=-1, and P=8 for Poisson Regression.

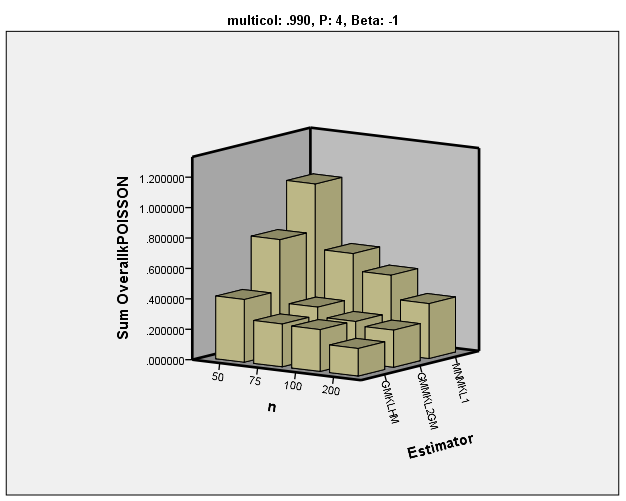


Figure 4.11: MSE of Best Performing Parameters when beta=-1, and P=8 for Poisson Regression.

### Comparison of Selected Best Parameters with Existing Parameters in Poisson Regression Model

When the selected best performing parameters for the KL, MKL1 and MKL2 were considered together with existing ridge parameters for the Poisson regression model, Table 4.11 shows that the MAKLMN had the highest frequency of 36 and was followed by AMKLHM with frequency of 33 when the number of explanatory variables is 4. When the number of explanatory variables is increased to 8, the GMKLHM had the highest frequency of 27. The AMMKL2HM and GMKLMD also had high frequency of 26 and 25 respectively. The parameters all performed well at lower samples sizes.

### Real Life Application for the Poisson Regression Model

The selected best ridge parameters for the KL, MKL1 and MKL2 were compared with the ridge estimator and the ordinary least squares estimator in a real life application for the Poisson regression model. The real-life results in

Table 4.12 shows that the MRKLHM version of the KL ridge parameter performed the best.

Table 4.11: Overall Performance of the Ridge Parameters for Poisson Regression

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | | n | | | | | | | | Total | |
| 50 | | 75 | | 100 | | 200 | |
| 4 | FAYINDE3 | | 0 | | 0 | | 0 | | 1 | | 1 | |
| ASIMOTA | | 4 | | 4 | | 3 | | 2 | | 13 | |
| FAYINDE1 | | 2 | | 3 | | 4 | | 3 | | 12 | |
| FAYINDE2 | | 2 | | 1 | | 1 | | 0 | | 4 | |
| FAYINDE3 | | 1 | | 1 | | 1 | | 0 | | 3 | |
| AMKLHM | | 11 | | 9 | | 7 | | 6 | | 33 | |
| MAKLMN | | 11 | | 9 | | 9 | | 7 | | 36 | |
| GMKLMD | | 5 | | 7 | | 4 | | 4 | | 20 | |
| GMKLHM | | 8 | | 4 | | 3 | | 4 | | 19 | |
| AMMKL2HM | | 3 | | 4 | | 4 | | 2 | | 13 | |
| GMMKL2 | | 1 | | 0 | | 1 | | 2 | | 4 | |
| GMMKL2GM | | 0 | | 0 | | 1 | | 1 | | 2 | |
| Total | | 48 | | 42 | | 38 | | 32 | | 160 | |
|  | n | | | | | | | | | |  | |
| Estimators | | 50 | | 75 | | 100 | | 200 | |  | |
| 8 | RIDGE | | 0 | | 0 | | 0 | | 2 | | 2 | |
| FAYINDE3 | | 0 | | 0 | | 1 | | 1 | | 2 | |
| ASIMOTA | | 5 | | 2 | | 3 | | 4 | | 14 | |
| FAYINDE1 | | 2 | | 1 | | 2 | | 4 | | 9 | |
| FAYINDE2 | | 2 | | 1 | | 0 | | 1 | | 4 | |
| FAYINDE3 | | 2 | | 1 | | 0 | | 0 | | 3 | |
| AMKLHM | | 3 | | 6 | | 7 | | 5 | | 21 | |
| MAKLMN | | 4 | | 4 | | 3 | | 1 | | 12 | |
| GMKLMD | | 8 | | 8 | | 8 | | 1 | | 25 | |
| GMKLHM | | 11 | | 8 | | 7 | | 1 | | 27 | |
| AMMKL2HM | | 9 | | 8 | | 7 | | 2 | | 26 | |
| GMMKL2 | | 6 | | 7 | | 4 | | 3 | | 20 | |
| GMMKL2GM | | 5 | | 7 | | 3 | | 1 | | 16 | |
| Total | 57 | | 53 | | 45 | | 26 | | 181 | |

Table 4.12: MSE of Selected Parameter for Real Life Application in the Poisson Regression Model.

|  |  |  |
| --- | --- | --- |
| Estimators | MSE | Rank of MSE |
| OLS | 1.0290 | 5.000 |
| RIDGE | 0.7269 | 4.000 |
| MRKLHM | 0.4160 | 1.000 |
| MKL1AM | 0.4884 | 2.000 |
| MAMKL2 | 0.5967 | 3.000 |

## Summary of Results of the Estimators with Logistic Regression Model

The summary of the three ridge parameters, KL, MKL1 and MKL2 in the logistic regression model are provided as follows.

### KL Result with Logistic Regression Model

The frequency of the KL ridge parameters whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity is presented in Table 4.13 as follows: The result shows that when the number of explanatory variables is 4 the MAKLMN had the highest frequency of 20, the performance was closely marked by the MRKLHM and AMKLHM with frequency 19. When the number of explanatory variables is increased to 8, the MAKLMN and the MRKLHM had the highest frequency of 20. The parameters had consistent frequency at all sample sizes across the number of explanatory variables. Figure 4.12 shows the performance of the best KL Ridge parameter for the logistic regression model. The MAKLMN and MRKLHM were consistent and had the highest frequency across the sample size and number of explanatory variables.

Table 4.13: Frequency of KL Parameters that Rank Between one and ten across all Sample Size of the KL Estimator in Logistic Regression Model

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total |  |
| 50 | 75 | 100 | 200 | Rank |
| 4 | GKL | 0 | 0 | 0 | 1 | 1 | 19 |
| GKLMD | 3 | 3 | 2 | 2 | 10 | 10 |
| GKLHM | 3 | 3 | 2 | 1 | 9 | 11 |
| AMKL | 0 | 0 | 1 | 2 | 3 | 15 |
| AMKLMN | 2 | 4 | 1 | 1 | 8 | 13 |
| AMKLMD | 0 | 0 | 5 | 4 | 9 | 11 |
| **AMKLHM** | **5** | **5** | **5** | **4** | **19** | **2** |
| **MAKLMN** | **5** | **5** | **5** | **5** | **20** | **1** |
| MAKLHM | 0 | 0 | 0 | 1 | 1 | 19 |
| MRKL | 0 | 0 | 0 | 1 | 1 | 19 |
| MRKLMN | 0 | 0 | 1 | 1 | 2 | 18 |
| MRKLMD | 1 | 0 | 4 | 3 | 8 | 13 |
| **MRKLHM** | **5** | **5** | **5** | **4** | **19** | **2** |
| MDKL | 0 | 0 | 1 | 2 | 3 | 15 |
| MDKLMN | 5 | 5 | 0 | 1 | 11 | 9 |
| MDKLMD | 0 | 0 | 1 | 2 | 3 | 15 |
| MDKLHM | 3 | 4 | 4 | 2 | 13 | 6 |
| GMKL | 3 | 3 | 3 | 4 | 13 | 6 |
| GMKLAM | 1 | 0 | 0 | 0 | 1 | 19 |
| GMKLMN | 0 | 0 | 0 | 1 | 1 | 19 |
| **GMKLMD** | **5** | **5** | **4** | **3** | **17** | **4** |
| GMKLGM | 4 | 3 | 3 | 2 | 12 | 8 |
| **GMKLHM** | **5** | **5** | **3** | **3** | **16** | **5** |
| Total | 50 | 50 | 50 | 50 | 200 |  |
|  |  | n | | | |  |  |
| Estimators | 50 | 75 | 100 | 200 |  |  |
| 8 | GKLMN | 2 | 1 | 0 | 0 | 3 | 21 |
| AMKL | 0 | 0 | 3 | 3 | 6 | 11 |
| AMKLMN | 5 | 1 | 0 | 0 | 6 | 11 |
| **AMKLMD** | **0** | **3** | **5** | **3** | **11** | **5** |
| AMKLGM | 0 | 0 | 3 | 5 | 8 | 9 |
| **AMKLHM** | **2** | **4** | **5** | **4** | **15** | **3** |
| MNKL | 0 | 1 | 0 | 0 | 1 | 28 |
| MNKLMN | 0 | 1 | 0 | 0 | 1 | 28 |
| MAKL | 0 | 0 | 0 | 1 | 1 | 28 |
| **MAKLMN** | **5** | **5** | **5** | **5** | **20** | **1** |
| MAKLMD | 0 | 0 | 0 | 3 | 3 | 21 |
| MAKLHM | 0 | 0 | 0 | 3 | 3 | 21 |
| MRKL | 0 | 0 | 1 | 4 | 5 | 16 |
| MRKLMN | 1 | 1 | 0 | 1 | 3 | 21 |
| **MRKLMD** | **0** | **4** | **5** | **3** | **12** | **4** |
| MRKLGM | 0 | 0 | 4 | 5 | 9 | 7 |
| **MRKLHM** | **5** | **5** | **5** | **5** | **20** | **1** |
| MDKL | 0 | 0 | 1 | 0 | 1 | 28 |
| MDKLMN | 5 | 1 | 0 | 0 | 6 | 11 |
| MDKLMD | 0 | 0 | 2 | 0 | 2 | 26 |
| MDKLGM | 0 | 0 | 0 | 1 | 1 | 28 |
| MDKLHM | 0 | 4 | 3 | 0 | 7 | 10 |
| GMKL | 0 | 3 | 2 | 1 | 6 | 11 |
| GMKLAM | 0 | 1 | 1 | 2 | 4 | 20 |
| GMKLMN | 2 | 1 | 0 | 0 | 3 | 21 |
| GMKLMD | 1 | 4 | 0 | 0 | 5 | 16 |
| GMKLGM | 0 | 4 | 5 | 1 | 10 | 6 |
| GMKLHM | 5 | 4 | 0 | 0 | 9 | 7 |
| HMKLAM | 5 | 0 | 0 | 0 | 5 | 16 |
| HMKLMN | 0 | 1 | 0 | 0 | 1 | 28 |
| HMKLMA | 5 | 1 | 0 | 0 | 6 | 11 |
| HMKLMR | 5 | 0 | 0 | 0 | 5 | 16 |
| HMKLGM | 2 | 0 | 0 | 0 | 2 | 26 |
|  | Total | 50 | 50 | 50 | 50 | 200 |  |

Figure 4.12:Best KL Ridge Parameter for Logistic Regression Model

### MKL1 Result with Logistic Regression Model

The frequency of the MKL1 ridge parameters whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity is presented in Table 4.14. The results of MKL1 ridge parameters show that the GMKL1MN, AMMKL1MN, MNMKL1, MNMKL1MN, MAMKL1MN, MRMKL1MN, MDMKL1MN, GMMKL1MN all had the highest and equal frequency of 20 when the number of explanatory variables is 4 and 8. The parameters all performed well at all the different sample sizes. Figure 4.13 shows the performance of the best MKL1 ridge parameter for the logistic regression model. The ridge parameters performed equally at all sample size and number of explanatory variables.

Table 4.14: Frequency of MKL1 Parameters that rank between one and ten across all Sample Size of the MKL Estimator in Logistic Regression Model

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total |
| 50 | 75 | 100 | 200 |
| 4 | GMKL1 | 1 | 1 | 1 | 1 | 4 |
| GMKL1MN | 5 | 5 | 5 | 5 | 20 |
| AMMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MNMKL1 | 5 | 5 | 5 | 5 | 20 |
| MNMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MAMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MRMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MDMKL1MN | 5 | 5 | 5 | 5 | 20 |
| GMMKL1MN | 5 | 5 | 5 | 5 | 20 |
| Total | 41 | 41 | 41 | 41 | 164 |
|  |  | n | | | |  |
| Estimator | 50 | 75 | 100 | 200 |  |
| 8 | GMKL1 | 4 | 2 | 2 | 1 | 9 |
| GMKL1MN | 5 | 5 | 5 | 5 | 20 |
| AMMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MNMKL1 | 5 | 5 | 5 | 5 | 20 |
| MNMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MAMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MRMKL1MN | 5 | 5 | 5 | 5 | 20 |
| MDMKL1MN | 5 | 5 | 5 | 5 | 20 |
| GMMKL1MN | 5 | 5 | 5 | 5 | 20 |
|  | Total | 44 | 42 | 42 | 41 | 169 |

Figure 4.13: Best Performing MKL1 Ridge Parameter for Logistic Regression Model

### MKL2 Result with Logistic Regression Model

The frequency of the MKL2 ridge parameters whose MSE ranked between 1 and 10 when counted over the levels of multicollinearity is presented in Table 4.15. The simulation result show that the AMMKL2HM and GMMKL2 had the highest frequency of 19 when the number of explanatory variables are 4 and when the number of explanatory variables are 8 the MRMKL2MD and AMMKL2GM had the highest frequency of 17. All the parameters remained consistent at all sample sizes while the frequency of AMMKL2GM increased as the sample size also increased. Figure 4.14 show performance of best performing MKL2 Ridge parameter for the Poisson Regression model. The MRMKL2HM had the highest frequency across the sample size and number of explanatory variables.

Table 4.15: Frequency of Best Performing Parameters for MKL2 Ridge Parameter in Logistic Regression Model

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | Total | Rank |
| 50 | 75 | 100 | 200 |
| 4 | AMMKL2 | 0 | 0 | 0 | 2 | 2 | 18 |
| AMMKL2MN | 3 | 4 | 3 | 3 | 13 | 8 |
| **AMMKL2MD** | **4** | **3** | **4** | **3** | **14** | **5** |
| AMMKL2GM | 0 | 0 | 0 | 1 | 1 | 20 |
| **AMMKL2HM** | **5** | **5** | **5** | **4** | **19** | **1** |
| MNMKL2MN | 0 | 0 | 0 | 1 | 1 | 20 |
| MNMKL2MD | 0 | 0 | 0 | 1 | 1 | 20 |
| MNMKL2HM | 0 | 0 | 0 | 1 | 1 | 20 |
| MAMKL2MD | 1 | 0 | 0 | 0 | 1 | 20 |
| MAMKL2HM | 3 | 2 | 3 | 2 | 10 | 10 |
| MRMKL2MD | 4 | 3 | 3 | 2 | 12 | 9 |
| **MRMKL2HM** | **5** | **5** | **4** | **3** | **17** | **3** |
| MDMKL2 | 0 | 1 | 1 | 2 | 4 | 15 |
| MDMKL2MN | 2 | 2 | 2 | 3 | 9 | 11 |
| MDMKL2GM | 0 | 1 | 0 | 0 | 1 | 20 |
| MDMKL2HM | 1 | 3 | 2 | 2 | 8 | 12 |
| **GMMKL2** | **5** | **4** | **5** | **5** | **19** | **1** |
| GMMKL2AM | 1 | 1 | 2 | 2 | 6 | 14 |
| GMMKL2MN | 1 | 2 | 2 | 2 | 7 | 13 |
| GMMKL2MA | 1 | 0 | 1 | 1 | 3 | 17 |
| GMMKL2MR | 1 | 1 | 1 | 1 | 4 | 15 |
| **GMMKL2MD** | **4** | **5** | **4** | **3** | **16** | **4** |
| **GMMKL2GM** | **5** | **4** | **3** | **2** | **14** | **5** |
| **GMMKL2HM** | **4** | **4** | **4** | **2** | **14** | **5** |
| HMMKL2MN | 0 | 0 | 1 | 1 | 2 | 18 |
| HMMKL2HM | 0 | 0 | 0 | 1 | 1 | 20 |
| Total | 50 | 50 | 50 | 50 | 200 |  |
|  | n | | | | |  |
| Estimators | 50 | 75 | 100 | 200 |  |  |
| 8 | AMMKL2 | 0 | 3 | 4 | 3 | 10 | 9 |
| AMMKL2MN | 1 | 1 | 0 | 1 | 3 | 16 |
| AMMKL2MD | 4 | 4 | 3 | 2 | 13 | 6 |
| **AMMKL2GM** | **3** | **4** | **5** | **5** | **17** | **1** |
| AMMKL2HM | 4 | 2 | 2 | 3 | 11 | 7 |
| MAMKL2 | 0 | 0 | 0 | 1 | 1 | 22 |
| MAMKL2MD | 2 | 2 | 3 | 3 | 10 | 9 |
| MAMKL2GM | 0 | 0 | 0 | 1 | 1 | 22 |
| MAMKL2HM | 2 | 3 | 3 | 3 | 11 | 7 |
| MRMKL2 | 0 | 2 | 3 | 4 | 9 | 11 |
| **MRMKL2MD** | **4** | **4** | **5** | **4** | **17** | **1** |
| **MRMKL2GM** | **3** | **4** | **4** | **4** | **15** | **4** |
| **MRMKL2HM** | **4** | **4** | **4** | **4** | **16** | **3** |
| MDMKL2 | 0 | 2 | 1 | 0 | 3 | 16 |
| MDMKL2MN | 1 | 0 | 0 | 0 | 1 | 22 |
| MDMKL2GM | 1 | 3 | 2 | 0 | 6 | 15 |
| MDMKL2HM | 2 | 1 | 0 | 0 | 3 | 16 |
| GMMKL2 | 4 | 2 | 1 | 0 | 7 | 14 |
| GMMKL2AM | 2 | 4 | 4 | 5 | 15 | 4 |
| GMMKL2MN | 1 | 0 | 0 | 0 | 1 | 22 |
| GMMKL2MA | 0 | 0 | 1 | 2 | 3 | 16 |
| GMMKL2MR | 0 | 1 | 3 | 4 | 8 | 12 |
| GMMKL2MD | 2 | 1 | 0 | 0 | 3 | 16 |
| GMMKL2GM | 3 | 2 | 2 | 1 | 8 | 12 |
| GMMKL2HM | 2 | 1 | 0 | 0 | 3 | 16 |
| HMMKL2AM | 1 | 0 | 0 | 0 | 1 | 22 |
| HMMKL2MR | 1 | 0 | 0 | 0 | 1 | 22 |
| HMMKL2MD | 1 | 0 | 0 | 0 | 1 | 22 |
| HMMKL2GM | 1 | 0 | 0 | 0 | 1 | 22 |
| HMMKL2HM | 1 | 0 | 0 | 0 | 1 | 22 |
|  | Total | 50 | 50 | 50 | 50 | 200 |  |

Figure 4.14:Best performing MKL2 Ridge Parameter for the Logistic Regression Model

### Combined result of KL, MKL1 and MKL2 for Logistic Regression Model

Table 4.16 shows the parameters with the highest frequency for the KL, MKL1 and MKL2 in logistic regression model. The MAKLMN, GMKL1MN and AMMKL2HM were selected for the KL, MKL1 and MKL2 estimators respectively. Figure 4.15 and Figure 4.16 shows that at low levels of multicollinearity and when the number of explanatory variables is 4 and 8, the AMMKL2HM and MAKLMN had lower and consistent MSE. As the level of multicollinearity increases, the MAKLMN had high MSE across the sample sizes. In general, the AMMKL2HM version of MKL2 had the best and consistent performance across all levels of multicollinearity and sample size.

Table 4.16: Performance of KL, MKL1 and MKL2 Parameters in Logistic Regression Model.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | KL | | | MKL1 | | | MKL2 | | |
| S/No | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | | Ridge Parameter | Frequency | |
| P=4 | P=8 | P=4 | P=8 | P=4 | P=8 |
| 1 | MAKLMN | 20 | 20 | GMKL1MN | 20 | 20 | AMMKL2HM | 19 | 8 |
| 2 | MRKLHM | 19 | 20 | AMMKL1MN | 20 | 20 | GMMKL2 | 19 | 7 |
| 3 | AMKLHM | 19 | 15 | MNMKL1 | 20 | 20 | AMMKL2GM | 1 | 17 |
| 4 |  |  | | MNMKL1MN | 20 | 20 | MRMKL2MD | 12 | 17 |
| 5 |  |  | | MAMKL1MN | 20 | 20 |  |  | |
| 6 |  |  | | MRMKL1MN | 20 | 20 |  |  | |
| 7 |  |  | | MDMKL1MN | 20 | 20 |  |  | |
| 8 |  |  | | GMMKL1MN | 20 | 20 |  |  | |

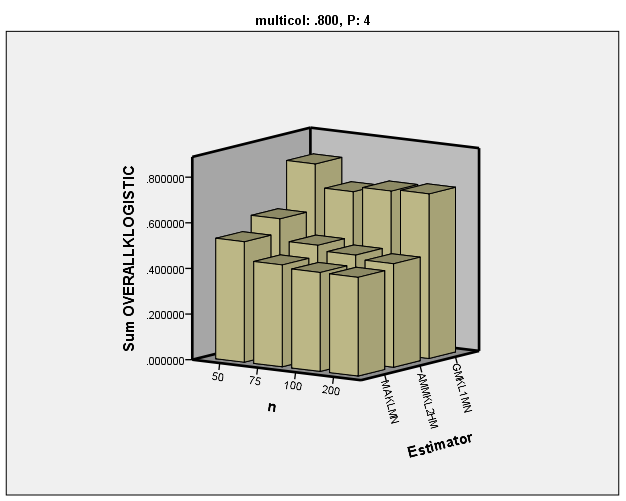


Figure 4.15: MSE of Best Performing Parameters when and P=4 for Logistic Regression.

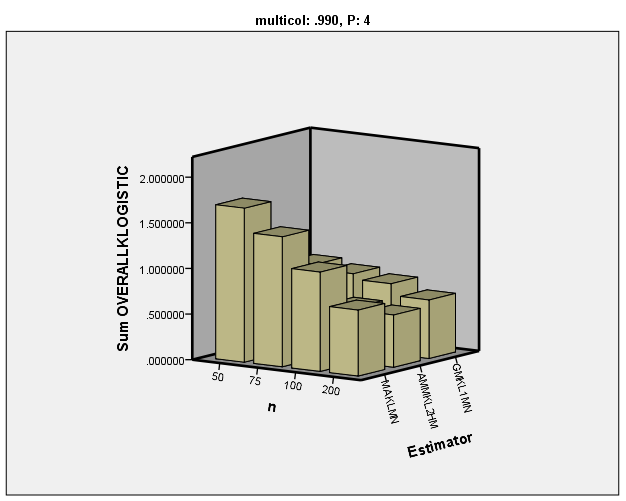


Figure 4.16: MSE of Best Performing Parameters when ρ=0.99 and P=4 for Logistic Regression.

### Comparison of Selected Best Parameter in Logistic Regression Model with Existing Parameters

The selected best performing ridge parameter for KL, MKL1 and MKL2 in the logistic regression model was compared with existing ridge parameter so as to compare their performance. As shown in Table 4.17, the AMMKL2HM was observed to have the highest frequency of 15 when the number of explanatory variables is 4. The parameter had the arithmetic mean form of the MKL2 ridge parameter. When the number of explanatory variables is 8, the AMMKL2GM had the highest frequency of 13 which also had the arithmetic mean for of the MKL2 ridge parameter. The parameters were consistent at all sample sizes.

Table 4.17: Overall Performance of the Ridge Parameters for Logistic Regression.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| P | Estimators | n | | | | | | | | Total | |
| 50 | | 75 | | 100 | | 200 | |
| 4 | ASIMOTA | | 2 | | 2 | | 2 | | 2 | | 8 | |
| FAYINDE | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE1 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE2 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE3 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| AMKLHM | | 2 | | 1 | | 2 | | 2 | | 7 | |
| MAKLMN | | 2 | | 2 | | 2 | | 3 | | 9 | |
| MRKLHM | | 2 | | 2 | | 2 | | 2 | | 8 | |
| AMMKL2GM | | 0 | | 0 | | 0 | | 1 | | 1 | |
| AMMKL2HM | | 4 | | 4 | | 4 | | 3 | | 15 | |
| MRMKL2MD | | 3 | | 2 | | 3 | | 2 | | 10 | |
| GMMKL2 | | 3 | | 3 | | 3 | | 4 | | 13 | |
| Total | | 22 | | 20 | | 22 | | 23 | | 87 | |
|  | n | | | | | | | | | |  | |
|  | | 50 | | 75 | | 100 | | 200 | |  | |
| Estimators | |  | |  | |  | |  | |  | |
| 8 | ASIMOTA | | 1 | | 1 | | 2 | | 2 | | 6 | |
| FAYINDE | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE1 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE2 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| FAYINDE3 | | 1 | | 1 | | 1 | | 1 | | 4 | |
| AMKLHM | | 0 | | 1 | | 1 | | 2 | | 4 | |
| MAKLMN | | 2 | | 1 | | 2 | | 3 | | 8 | |
| MRKLHM | | 0 | | 1 | | 2 | | 2 | | 5 | |
| MKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| AMMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| MNMKL1 | | 0 | | 1 | | 0 | | 0 | | 1 | |
| MNMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| MAMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| MRMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| MDMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| GMMKL1MN | | 0 | | 1 | | 0 | | 0 | | 1 | |
| AMMKL2GM | | 3 | | 4 | | 4 | | 2 | | 13 | |
| AMMKL2HM | | 3 | | 1 | | 0 | | 0 | | 4 | |
| MRMKL2MD | | 4 | | 2 | | 3 | | 1 | | 10 | |
| GMMKL2 | | 4 | | 0 | | 1 | | 0 | | 5 | |
|  | Total | | 21 | | 23 | | 19 | | 16 | | 79 | |

### Real Life Application for Logistic Regression Model

The real-life results for the logistic regression model are shown in Table 4.18. The selected best performing parameter for the KL, MKL1 and MKL2 is compared with ordinary least squares and the ridge parameter for the logistic regression model. The MRKLHM version of the KL ridge parameter had the lowest MSE and was closely marked by the MAMKL2 version of the MKL2 ridge parameter.

Table 4.18: MSE of Selected Parameter for Real life Application in logistic regression Model

|  |  |  |
| --- | --- | --- |
| Estimators | MSE | Rank of MSE |
| OLS | 21.3514 | 5.000 |
| RIDGE2 | 11.0435 | 4.000 |
| MRKLHM | 0.3967 | 1.000 |
| MKL1AM | 0.7023 | 3.000 |
| MAMKL2 | 0.6985 | 2.000 |



# SUMMARY, CONCLUSION AND RECOMMENDATIONS

# Introduction

In this chapter, we have the summary, conclusion and recommendations on the proposed ridge estimators and parameters for the linear and generalized regression models.

## Summary

In this research, a new one-parameter estimator for estimating parameters in the presence of multicollinearity was developed. New ridge parameters based on the Kibria-Lukman (KL) estimator and the Modified KL estimators were also introduced. The ridge parameters were all introduced into the logistic and Poisson regression models.

Background study on the linear regression models and the generalized regression models were discussed. The assumptions of the LRM and the GLM alongside the effect of violating the assumptions were extensively studied. The various means of detecting the presence of multicollinearity were also discussed in this study.

Reviews done on different forms of estimators involving the estimations with multicollinearity challenge was carried out both in LRM and the GLM. The varying forms and types of the KL ridge parameter and the MKL ridge parameters were introduced to the study. The newly developed estimator was theoretically compared with other one parameter estimators such as the ridge estimator by Hoerl and Kennard (1970) and the KL estimator by (Kibria and Lukman, 2020).

Simulation and real-life studies were carried out in order to determine the performance of the estimator and the various versions of the ridge parameters.

## Conclusion

A one-parameter estimator was developed to estimate parameters in the presence of multicollinearity. The simulation study was carried out using the R-studio 4.1.0. More statistical analysis were done using the IBM SPSS statistics 23 package and the following conclusions were made.

1. By replacing the with the in the KL estimator from Kibria and Lukman (2020), the new modified KL (MKL) estimator was developed as a new one-parameter estimator which can estimate parameters in the presence of multicollinearity more efficiently and consistently. The MKL was also introduced to the Poisson and logistic regression models.

New versions of the ridge parameter for the KL and MKL estimators were also proposed. The different versions of the mean, maximum, minimum, midrange, median, geometric mean and the harmonic mean for the eigen value and the estimator were obtained.

1. The ridge parameters for the KL and the MKL estimators were examined through simulation studies in LRM and the GLM and were observed to perform well at different instances.
2. The MSE of the ridge parameters were ranked across all levels of multicollinearity, variance, sample size and number of explanatory variables in the LRM and GLM. The parameters that ranked between 1 and 10 were selected. The parameter(s) that had the highest frequency after being ranked between 1 and 10 for the KL estimator was considered as the most efficient parameters for the KL estimator. The parameter(s) that had the highest frequency after being ranked between 1 and 10 for the MKL1 and MKL2 parameters were also considered the most efficient parameters for the MKL estimator. The different forms and types of these ridge parameters for KL, MKL1 and MKL2 were also extended to the Poisson and logistic regression models. Versions of MKL2 ridge parameter performed well for the Linear and Logistic regression models while the versions of KL ridge parameter performed well for the Poisson regression model.
3. The parameters that had the best performance for the KL, MKL1 and MKL2 were applied to real life data set so as to further investigate their performance and determine the most efficient and consistent estimators in the CLRM and the GLMs. The parameters performed well in real life study.

The objectives of the study were achieved as the new estimator and the different versions of the ridge parameters proposed performed well for the LRM and the GLM.

## Recommendations

The recommendations from this research are as follow:

1. The Modified KL estimator can be used for estimating regression parameters in the presence of multicollinearity for the linear regression model and the generalized linear regression models.
2. The MAMKL2 version for the MKL2 ridge parameter can specifically be adopted in the linear regression model for estimating regression parameters in the presence of multicollinearity.
3. For the Poison regression model, the GMKLHM version of the KL ridge parameter could be used to estimate parameters in the presence of multicollinearity.
4. The AMMKL2HM version for the MKL2 ridge parameter is recommended for use in the Logistic regression model for estimating parameters in the presence of multicollinearity.

## Contributions to Knowledge

The following contributions were made:

1. A new estimator was developed and named as the modified KL (MKL) estimator in the Linear regression model to estimate parameters more efficiently when multicollinearity is present.
2. The MKL estimator was extended to the Poisson and Logistic regression models to estimate parameters more efficiently when multicollinearity is a challenge.
3. Various forms and types of the ridge parameters such as AM, MN, MA, MR, MD, GM and HM for the KL, MKL1 and MKL2 ridge parameters were obtained in this study. The best versions of these parameters were identified in the linear, Poisson and the Logistic regression models.

## Further study

Further study can be carried out by introducing different forms of the MKL estimator such as the unbiased MKL estimator and Principal component MKL estimator. There can also be further extension of the estimator to other forms of generalized linear regression models such as Gamma regression, Inverse Gaussian regression, Negative Binomial regression among others. The different forms and types of the MKL ridge parameters can also be introduced to other forms the generalized linear regression model.

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APPENDIX 1: MSE of the Ridge Parameters when P=4, n=10 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GKMN | 1.0148 | .3524 | 1.0105 | 66.000 | 4.500 | 63.000 |
| AMKMN | .5694 | .3524 | .5268 | 34.000 | 4.500 | 38.000 |
| AMKMD | .3767 | .7998 | .3736 | 5.000 | 24.500 | 10.000 |
| MNK | 1.0543 | .3524 | 1.0554 | 68.000 | 4.500 | 69.000 |
| MNKMN | 1.0872 | .3524 | 1.0829 | 70.000 | 4.500 | 71.000 |
| MAK | .4523 | 1.2037 | .3060 | 12.000 | 66.500 | 2.000 |
| MAKMN | .3481 | .3524 | .5618 | 2.000 | 4.500 | 39.500 |
| MAKMD | .4717 | .7998 | .3400 | 20.000 | 24.500 | 3.000 |
| MAKGM | .5016 | .8316 | .3486 | 28.000 | 34.500 | 4.000 |
| MAKHM | .3384 | .6155 | .3030 | 1.000 | 14.500 | 1.000 |
| MRKMN | .5470 | .3524 | .4935 | 33.000 | 4.500 | 33.000 |
| MRKMD | .3751 | .7998 | .3993 | 4.000 | 24.500 | 5.000 |
| MRKLHM | .3708 | .6155 | .3705 | 3.000 | 14.500 | 8.000 |
| MDKMN | .6070 | .3524 | .5901 | 35.000 | 4.500 | 41.000 |
| GMKMN | .6664 | .3524 | .6365 | 42.000 | 4.500 | 43.000 |
| .9 | ASIMOTA | .3535 | .3535 | .3535 | 1.000 | 9.000 | 4.000 |
| GKMN | 1.8986 | .3080 | 1.8819 | 67.000 | 4.500 | 67.000 |
| AMKMN | .9102 | .3080 | .8161 | 33.000 | 4.500 | 42.000 |
| MNK | 1.9702 | .3080 | 1.9668 | 69.000 | 4.500 | 69.000 |
| MNKMN | 2.0279 | .3080 | 2.0163 | 70.000 | 4.500 | 71.000 |
| MAK | .8399 | 1.1950 | .3417 | 30.000 | 64.500 | 1.000 |
| MAKMN | .5191 | .3080 | .5403 | 3.000 | 4.500 | 22.500 |
| MAKMD | .7080 | .5669 | .3519 | 22.000 | 21.500 | 3.000 |
| MAKGM | .8701 | .6229 | .3641 | 31.000 | 31.500 | 5.000 |
| MAKHM | .4702 | .4669 | .3446 | 2.000 | 13.500 | 2.000 |
| MRKMN | .9133 | .3080 | .5280 | 34.000 | 4.500 | 21.000 |
| MRKMD | .5199 | .5669 | .4472 | 4.000 | 21.500 | 8.000 |
| MRKLHM | .5232 | .4669 | .5023 | 5.000 | 13.500 | 18.000 |
| MDKMN | .9297 | .3080 | .8832 | 35.000 | 4.500 | 43.000 |
| GMKMN | 1.1045 | .3080 | 1.0311 | 43.000 | 4.500 | 45.000 |
| .95 | ASIMOTA | .3367 | .3367 | .3367 | 1.000 | 1.000 | 1.000 |
| GFA | .7444 | .7444 | .7444 | 4.000 | 39.000 | 28.000 |
| FAPR | .6993 | .6993 | .6993 | 2.000 | 37.000 | 23.000 |
| FAMR | .7069 | .7069 | .7069 | 3.000 | 38.000 | 24.000 |
| MAK | 1.8359 | 1.1905 | .4162 | 38.000 | 64.500 | 2.000 |
| MAKMD | 1.1660 | .4914 | .4196 | 23.000 | 21.500 | 3.000 |
| MAKGM | 1.7468 | .5191 | .4234 | 34.000 | 30.500 | 4.000 |
| MAKHM | .7451 | .4539 | .4410 | 5.000 | 13.500 | 5.000 |
| .99 | ASIMOTA | .4403 | .4403 | .4403 | 2.000 | 2.000 | 2.000 |
| GFA | 1.1760 | 1.1760 | 1.1760 | 5.000 | 59.000 | 24.000 |
| FAPR | .5938 | .5938 | .5938 | 4.000 | 15.000 | 4.000 |
| FASR | .4939 | .4939 | .4939 | 3.000 | 5.000 | 3.000 |
| FAMR | .4030 | .4030 | .4030 | 1.000 | 1.000 | 1.000 |
| MAKAM | 21.9960 | .9016 | .6421 | 69.000 | 44.500 | 5.000 |
| HMKMD | 8.8067 | .4512 | 8.9449 | 35.000 | 4.000 | 50.000 |
| HMKGM | 8.2671 | .4446 | 8.3046 | 32.000 | 3.000 | 49.000 |
| .999 | ASIMOTA | .7467 | .7467 | .7467 | 4.000 | 5.000 | 9.000 |
| GFA | 1.6376 | 1.6376 | 1.6376 | 5.000 | 43.000 | 15.000 |
| FAPR | .3935 | .3935 | .3935 | 3.000 | 4.000 | 3.000 |
| FASR | .3327 | .3327 | .3327 | 2.000 | 2.000 | 2.000 |
| FAMR | .3284 | .3284 | .3284 | 1.000 | 1.000 | 1.000 |
| MAKMR | 295.3992 | 1.1169 | .6801 | 70.000 | 27.500 | 4.000 |
| HMKGM | 78.7683 | .3805 | 77.8864 | 23.000 | 3.000 | 49.000 |

APPENDIX 2: MSE of the Ridge Parameters when P=4, n=10 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.1793 | 1.1793 | 1.1793 | 1.000 | 45.000 | 19.000 |
| GKMN | 8.7697 | .7289 | 8.6571 | 65.000 | 4.500 | 65.000 |
| AMKMN | 3.4799 | .7289 | 2.9126 | 25.000 | 4.500 | 39.000 |
| AMKHM | 1.8643 | .8279 | 1.6228 | 4.000 | 12.500 | 26.000 |
| MNK | 9.1478 | .7289 | 9.1122 | 67.000 | 4.500 | 68.000 |
| MNKMN | 9.3873 | .7289 | 9.3204 | 68.000 | 4.500 | 69.000 |
| MAKAM | 6.5039 | 1.1337 | .8779 | 47.000 | 39.500 | 3.000 |
| MAKMN | 1.8957 | .7289 | .9604 | 6.000 | 4.500 | 7.500 |
| MAKMR | 7.9590 | 1.2062 | .9091 | 55.000 | 50.500 | 4.000 |
| MAKMD | 3.3138 | .9273 | .8611 | 21.000 | 20.500 | 2.000 |
| MAKGM | 3.6178 | .9469 | .8585 | 26.000 | 29.500 | 1.000 |
| MAKHM | 1.8873 | .8279 | .9188 | 5.000 | 12.500 | 5.000 |
| MRKMN | 3.6635 | .7289 | 1.0311 | 27.000 | 4.500 | 12.000 |
| MRKMD | 1.7876 | .9273 | 1.1433 | 3.000 | 20.500 | 18.000 |
| MRKLHM | 1.6821 | .8279 | 1.4394 | 2.000 | 12.500 | 23.000 |
| MDKMN | 3.4401 | .7289 | 3.1004 | 24.000 | 4.500 | 40.000 |
| GMKMN | 4.5027 | .7289 | 4.0249 | 38.000 | 4.500 | 43.000 |
| .900 | ASIMOTA | 2.6103 | 2.6103 | 2.6103 | 3.000 | 66.000 | 32.000 |
| GFA | 1.3568 | 1.3568 | 1.3568 | 1.000 | 64.000 | 17.000 |
| GKGM | 14.4728 | .8344 | 14.6072 | 52.000 | 4.500 | 57.000 |
| AMKGM | 4.0243 | .8344 | 1.6138 | 14.000 | 4.500 | 21.000 |
| AMKHM | 2.9513 | .8724 | 2.3886 | 4.000 | 21.500 | 28.000 |
| MNKGM | 16.0303 | .8344 | 16.2303 | 60.000 | 4.500 | 65.000 |
| MAK | 17.7476 | 1.2038 | .9985 | 68.000 | 59.500 | 5.000 |
| MAKAM | 16.8889 | 1.0350 | .9485 | 65.000 | 39.500 | 1.000 |
| MAKMN | 3.1318 | .9451 | .9778 | 6.000 | 30.500 | 3.500 |
| MAKMA | 21.1693 | 1.2038 | .9778 | 71.000 | 59.500 | 3.500 |
| MAKMR | 19.3265 | 1.1640 | .9495 | 70.000 | 50.500 | 2.000 |
| MAKGM | 8.5027 | .8344 | 1.0374 | 34.000 | 4.500 | 6.000 |
| MRKMD | 3.0019 | .8415 | 1.4997 | 5.000 | 13.500 | 20.000 |
| MRKGM | 3.9931 | .8344 | 1.3737 | 12.000 | 4.500 | 18.000 |
| MRKLHM | 2.5916 | .8724 | 2.0549 | 2.000 | 21.500 | 23.000 |
| MDKGM | 4.8388 | .8344 | 2.6084 | 18.000 | 4.500 | 31.000 |
| GMK | 3.9947 | .8344 | 2.9865 | 13.000 | 4.500 | 34.000 |
| GMKGM | 3.8920 | .8344 | 3.2421 | 10.000 | 4.500 | 36.000 |
| .950 | ASIMOTA | 1.4788 | 1.4788 | 1.4788 | 1.000 | 64.000 | 14.000 |
| GFA | 1.5468 | 1.5468 | 1.5468 | 2.000 | 65.000 | 17.000 |
| FAPR | 4.0996 | 4.0996 | 4.0996 | 3.000 | 67.000 | 32.000 |
| MAK | 42.0110 | 1.1969 | 1.0796 | 69.000 | 51.500 | 5.000 |
| MAKAM | 40.7009 | .9686 | 1.0358 | 68.000 | 15.500 | 4.000 |
| MAKMN | 5.6338 | 1.3804 | 1.0182 | 7.000 | 59.500 | 2.500 |
| MAKMA | 46.1790 | 1.1969 | 1.0182 | 71.000 | 51.500 | 2.500 |
| MAKMR | 44.0811 | 1.1409 | 1.0071 | 70.000 | 33.500 | 1.000 |
| MRKMD | 5.1449 | 1.0281 | 2.1429 | 5.000 | 24.500 | 21.000 |
| MRKLHM | 4.3830 | 1.1644 | 3.1496 | 4.000 | 42.500 | 24.000 |
| HMKMD | 16.0158 | .7671 | 16.2502 | 34.000 | 1.000 | 50.000 |
| HMKGM | 15.4697 | .8540 | 15.5991 | 32.000 | 3.000 | 49.000 |
| HMKHM | 17.1440 | .7870 | 17.2736 | 35.500 | 2.000 | 51.000 |
| .990 | ASIMOTA | .5739 | .5739 | .5739 | 1.000 | 1.000 | 1.000 |
| GFA | 1.8761 | 1.8761 | 1.8761 | 2.000 | 40.000 | 12.000 |
| FAPR | 2.5762 | 2.5762 | 2.5762 | 3.000 | 41.000 | 17.000 |
| FASR | 6.8531 | 6.8531 | 6.8531 | 5.000 | 69.000 | 25.000 |
| FAMR | 4.1435 | 4.1435 | 4.1435 | 4.000 | 59.000 | 19.000 |
| MAK | 262.6462 | 1.1916 | 1.1412 | 69.000 | 26.500 | 5.000 |
| MAKMN | 25.4366 | 4.3239 | 1.0828 | 10.000 | 63.500 | 2.500 |
| MAKMA | 266.0415 | 1.1916 | 1.0828 | 71.000 | 26.500 | 2.500 |
| MAKMR | 263.4016 | 1.1228 | 1.0934 | 70.000 | 16.500 | 4.000 |
| HMKGM | 72.8668 | .6210 | 72.8196 | 28.000 | 2.000 | 49.000 |
| .999 | ASIMOTA | .7649 | .7649 | .7649 | 2.000 | 2.000 | 6.000 |
| GFA | 2.0813 | 2.0813 | 2.0813 | 5.000 | 33.000 | 16.000 |
| FAPR | .9823 | .9823 | .9823 | 3.000 | 12.000 | 8.000 |
| FASR | 1.2265 | 1.2265 | 1.2265 | 4.000 | 32.000 | 14.000 |
| FAMR | .6864 | .6864 | .6864 | 1.000 | 1.000 | 1.000 |
| MAKAM | 2797.6404 | .8938 | .7603 | 68.000 | 7.500 | 3.000 |
| MAKMN | 245.2628 | 34.1287 | .7618 | 10.000 | 64.500 | 4.500 |
| MAKMA | 2806.3532 | 1.1903 | .7618 | 71.000 | 27.500 | 4.500 |
| MAKMR | 2803.4313 | 1.1188 | .7543 | 69.000 | 17.500 | 2.000 |
| HMKGM | 707.2739 | .8552 | 698.7350 | 23.000 | 3.000 | 49.000 |

APPENDIX 3: MSE of the Ridge Parameters when P=4, n=10 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.7541 | 1.7541 | 1.7541 | 1.000 | 64.000 | 10.000 |
| AMKHM | 4.6325 | 1.2534 | 3.7831 | 4.000 | 41.500 | 27.000 |
| MAK | 23.8272 | 1.2424 | 1.5665 | 63.000 | 33.500 | 5.000 |
| MAKAM | 18.9332 | 1.2149 | 1.4913 | 47.000 | 24.500 | 4.000 |
| MAKMN | 4.8671 | 1.4884 | 1.3874 | 5.000 | 59.500 | 1.500 |
| MAKMA | 27.6946 | 1.2424 | 1.3874 | 69.000 | 33.500 | 1.500 |
| MAKMR | 23.1741 | 1.2539 | 1.4267 | 60.000 | 49.500 | 3.000 |
| MRKMD | 4.6149 | 1.1803 | 2.4467 | 3.000 | 15.500 | 20.000 |
| MRKLHM | 4.1030 | 1.2534 | 3.2833 | 2.000 | 41.500 | 23.000 |
| HMKMA | 11.6519 | 1.0994 | 11.4129 | 32.000 | 1.000 | 44.000 |
| HMKMR | 11.6737 | 1.1660 | 11.5652 | 33.000 | 2.000 | 45.000 |
| .900 | ASIMOTA | 7.1375 | 7.1375 | 7.1375 | 3.000 | 66.000 | 34.000 |
| GFA | 1.9583 | 1.9583 | 1.9583 | 1.000 | 56.000 | 8.000 |
| AMKHM | 7.5772 | 1.6839 | 5.7190 | 4.000 | 51.500 | 29.000 |
| MAK | 58.0413 | 1.2197 | 1.6497 | 70.000 | 26.500 | 5.000 |
| MAKAM | 50.6900 | 1.0972 | 1.6484 | 67.000 | 6.500 | 4.000 |
| MAKMN | 8.1926 | 2.2274 | 1.4404 | 5.000 | 60.500 | 1.500 |
| MAKMA | 63.4999 | 1.2197 | 1.4404 | 71.000 | 26.500 | 1.500 |
| MAKMR | 58.0394 | 1.1919 | 1.5098 | 69.000 | 17.500 | 3.000 |
| MRKLHM | 6.5142 | 1.6839 | 4.7817 | 2.000 | 51.500 | 23.000 |
| HMKMD | 22.8131 | 1.0744 | 23.1299 | 32.000 | 2.000 | 48.000 |
| HMKHM | 24.5136 | 1.0743 | 24.7001 | 34.500 | 1.000 | 49.000 |
| .950 | ASIMOTA | 3.7776 | 3.7776 | 3.7776 | 2.000 | 65.000 | 18.000 |
| GFA | 2.1411 | 2.1411 | 2.1411 | 1.000 | 48.000 | 8.000 |
| FAPR | 9.5663 | 9.5663 | 9.5663 | 3.000 | 67.000 | 29.000 |
| AMKHM | 13.6471 | 2.5849 | 9.5604 | 5.000 | 52.500 | 28.000 |
| MAK | 132.3353 | 1.2084 | 1.7025 | 70.000 | 25.500 | 4.000 |
| MAKAM | 121.8651 | 1.0089 | 1.7822 | 68.000 | 6.500 | 5.000 |
| MAKMN | 15.0292 | 3.4600 | 1.4986 | 7.000 | 60.500 | 1.500 |
| MAKMA | 138.4028 | 1.2084 | 1.4986 | 71.000 | 25.500 | 1.500 |
| MAKMR | 132.1176 | 1.1568 | 1.5823 | 69.000 | 15.500 | 3.000 |
| MRKLHM | 11.3802 | 2.5849 | 7.5645 | 4.000 | 52.500 | 24.000 |
| HMKMD | 44.1565 | .9933 | 44.7419 | 33.000 | 2.000 | 49.000 |
| HMKGM | 42.6413 | .9445 | 42.9216 | 31.000 | 1.000 | 48.000 |
| .990 | ASIMOTA | .8455 | .8455 | .8455 | 1.000 | 1.000 | 1.000 |
| GFA | 2.4062 | 2.4062 | 2.4062 | 2.000 | 31.000 | 9.000 |
| FAPR | 5.6043 | 5.6043 | 5.6043 | 3.000 | 41.000 | 17.000 |
| FASR | 30.0458 | 30.0458 | 30.0458 | 5.000 | 69.000 | 31.000 |
| FAMR | 16.6134 | 16.6134 | 16.6134 | 4.000 | 68.000 | 23.000 |
| MAK | 759.6313 | 1.1998 | 1.6234 | 70.000 | 25.500 | 5.000 |
| MAKMN | 69.8961 | 11.6193 | 1.5176 | 10.000 | 62.500 | 2.500 |
| MAKMA | 763.8076 | 1.1998 | 1.5176 | 71.000 | 25.500 | 2.500 |
| MAKMR | 756.2241 | 1.1286 | 1.5844 | 69.000 | 15.500 | 4.000 |
| .999 | ASIMOTA | .8029 | .8029 | .8029 | 1.000 | 1.000 | 1.000 |
| GFA | 2.5523 | 2.5523 | 2.5523 | 4.000 | 32.000 | 14.000 |
| FAPR | 1.9422 | 1.9422 | 1.9422 | 3.000 | 31.000 | 13.000 |
| FASR | 5.1765 | 5.1765 | 5.1765 | 5.000 | 33.000 | 19.000 |
| FAMR | 1.7116 | 1.7116 | 1.7116 | 2.000 | 29.000 | 11.000 |
| MAK | 7829.9313 | 1.1978 | 1.0646 | 70.000 | 24.500 | 5.000 |
| MAKMN | 680.5194 | 94.4290 | 1.0491 | 10.000 | 64.500 | 2.500 |
| MAKMA | 7831.2937 | 1.1978 | 1.0491 | 71.000 | 24.500 | 2.500 |
| MAKMR | 7823.1796 | 1.1223 | 1.0564 | 69.000 | 14.500 | 4.000 |

APPENDIX 4: MSE of the Ridge Parameters when P=4, n=10 and v= 49

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 2.2448 | 2.2448 | 2.2448 | 1.000 | 56.000 | 4.000 |
| AMKHM | 8.7669 | 1.8918 | 6.9855 | 3.000 | 51.500 | 28.000 |
| MAKAM | 37.7054 | 1.3351 | 2.3565 | 47.000 | 25.500 | 5.000 |
| MAKMN | 9.3130 | 2.6307 | 1.9572 | 5.000 | 60.500 | 1.500 |
| MAKMA | 55.0829 | 1.2789 | 1.9572 | 69.000 | 7.500 | 1.500 |
| MAKMR | 46.1474 | 1.3241 | 2.1395 | 62.000 | 17.500 | 3.000 |
| MRKMD | 8.8701 | 1.5588 | 4.3687 | 4.000 | 43.500 | 20.000 |
| MRKLHM | 7.7180 | 1.8918 | 6.0166 | 2.000 | 51.500 | 23.000 |
| HMKAM | 22.8037 | 1.2459 | 22.7841 | 34.000 | 3.000 | 46.000 |
| HMKMA | 22.6170 | 1.1054 | 22.1044 | 32.000 | 1.000 | 44.000 |
| HMKMR | 22.6650 | 1.1816 | 22.4115 | 33.000 | 2.000 | 45.000 |
| .900 | ASIMOTA | 13.9340 | 13.9340 | 13.9340 | 3.000 | 66.000 | 34.000 |
| GFA | 2.5011 | 2.5011 | 2.5011 | 1.000 | 48.000 | 5.000 |
| AMKHM | 14.4998 | 2.9014 | 10.6691 | 4.000 | 52.500 | 28.000 |
| MAK | 119.3322 | 1.2429 | 2.4945 | 70.000 | 25.500 | 4.000 |
| MAKMN | 15.7608 | 4.1551 | 2.0436 | 5.000 | 61.500 | 1.500 |
| MAKMA | 127.6246 | 1.2429 | 2.0436 | 71.000 | 25.500 | 1.500 |
| MAKMR | 116.6768 | 1.2326 | 2.2657 | 69.000 | 16.500 | 3.000 |
| MRKLHM | 12.3715 | 2.9014 | 8.8182 | 2.000 | 52.500 | 23.000 |
| HMKMA | 42.4886 | 1.1032 | 40.8831 | 29.000 | 1.000 | 44.000 |
| HMKMR | 42.4457 | 1.1739 | 41.1563 | 27.000 | 2.000 | 45.000 |
| .950 | ASIMOTA | 7.2314 | 7.2314 | 7.2314 | 2.000 | 66.000 | 18.000 |
| GFA | 2.7069 | 2.7069 | 2.7069 | 1.000 | 39.000 | 5.000 |
| FAPR | 16.7602 | 16.7602 | 16.7602 | 3.000 | 67.000 | 28.000 |
| GKAM | 147.8570 | 1.0683 | 147.8190 | 44.000 | 4.500 | 56.000 |
| AMK | 191.9470 | 1.0683 | 5.0111 | 61.000 | 4.500 | 14.000 |
| AMKAM | 163.7295 | 1.0683 | 5.3095 | 48.000 | 4.500 | 15.000 |
| AMKHM | 26.3425 | 4.7153 | 18.0611 | 5.000 | 52.500 | 29.000 |
| MNKAM | 165.0308 | 1.0683 | 167.0767 | 53.000 | 4.500 | 64.000 |
| MAK | 268.9280 | 1.2251 | 2.5247 | 70.000 | 24.500 | 4.000 |
| MAKAM | 244.4363 | 1.0683 | 2.8358 | 67.000 | 4.500 | 6.000 |
| MAKMN | 29.1113 | 6.5847 | 2.1419 | 7.000 | 61.500 | 1.500 |
| MAKMA | 277.7053 | 1.2251 | 2.1419 | 71.000 | 24.500 | 1.500 |
| MAKMR | 265.0863 | 1.1799 | 2.3724 | 69.000 | 15.500 | 3.000 |
| MRKAM | 193.6697 | 1.0683 | 4.2801 | 62.000 | 4.500 | 12.000 |
| MRKLHM | 21.8749 | 4.7153 | 14.1529 | 4.000 | 52.500 | 25.000 |
| MDKAM | 117.0052 | 1.0683 | 19.3251 | 37.000 | 4.500 | 33.000 |
| GMKAM | 65.7724 | 1.0683 | 15.9306 | 23.000 | 4.500 | 27.000 |
| .990 | ASIMOTA | 1.2550 | 1.2550 | 1.2550 | 1.000 | 28.000 | 1.000 |
| GFA | 3.0014 | 3.0014 | 3.0014 | 2.000 | 30.000 | 7.000 |
| FAPR | 9.4919 | 9.4919 | 9.4919 | 3.000 | 41.000 | 17.000 |
| FASR | 70.3036 | 70.3036 | 70.3036 | 5.000 | 69.000 | 31.000 |
| FAMR | 41.8334 | 41.8334 | 41.8334 | 4.000 | 68.000 | 26.000 |
| GKAM | 708.2352 | .9370 | 705.8882 | 43.000 | 4.500 | 56.000 |
| AMK | 1338.7480 | .9370 | 6.1187 | 62.000 | 4.500 | 15.000 |
| AMKAM | 1255.6134 | .9370 | 7.4960 | 60.000 | 4.500 | 16.000 |
| MNKAM | 790.8374 | .9370 | 800.6174 | 52.000 | 4.500 | 64.000 |
| MAK | 1505.7725 | 1.2118 | 2.2962 | 70.000 | 23.500 | 5.000 |
| MAKAM | 1467.5954 | .9370 | 2.6525 | 68.000 | 4.500 | 6.000 |
| MAKMN | 136.5941 | 22.5720 | 2.1183 | 10.000 | 62.500 | 2.500 |
| MAKMA | 1511.3974 | 1.2118 | 2.1183 | 71.000 | 23.500 | 2.500 |
| MAKMR | 1496.3995 | 1.1370 | 2.2757 | 69.000 | 13.500 | 4.000 |
| MRKAM | 1369.9958 | .9370 | 4.7568 | 63.000 | 4.500 | 12.000 |
| MDKAM | 681.5900 | .9370 | 91.7879 | 38.000 | 4.500 | 35.000 |
| GMKAM | 467.9978 | .9370 | 44.4359 | 34.000 | 4.500 | 27.000 |
| .999 | ASIMOTA | .8600 | .8600 | .8600 | 1.000 | 1.000 | 1.000 |
| GFA | 3.1900 | 3.1900 | 3.1900 | 2.000 | 29.000 | 12.000 |
| FAPR | 3.2300 | 3.2300 | 3.2300 | 3.000 | 31.000 | 13.000 |
| FASR | 14.5000 | 14.5000 | 14.5000 | 5.000 | 33.000 | 19.000 |
| FAMR | 3.7000 | 3.7000 | 3.7000 | 4.000 | 32.000 | 14.000 |
| MAK | 15400.0000 | 1.2100 | 1.4700 | 70.500 | 24.000 | 4.000 |
| MAKMN | 1330.0000 | 185.0000 | 1.4400 | 10.000 | 64.500 | 2.500 |
| MAKMA | 15400.0000 | 1.2100 | 1.4400 | 70.500 | 24.000 | 2.500 |
| MAKMR | 15400.0000 | 1.1300 | 1.4900 | 70.500 | 14.500 | 5.000 |

APPENDIX 5: MSE of the Ridge Parameters when P=4, n=10 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 2.8725 | 2.8725 | 2.8725 | 1.000 | 48.000 | 1.000 |
| AMKHM | 17.5377 | 3.2490 | 13.7642 | 3.000 | 52.500 | 27.000 |
| MAK | 101.8637 | 1.3551 | 4.1596 | 67.000 | 7.500 | 5.000 |
| MAKMN | 18.7435 | 5.0621 | 3.1283 | 5.000 | 61.500 | 2.500 |
| MAKMA | 113.3803 | 1.3551 | 3.1283 | 69.000 | 7.500 | 2.500 |
| MAKMR | 95.0489 | 1.4713 | 3.6173 | 62.000 | 17.500 | 4.000 |
| MRKMD | 17.8969 | 2.3618 | 8.4189 | 4.000 | 43.500 | 20.000 |
| MRKLHM | 15.3726 | 3.2490 | 11.7902 | 2.000 | 52.500 | 23.000 |
| HMKAM | 46.3049 | 1.3034 | 46.2231 | 33.000 | 3.000 | 46.000 |
| HMKMA | 45.9119 | 1.1176 | 44.8164 | 31.000 | 1.000 | 44.000 |
| HMKMR | 46.0162 | 1.2138 | 45.4531 | 32.000 | 2.000 | 45.000 |
| .900 | ASIMOTA | 28.3853 | 28.3853 | 28.3853 | 3.000 | 66.000 | 34.000 |
| GFA | 3.2587 | 3.2587 | 3.2587 | 1.000 | 39.000 | 1.000 |
| AMKHM | 29.1955 | 5.4890 | 21.1581 | 4.000 | 52.500 | 29.000 |
| MAK | 249.9691 | 1.2910 | 4.2383 | 70.000 | 7.500 | 5.000 |
| MAKMN | 31.8380 | 8.2563 | 3.2947 | 5.000 | 61.500 | 2.500 |
| MAKMA | 264.1639 | 1.2910 | 3.2947 | 71.000 | 7.500 | 2.500 |
| MAKMR | 241.5264 | 1.3178 | 3.8417 | 69.000 | 15.500 | 4.000 |
| MRKLHM | 24.8121 | 5.4890 | 17.3774 | 2.000 | 52.500 | 23.000 |
| HMKAM | 86.4293 | 1.2772 | 84.6110 | 28.000 | 3.000 | 46.000 |
| HMKMA | 86.5110 | 1.1123 | 83.1656 | 29.000 | 1.000 | 44.000 |
| HMKMR | 86.4241 | 1.1971 | 83.7268 | 27.000 | 2.000 | 45.000 |
| .950 | ASIMOTA | 14.5772 | 14.5772 | 14.5772 | 2.000 | 66.000 | 19.000 |
| GFA | 3.5638 | 3.5638 | 3.5638 | 1.000 | 31.000 | 3.000 |
| FAPR | 30.3948 | 30.3948 | 30.3948 | 3.000 | 67.000 | 27.000 |
| AMKHM | 53.3064 | 9.2421 | 36.0955 | 5.000 | 52.500 | 30.000 |
| MAK | 559.7447 | 1.2599 | 4.2261 | 70.000 | 23.500 | 5.000 |
| MAKMN | 59.0298 | 13.2312 | 3.4700 | 7.000 | 61.500 | 1.500 |
| MAKMA | 574.3209 | 1.2599 | 3.4700 | 71.000 | 23.500 | 1.500 |
| MAKMR | 548.2113 | 1.2280 | 4.0136 | 69.000 | 14.500 | 4.000 |
| MRKLHM | 44.1730 | 9.2421 | 28.1326 | 4.000 | 52.500 | 25.000 |
| HMKMA | 167.2690 | 1.1096 | 159.9250 | 28.000 | 1.000 | 44.000 |
| HMKMR | 167.1082 | 1.1887 | 160.4573 | 27.000 | 2.000 | 45.000 |
| .990 | ASIMOTA | 2.1276 | 2.1276 | 2.1276 | 1.000 | 28.000 | 1.000 |
| GFA | 4.0623 | 4.0623 | 4.0623 | 2.000 | 30.000 | 6.000 |
| FAPR | 16.7280 | 16.7280 | 16.7280 | 3.000 | 41.000 | 17.000 |
| FASR | 174.4790 | 174.4790 | 174.4790 | 5.000 | 69.000 | 32.000 |
| FAMR | 110.9251 | 110.9251 | 110.9251 | 4.000 | 68.000 | 30.000 |
| GKAM | 1445.0921 | .9732 | 1440.2947 | 42.000 | 4.500 | 56.000 |
| AMK | 2783.6993 | .9732 | 10.6686 | 62.000 | 4.500 | 14.000 |
| AMKAM | 2597.2394 | .9732 | 13.7982 | 60.000 | 4.500 | 16.000 |
| MNKAM | 1613.8565 | .9732 | 1633.8106 | 52.000 | 4.500 | 64.000 |
| MAK | 3091.5172 | 1.2369 | 3.7044 | 70.000 | 23.500 | 4.000 |
| MAKAM | 3010.5768 | .9732 | 4.5142 | 68.000 | 4.500 | 7.000 |
| MAKMN | 278.3411 | 45.8589 | 3.3737 | 10.000 | 62.500 | 2.500 |
| MAKMA | 3100.4634 | 1.2369 | 3.3737 | 71.000 | 23.500 | 2.500 |
| MAKMR | 3069.7118 | 1.1542 | 3.7263 | 69.000 | 13.500 | 5.000 |
| MRKAM | 2822.7571 | .9732 | 8.4767 | 63.000 | 4.500 | 12.000 |
| MDKAM | 1394.5003 | .9732 | 186.8641 | 38.000 | 4.500 | 35.000 |
| GMKAM | 964.4409 | .9732 | 87.4921 | 34.000 | 4.500 | 26.000 |
| .999 | ASIMOTA | .9840 | .9840 | .9840 | 1.000 | 9.000 | 1.000 |
| GFA | 4.4500 | 4.4500 | 4.4500 | 2.000 | 29.000 | 12.000 |
| FAPR | 5.7500 | 5.7500 | 5.7500 | 3.000 | 30.000 | 13.000 |
| FASR | 30.7000 | 30.7000 | 30.7000 | 5.000 | 33.000 | 19.000 |
| FAMR | 9.1300 | 9.1300 | 9.1300 | 4.000 | 32.000 | 18.000 |
| GKAM | 14100.0000 | .9150 | 14000.0000 | 38.000 | 4.500 | 55.000 |
| AMK | 31000.0000 | .9150 | 7.0700 | 63.000 | 4.500 | 15.000 |
| AMKAM | 30600.0000 | .9150 | 8.5900 | 60.000 | 4.500 | 17.000 |
| MNKAM | 15800.0000 | .9150 | 16000.0000 | 50.000 | 4.500 | 63.000 |
| MAK | 31400.0000 | 1.2300 | 2.3300 | 70.000 | 24.000 | 4.000 |
| MAKAM | 31300.0000 | .9150 | 2.6500 | 68.000 | 4.500 | 6.000 |
| MAKMN | 2720.0000 | 377.0000 | 2.2600 | 10.000 | 64.500 | 2.500 |
| MAKMA | 31400.0000 | 1.2300 | 2.2600 | 70.000 | 24.000 | 2.500 |
| MAKMR | 31400.0000 | 1.1400 | 2.3800 | 70.000 | 14.500 | 5.000 |
| MRKAM | 31000.0000 | .9150 | 4.4100 | 63.000 | 4.500 | 11.000 |
| MDKAM | 14000.0000 | .9150 | 1880.0000 | 35.000 | 4.500 | 35.000 |
| GMKAM | 13900.0000 | .9150 | 445.0000 | 34.000 | 4.500 | 27.000 |

APPENDIX 6: MSE of the Ridge Parameters when P=4, n=20 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GKMN | .4071 | .2567 | .4040 | 54.000 | 4.500 | 56.000 |
| AMKMN | .2575 | .2567 | .2365 | 20.000 | 4.500 | 19.000 |
| AMKGM | .1860 | .6118 | .1871 | 5.000 | 35.500 | 6.000 |
| MNK | .4775 | .2567 | .4791 | 62.000 | 4.500 | 65.000 |
| MNKMN | .4935 | .2567 | .4928 | 66.000 | 4.500 | 69.000 |
| MAKMN | .1582 | .2567 | .4746 | 1.000 | 4.500 | 62.500 |
| MAKMD | .2068 | .4708 | .1816 | 11.000 | 24.500 | 5.000 |
| MAKHM | .1876 | .4210 | .1738 | 6.000 | 16.500 | 1.000 |
| MRKMN | .2294 | .2567 | .3271 | 16.000 | 4.500 | 39.000 |
| MRKMD | .1732 | .4708 | .1755 | 2.000 | 24.500 | 2.000 |
| MRKGM | .1826 | .6118 | .1801 | 4.000 | 35.500 | 4.000 |
| MRKLHM | .1753 | .4210 | .1762 | 3.000 | 16.500 | 3.000 |
| MDKMN | .3016 | .2567 | .2943 | 34.000 | 4.500 | 35.000 |
| GMKMN | .3162 | .2567 | .3030 | 35.000 | 4.500 | 36.000 |
| .900 | GKMN | .8080 | .1759 | .7987 | 58.000 | 4.500 | 61.000 |
| AMKMN | .4519 | .1759 | .4048 | 32.000 | 4.500 | 33.000 |
| AMKGM | .2607 | .3528 | .2465 | 5.000 | 29.500 | 9.000 |
| MNK | .8764 | .1759 | .8784 | 67.000 | 4.500 | 69.000 |
| MNKMN | .9148 | .1759 | .9111 | 69.000 | 4.500 | 71.000 |
| MAK | .3309 | 1.2406 | .2089 | 15.000 | 66.500 | 4.000 |
| MAKMN | .2434 | .1759 | .4090 | 2.000 | 4.500 | 36.500 |
| MAKMD | .2706 | .2522 | .1901 | 7.000 | 20.500 | 2.000 |
| MAKGM | .4032 | .3528 | .1984 | 22.000 | 29.500 | 3.000 |
| MAKHM | .2378 | .2299 | .1894 | 1.000 | 12.500 | 1.000 |
| MRKMN | .4115 | .1759 | .3227 | 28.000 | 4.500 | 23.000 |
| MRKMD | .2461 | .2522 | .2380 | 3.000 | 20.500 | 6.000 |
| MRKGM | .2677 | .3528 | .2253 | 6.000 | 29.500 | 5.000 |
| MRKLHM | .2589 | .2299 | .2497 | 4.000 | 12.500 | 10.000 |
| MDKMN | .5116 | .1759 | .4925 | 36.000 | 4.500 | 41.000 |
| GMKMN | .5459 | .1759 | .5145 | 40.000 | 4.500 | 45.000 |
| .950 | ASIMOTA | .3244 | .3244 | .3244 | 1.000 | 35.000 | 10.000 |
| GKMN | 1.6398 | .1745 | 1.6172 | 64.000 | 4.500 | 65.000 |
| AMKMN | .8080 | .1745 | .7130 | 32.000 | 4.500 | 41.000 |
| AMKGM | .4230 | .2295 | .3493 | 4.000 | 29.500 | 14.000 |
| MNK | 1.7337 | .1745 | 1.7304 | 68.000 | 4.500 | 69.000 |
| MNKMN | 1.7967 | .1745 | 1.7851 | 70.000 | 4.500 | 71.000 |
| MAK | .5327 | 1.2405 | .2706 | 17.000 | 64.500 | 5.000 |
| MAKAM | 1.5709 | .8895 | .2702 | 62.000 | 44.500 | 4.000 |
| MAKMN | .4340 | .1745 | .3800 | 5.000 | 4.500 | 18.500 |
| MAKMD | .4354 | .1938 | .2498 | 7.000 | 20.500 | 2.000 |
| MAKGM | .7684 | .2295 | .2385 | 30.000 | 29.500 | 1.000 |
| MAKHM | .3690 | .1910 | .2589 | 2.000 | 12.500 | 3.000 |
| MRKMN | .7886 | .1745 | .3691 | 31.000 | 4.500 | 16.000 |
| MRKMD | .4010 | .1938 | .3740 | 3.000 | 20.500 | 17.000 |
| MDKMN | .8483 | .1745 | .8035 | 36.000 | 4.500 | 42.000 |
| GMKMN | .9760 | .1745 | .9046 | 41.000 | 4.500 | 47.000 |
| .990 | ASIMOTA | .1461 | .1461 | .1461 | 1.000 | 1.000 | 1.000 |
| GFA | .7921 | .7921 | .7921 | 5.000 | 41.000 | 24.000 |
| FAPR | .5615 | .5615 | .5615 | 4.000 | 40.000 | 21.000 |
| FASR | .5342 | .5342 | .5342 | 3.000 | 39.000 | 20.000 |
| FAMR | .4153 | .4153 | .4153 | 2.000 | 38.000 | 12.000 |
| MAK | 5.3364 | 1.2405 | .3465 | 43.000 | 64.500 | 4.000 |
| MAKAM | 9.1782 | .8645 | .3015 | 69.000 | 45.500 | 2.000 |
| MAKMR | 9.3542 | 1.1513 | .3244 | 71.000 | 54.500 | 3.000 |
| MAKGM | 4.9478 | .1806 | .3481 | 42.000 | 7.500 | 5.000 |
| HMKMD | 4.1249 | .1653 | 4.2247 | 38.000 | 2.000 | 50.000 |
| HMKHM | 4.3583 | .1772 | 4.4140 | 40.500 | 3.000 | 51.000 |
| .999 | ASIMOTA | .3902 | .3902 | .3902 | 4.000 | 13.000 | 4.000 |
| GFA | 1.1428 | 1.1428 | 1.1428 | 5.000 | 25.000 | 15.000 |
| FAPR | .2348 | .2348 | .2348 | 3.000 | 4.000 | 3.000 |
| FASR | .1337 | .1337 | .1337 | 2.000 | 3.000 | 2.000 |
| FAMR | .1216 | .1216 | .1216 | 1.000 | 1.000 | 1.000 |
| MAKAM | 148.1915 | .8590 | .4842 | 69.000 | 19.500 | 5.000 |
| HMKGM | 35.5005 | .1316 | 35.7748 | 26.000 | 2.000 | 49.000 |

APPENDIX 7: MSE of the Ridge Parameters when P=4, n=20 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .6732 | .6732 | .6732 | 2.000 | 25.000 | 18.000 |
| GKMN | 3.3907 | .3834 | 3.3531 | 62.000 | 4.500 | 61.000 |
| AMKMN | 1.5090 | .3834 | 1.3097 | 33.000 | 4.500 | 39.000 |
| AMKGM | .7442 | .6738 | .6913 | 5.000 | 29.500 | 19.000 |
| MNK | 3.6130 | .3834 | 3.6065 | 67.000 | 4.500 | 67.000 |
| MNKMN | 3.7191 | .3834 | 3.6967 | 68.000 | 4.500 | 68.000 |
| MAK | 1.4008 | 1.2466 | .4670 | 31.000 | 60.500 | 4.000 |
| MAKAM | 1.8892 | 1.0106 | .4804 | 42.000 | 38.500 | 5.000 |
| MAKMN | .7709 | .3834 | .6426 | 7.000 | 4.500 | 15.500 |
| MAKMD | .6896 | .5581 | .4398 | 4.000 | 20.500 | 2.000 |
| MAKGM | .9641 | .6738 | .4336 | 15.000 | 29.500 | 1.000 |
| MAKHM | .6194 | .5186 | .4511 | 1.000 | 12.500 | 3.000 |
| MRKMN | 1.5177 | .3834 | .6143 | 34.000 | 4.500 | 14.000 |
| MRKGM | .6854 | .6738 | .6104 | 3.000 | 29.500 | 13.000 |
| MDKMN | 1.5468 | .3834 | 1.4424 | 35.000 | 4.500 | 40.000 |
| GMKMN | 1.8849 | .3834 | 1.7271 | 41.000 | 4.500 | 42.000 |
| .900 | GFA | .8060 | .8060 | .8060 | 1.000 | 36.000 | 18.000 |
| GKMN | 7.0195 | .4015 | 6.9222 | 63.000 | 4.500 | 63.000 |
| AMKMN | 2.7952 | .4015 | 2.3249 | 29.000 | 4.500 | 39.000 |
| AMKGM | 1.2450 | .4682 | .8542 | 5.000 | 29.500 | 20.000 |
| MNK | 7.4373 | .4015 | 7.4148 | 67.000 | 4.500 | 68.000 |
| MNKMN | 7.6644 | .4015 | 7.6078 | 69.000 | 4.500 | 69.000 |
| MAK | 4.2178 | 1.2438 | .4967 | 43.000 | 60.500 | 3.000 |
| MAKAM | 5.6599 | .9362 | .4804 | 49.000 | 40.500 | 2.000 |
| MAKMN | 1.3693 | .4015 | .5960 | 9.000 | 4.500 | 9.500 |
| MAKMR | 6.6883 | 1.1646 | .5253 | 61.000 | 50.500 | 5.000 |
| MAKMD | 1.3097 | .4328 | .5140 | 8.000 | 20.500 | 4.000 |
| MAKGM | 2.4239 | .4682 | .4767 | 25.000 | 29.500 | 1.000 |
| MAKHM | 1.0577 | .4307 | .5499 | 2.000 | 12.500 | 6.000 |
| MRKMN | 2.9390 | .4015 | .6187 | 31.000 | 4.500 | 12.000 |
| MRKMD | 1.1053 | .4328 | .9550 | 3.000 | 20.500 | 21.000 |
| MRKGM | 1.1760 | .4682 | .7221 | 4.000 | 29.500 | 17.000 |
| MDKMN | 2.7538 | .4015 | 2.4698 | 28.000 | 4.500 | 40.000 |
| GMKMN | 3.5968 | .4015 | 3.2063 | 38.000 | 4.500 | 43.000 |
| .950 | GFA | .9408 | .9408 | .9408 | 1.000 | 45.000 | 19.000 |
| AMKMD | 2.1332 | .5246 | 1.6295 | 5.000 | 15.500 | 23.000 |
| MAK | 13.6651 | 1.2428 | .5786 | 61.000 | 60.500 | 3.000 |
| MAKAM | 15.7749 | .8983 | .5368 | 68.000 | 40.500 | 1.000 |
| MAKMR | 17.2094 | 1.1576 | .5620 | 70.000 | 50.500 | 2.000 |
| MAKGM | 6.7726 | .4197 | .6087 | 33.000 | 5.500 | 4.000 |
| MAKHM | 2.1161 | .5578 | .7252 | 4.000 | 23.500 | 15.000 |
| MRKMD | 1.7644 | .5246 | 1.2931 | 2.000 | 15.500 | 21.000 |
| MRKLHM | 2.0951 | .5578 | 1.6460 | 3.000 | 23.500 | 25.000 |
| HMKHM | 7.5022 | .4188 | 7.5848 | 37.500 | 1.000 | 49.000 |
| .990 | ASIMOTA | .3679 | .3679 | .3679 | 1.000 | 2.000 | 1.000 |
| GFA | 1.2007 | 1.2007 | 1.2007 | 2.000 | 31.000 | 12.000 |
| FAPR | 2.9603 | 2.9603 | 2.9603 | 3.000 | 66.000 | 22.000 |
| FAMR | 7.4175 | 7.4175 | 7.4175 | 4.000 | 68.000 | 34.000 |
| MAKAM | 123.9504 | .8672 | .7205 | 69.000 | 16.500 | 5.000 |
| MAKMN | 12.1117 | 1.9836 | .7135 | 10.000 | 61.500 | 3.500 |
| MAKMA | 127.4269 | 1.2422 | .7135 | 71.000 | 36.500 | 3.500 |
| MAKMR | 126.2238 | 1.1525 | .7039 | 70.000 | 25.500 | 2.000 |
| MRKMD | 7.8542 | 1.6685 | 3.4162 | 5.000 | 45.500 | 23.000 |
| HMKMD | 35.7440 | .5035 | 36.4329 | 31.000 | 3.000 | 50.000 |
| HMKGM | 31.8776 | .3059 | 32.5602 | 28.000 | 1.000 | 49.000 |
| HMKHM | 37.8459 | .6423 | 38.1433 | 34.500 | 4.000 | 51.000 |
| .999 | ASIMOTA | .4190 | .4190 | .4190 | 1.000 | 2.000 | 6.000 |
| GFA | 1.4060 | 1.4060 | 1.4060 | 5.000 | 33.000 | 15.000 |
| FAPR | .9079 | .9079 | .9079 | 3.000 | 12.000 | 13.000 |
| FASR | 1.1442 | 1.1442 | 1.1442 | 4.000 | 14.000 | 14.000 |
| FAMR | .6436 | .6436 | .6436 | 2.000 | 3.000 | 7.000 |
| MAK | 1477.0585 | 1.2420 | .3838 | 69.000 | 28.500 | 1.000 |
| MAKAM | 1474.3549 | .8600 | .3860 | 68.000 | 7.500 | 3.000 |
| MAKMN | 122.9929 | 18.2250 | .3909 | 10.000 | 64.500 | 4.500 |
| MAKMA | 1478.4820 | 1.2420 | .3909 | 71.000 | 28.500 | 4.500 |
| MAKMR | 1477.0961 | 1.1514 | .3846 | 70.000 | 18.500 | 2.000 |
| HMKGM | 318.4390 | .3625 | 320.2870 | 26.000 | 1.000 | 49.000 |

APPENDIX 8: MSE of the Ridge Parameters when P=4, n=20 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .9618 | .9618 | .9618 | 1.000 | 34.000 | 14.000 |
| GKMN | 9.3322 | .6342 | 9.2162 | 63.000 | 4.500 | 62.000 |
| AMKMN | 3.6842 | .6342 | 3.0471 | 29.000 | 4.500 | 39.000 |
| AMKGM | 1.5868 | .7957 | 1.2914 | 5.000 | 28.500 | 20.000 |
| MNK | 9.9565 | .6342 | 9.9318 | 67.000 | 4.500 | 67.000 |
| MNKMN | 10.2347 | .6342 | 10.1670 | 68.000 | 4.500 | 68.000 |
| MAK | 6.3729 | 1.2569 | .7905 | 47.000 | 60.500 | 5.000 |
| MAKAM | 5.5088 | 1.0521 | .7451 | 44.000 | 38.500 | 2.000 |
| MAKMN | 1.7418 | .6342 | .8435 | 8.000 | 4.500 | 7.500 |
| MAKMR | 7.4118 | 1.2020 | .7767 | 48.000 | 50.500 | 4.000 |
| MAKMD | 1.6472 | .7302 | .7696 | 6.000 | 20.500 | 3.000 |
| MAKGM | 2.5545 | .7957 | .7386 | 21.000 | 28.500 | 1.000 |
| MAKHM | 1.4048 | .7111 | .8015 | 3.000 | 12.500 | 6.000 |
| MRKMN | 3.7983 | .6342 | .9040 | 32.000 | 4.500 | 11.000 |
| MRKMD | 1.5332 | .7302 | 1.3379 | 4.000 | 20.500 | 21.000 |
| MRKGM | 1.4011 | .7957 | 1.0882 | 2.000 | 28.500 | 18.000 |
| MDKMN | 3.7250 | .6342 | 3.3644 | 30.000 | 4.500 | 40.000 |
| GMKMN | 4.8024 | .6342 | 4.2845 | 39.000 | 4.500 | 41.000 |
| .900 | GFA | 1.1044 | 1.1044 | 1.1044 | 1.000 | 45.000 | 16.000 |
| GKGM | 16.7209 | .6963 | 16.9371 | 50.000 | 4.500 | 55.000 |
| AMKMD | 2.8553 | .7908 | 2.2369 | 5.000 | 14.500 | 24.000 |
| AMKGM | 3.1851 | .6963 | 1.5658 | 7.000 | 4.500 | 20.000 |
| MNKGM | 18.9699 | .6963 | 19.2292 | 60.000 | 4.500 | 64.000 |
| MAK | 19.7785 | 1.2501 | .8722 | 64.000 | 60.500 | 5.000 |
| MAKAM | 18.5846 | .9643 | .8326 | 59.000 | 39.500 | 2.000 |
| MAKMN | 3.2640 | .8495 | .8546 | 9.000 | 31.500 | 3.500 |
| MAKMA | 24.4150 | 1.2501 | .8546 | 69.000 | 60.500 | 3.500 |
| MAKMR | 21.9937 | 1.1757 | .8203 | 68.000 | 50.500 | 1.000 |
| MAKGM | 7.5796 | .6963 | .9326 | 25.000 | 4.500 | 6.000 |
| MAKHM | 2.8087 | .8288 | 1.0398 | 4.000 | 23.500 | 12.000 |
| MRKMD | 2.3725 | .7908 | 1.7840 | 2.000 | 14.500 | 21.000 |
| MRKGM | 2.9890 | .6963 | 1.2488 | 6.000 | 4.500 | 18.000 |
| MRKLHM | 2.7962 | .8288 | 2.1895 | 3.000 | 23.500 | 22.000 |
| MDKGM | 4.4894 | .6963 | 2.9945 | 15.000 | 4.500 | 31.000 |
| GMK | 3.7588 | .6963 | 3.2873 | 11.000 | 4.500 | 32.000 |
| GMKGM | 3.9779 | .6963 | 3.6632 | 12.000 | 4.500 | 34.000 |
| .950 | GFA | 1.2403 | 1.2403 | 1.2403 | 1.000 | 39.000 | 12.000 |
| AMKMD | 5.3117 | 1.1820 | 3.4474 | 4.000 | 33.500 | 25.000 |
| AMKHM | 5.8869 | 1.2867 | 4.4016 | 5.000 | 52.500 | 30.000 |
| MAK | 54.2122 | 1.2474 | 1.0006 | 69.000 | 44.500 | 5.000 |
| MAKAM | 51.8438 | .9152 | .9624 | 68.000 | 15.500 | 4.000 |
| MAKMN | 6.4168 | 1.3394 | .9221 | 7.000 | 60.500 | 2.500 |
| MAKMA | 59.4924 | 1.2474 | .9221 | 71.000 | 44.500 | 2.500 |
| MAKMR | 56.6235 | 1.1638 | .9118 | 70.000 | 25.500 | 1.000 |
| MRKMD | 4.2216 | 1.1820 | 2.5228 | 2.000 | 33.500 | 21.000 |
| MRKLHM | 4.9186 | 1.2867 | 3.4029 | 3.000 | 52.500 | 24.000 |
| HMKMD | 19.5251 | .6040 | 19.9099 | 32.000 | 1.000 | 48.000 |
| HMKGM | 17.7436 | .7315 | 18.2913 | 30.000 | 3.000 | 47.000 |
| HMKHM | 20.5229 | .6280 | 20.7081 | 33.500 | 2.000 | 49.000 |
| .990 | ASIMOTA | .8123 | .8123 | .8123 | 1.000 | 2.000 | 1.000 |
| GFA | 1.4759 | 1.4759 | 1.4759 | 2.000 | 31.000 | 7.000 |
| FAPR | 6.6201 | 6.6201 | 6.6201 | 3.000 | 66.000 | 22.000 |
| MAK | 376.0353 | 1.2456 | .9882 | 69.000 | 26.500 | 5.000 |
| MAKMN | 32.8379 | 5.2919 | .9308 | 9.000 | 61.500 | 2.500 |
| MAKMA | 379.6737 | 1.2456 | .9308 | 71.000 | 26.500 | 2.500 |
| MAKMR | 376.0878 | 1.1548 | .9404 | 70.000 | 15.500 | 4.000 |
| MRKMD | 21.7719 | 4.4053 | 8.3300 | 4.000 | 45.500 | 23.000 |
| MRKLHM | 22.8923 | 4.9709 | 12.8186 | 5.000 | 53.500 | 29.000 |
| HMKGM | 88.2813 | .4770 | 90.0689 | 28.000 | 1.000 | 49.000 |
| .999 | ASIMOTA | .4771 | .4771 | .4771 | 1.000 | 1.000 | 1.000 |
| GFA | 1.6405 | 1.6405 | 1.6405 | 2.000 | 30.000 | 13.000 |
| FAPR | 1.9323 | 1.9323 | 1.9323 | 3.000 | 31.000 | 17.000 |
| FASR | 7.5632 | 7.5632 | 7.5632 | 5.000 | 41.000 | 19.000 |
| FAMR | 2.4292 | 2.4292 | 2.4292 | 4.000 | 32.000 | 18.000 |
| MAK | 4143.4872 | 1.2452 | .5564 | 70.000 | 25.500 | 4.000 |
| MAKAM | 4133.1180 | .8619 | .5439 | 68.000 | 6.500 | 2.000 |
| MAKMR | 4140.8125 | 1.1527 | .5448 | 69.000 | 15.500 | 3.000 |
| HMKGM | 884.3001 | .8220 | 889.2893 | 25.000 | 2.000 | 49.000 |

APPENDIX 9: MSE of the Ridge Parameters when P=4, n=20 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.2114 | 1.2114 | 1.2114 | 1.000 | 46.000 | 7.000 |
| GKGM | 16.0269 | .9776 | 16.2116 | 53.000 | 4.500 | 54.000 |
| AMKGM | 2.8101 | .9776 | 2.0885 | 5.000 | 4.500 | 20.000 |
| MNKGM | 18.2009 | .9776 | 18.4132 | 61.000 | 4.500 | 63.000 |
| MAK | 15.1077 | 1.2722 | 1.1737 | 48.000 | 60.500 | 5.000 |
| MAKAM | 11.3963 | 1.1139 | 1.1177 | 44.000 | 37.500 | 4.000 |
| MAKMN | 3.1395 | 1.0091 | 1.0931 | 8.000 | 29.500 | 2.500 |
| MAKMA | 19.3110 | 1.2722 | 1.0931 | 65.000 | 60.500 | 2.500 |
| MAKMR | 15.3987 | 1.2331 | 1.0877 | 50.000 | 50.500 | 1.000 |
| MAKGM | 5.1099 | .9776 | 1.1752 | 21.000 | 4.500 | 6.000 |
| MAKHM | 2.6160 | .9987 | 1.2908 | 3.000 | 20.500 | 10.000 |
| MRKMD | 2.6334 | .9872 | 2.1853 | 4.000 | 12.500 | 21.000 |
| MRKGM | 2.4489 | .9776 | 1.7171 | 2.000 | 4.500 | 18.000 |
| MDKGM | 4.1482 | .9776 | 3.5253 | 13.000 | 4.500 | 31.000 |
| GMK | 4.0475 | .9776 | 3.8801 | 12.000 | 4.500 | 32.000 |
| GMKGM | 4.4498 | .9776 | 4.3189 | 17.000 | 4.500 | 36.000 |
| .900 | GFA | 1.3783 | 1.3783 | 1.3783 | 1.000 | 48.000 | 6.000 |
| AMKMD | 5.2269 | 1.3262 | 3.8198 | 4.000 | 43.500 | 24.000 |
| MAK | 45.6607 | 1.2594 | 1.2923 | 67.000 | 35.500 | 4.000 |
| MAKAM | 39.2852 | 1.0060 | 1.3125 | 62.000 | 6.500 | 5.000 |
| MAKMN | 6.0519 | 1.5199 | 1.1521 | 8.000 | 60.500 | 1.500 |
| MAKMA | 51.6271 | 1.2594 | 1.1521 | 69.000 | 35.500 | 1.500 |
| MAKMR | 46.5425 | 1.1922 | 1.1892 | 68.000 | 25.500 | 3.000 |
| MAKHM | 5.5495 | 1.4242 | 1.7619 | 5.000 | 52.500 | 16.000 |
| MRKMD | 4.2392 | 1.3262 | 2.9321 | 2.000 | 43.500 | 21.000 |
| MRKLHM | 4.9959 | 1.4242 | 3.6895 | 3.000 | 52.500 | 23.000 |
| HMKMD | 18.9939 | .8947 | 19.3503 | 33.000 | 2.000 | 47.000 |
| HMKHM | 19.8343 | .8739 | 20.0140 | 36.500 | 1.000 | 48.000 |
| .950 | GFA | 1.5295 | 1.5295 | 1.5295 | 1.000 | 39.000 | 6.000 |
| AMKMD | 10.0838 | 2.1658 | 6.1008 | 4.000 | 44.500 | 25.000 |
| AMKHM | 11.1120 | 2.3777 | 7.9507 | 5.000 | 52.500 | 28.000 |
| MAK | 117.8311 | 1.2543 | 1.4224 | 69.000 | 26.500 | 4.000 |
| MAKAM | 108.1341 | .9403 | 1.4865 | 68.000 | 6.500 | 5.000 |
| MAKMN | 12.1823 | 2.5001 | 1.2459 | 6.000 | 60.500 | 1.500 |
| MAKMA | 124.1545 | 1.2543 | 1.2459 | 71.000 | 26.500 | 1.500 |
| MAKMR | 118.1803 | 1.1729 | 1.3033 | 70.000 | 16.500 | 3.000 |
| MRKMD | 7.9503 | 2.1658 | 4.3180 | 2.000 | 44.500 | 21.000 |
| MRKLHM | 9.1215 | 2.3777 | 5.9540 | 3.000 | 52.500 | 24.000 |
| HMKMD | 38.0970 | .8311 | 38.8213 | 32.000 | 2.000 | 48.000 |
| HMKGM | 34.6143 | .7974 | 35.6504 | 30.000 | 1.000 | 47.000 |
| .990 | ASIMOTA | 1.4796 | 1.4796 | 1.4796 | 1.000 | 29.000 | 6.000 |
| GFA | 1.7792 | 1.7792 | 1.7792 | 2.000 | 30.000 | 7.000 |
| FAPR | 11.2890 | 11.2890 | 11.2890 | 3.000 | 66.000 | 22.000 |
| MAK | 756.1642 | 1.2507 | 1.2645 | 70.000 | 24.500 | 4.000 |
| MAKAM | 739.0938 | .8796 | 1.3753 | 68.000 | 5.500 | 5.000 |
| MAKMN | 63.9143 | 10.2500 | 1.1824 | 8.000 | 61.500 | 1.500 |
| MAKMA | 760.0573 | 1.2507 | 1.1824 | 71.000 | 24.500 | 1.500 |
| MAKMR | 752.8883 | 1.1582 | 1.2261 | 69.000 | 14.500 | 3.000 |
| MRKMD | 42.7315 | 8.5057 | 15.7044 | 4.000 | 45.500 | 25.000 |
| MRKLHM | 44.4055 | 9.6115 | 24.2977 | 5.000 | 53.500 | 29.000 |
| HMKGM | 172.8852 | .7329 | 176.3272 | 28.000 | 1.000 | 49.000 |
| .999 | ASIMOTA | .5642 | .5642 | .5642 | 1.000 | 1.000 | 1.000 |
| GFA | 1.9464 | 1.9464 | 1.9464 | 2.000 | 30.000 | 13.000 |
| FAPR | 3.2474 | 3.2474 | 3.2474 | 3.000 | 31.000 | 17.000 |
| FASR | 29.7113 | 29.7113 | 29.7113 | 5.000 | 43.000 | 19.000 |
| FAMR | 6.2167 | 6.2167 | 6.2167 | 4.000 | 32.000 | 18.000 |
| MAK | 8142.9361 | 1.2499 | .7610 | 70.000 | 24.500 | 5.000 |
| MAKMN | 667.5146 | 98.5986 | .7573 | 10.000 | 64.500 | 3.500 |
| MAKMA | 8144.1880 | 1.2499 | .7573 | 71.000 | 24.500 | 3.500 |
| MAKMR | 8136.5902 | 1.1548 | .7499 | 69.000 | 14.500 | 2.000 |

APPENDIX 10: MSE of the Ridge Parameters when P=4, n=20 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.5415 | 1.5415 | 1.5415 | 1.000 | 48.000 | 1.000 |
| AMKGM | 5.4023 | 1.3629 | 3.7402 | 5.000 | 35.500 | 20.000 |
| MAKAM | 24.1390 | 1.2444 | 1.8887 | 44.000 | 6.500 | 5.000 |
| MAKMN | 6.0854 | 1.8044 | 1.5820 | 8.000 | 60.500 | 2.500 |
| MAKMA | 40.9683 | 1.3045 | 1.5820 | 68.000 | 26.500 | 2.500 |
| MAKMR | 32.6915 | 1.2987 | 1.7182 | 52.000 | 18.500 | 4.000 |
| MAKHM | 5.2120 | 1.6081 | 2.3136 | 4.000 | 52.500 | 12.000 |
| MRKMD | 4.9394 | 1.5318 | 3.9378 | 3.000 | 43.500 | 21.000 |
| MRKGM | 4.6740 | 1.3629 | 3.0154 | 2.000 | 35.500 | 18.000 |
| HMKMA | 16.3999 | 1.1302 | 16.7502 | 32.000 | 1.000 | 42.000 |
| HMKMR | 16.6037 | 1.2139 | 17.0882 | 33.000 | 2.000 | 43.000 |
| .900 | GFA | 1.7785 | 1.7785 | 1.7785 | 1.000 | 39.000 | 3.000 |
| AMKMD | 10.2614 | 2.4619 | 7.1452 | 4.000 | 44.500 | 24.000 |
| MAK | 101.8107 | 1.2792 | 2.0700 | 68.000 | 26.500 | 5.000 |
| MAKMN | 11.9539 | 2.9424 | 1.7108 | 7.000 | 60.500 | 1.500 |
| MAKMA | 110.4204 | 1.2792 | 1.7108 | 69.000 | 26.500 | 1.500 |
| MAKMR | 99.5709 | 1.2268 | 1.9121 | 67.000 | 17.500 | 4.000 |
| MAKHM | 11.4393 | 2.6872 | 3.2927 | 5.000 | 52.500 | 16.000 |
| MRKMD | 8.2052 | 2.4619 | 5.3361 | 2.000 | 44.500 | 21.000 |
| MRKLHM | 9.6441 | 2.6872 | 6.8289 | 3.000 | 52.500 | 23.000 |
| HMKMD | 38.5686 | 1.0881 | 39.2703 | 33.000 | 1.000 | 47.000 |
| .950 | GFA | 1.9857 | 1.9857 | 1.9857 | 1.000 | 31.000 | 3.000 |
| AMKMD | 20.2326 | 4.2536 | 11.7141 | 4.000 | 44.500 | 26.000 |
| AMKHM | 22.2127 | 4.6929 | 15.4643 | 5.000 | 52.500 | 28.000 |
| MAK | 254.1667 | 1.2689 | 2.1859 | 70.000 | 24.500 | 5.000 |
| MAKMN | 24.4241 | 4.9640 | 1.8400 | 6.000 | 60.500 | 1.500 |
| MAKMA | 262.6832 | 1.2689 | 1.8400 | 71.000 | 24.500 | 1.500 |
| MAKMR | 250.0498 | 1.1922 | 2.0497 | 69.000 | 14.500 | 4.000 |
| MRKMD | 15.9055 | 4.2536 | 8.1235 | 2.000 | 44.500 | 21.000 |
| MRKLHM | 18.0508 | 4.6929 | 11.3510 | 3.000 | 52.500 | 24.000 |
| HMKGM | 70.4640 | .9368 | 72.5355 | 29.000 | 1.000 | 47.000 |
| .990 | ASIMOTA | 2.8990 | 2.8990 | 2.8990 | 2.000 | 30.000 | 7.000 |
| GFA | 2.3264 | 2.3264 | 2.3264 | 1.000 | 29.000 | 6.000 |
| FAPR | 19.9084 | 19.9084 | 19.9084 | 3.000 | 58.000 | 18.000 |
| GKAM | 692.5384 | .8950 | 692.3233 | 42.000 | 4.500 | 56.000 |
| AMK | 1401.7537 | .8950 | 5.5933 | 62.000 | 4.500 | 15.000 |
| AMKAM | 1317.1705 | .8950 | 6.8967 | 60.000 | 4.500 | 16.000 |
| MNKAM | 782.2491 | .8950 | 794.9806 | 52.000 | 4.500 | 64.000 |
| MAK | 1564.2858 | 1.2616 | 1.8007 | 70.000 | 23.500 | 4.000 |
| MAKAM | 1525.8296 | .8950 | 2.1171 | 68.000 | 4.500 | 5.000 |
| MAKMN | 129.9519 | 20.7804 | 1.6647 | 8.000 | 62.500 | 1.500 |
| MAKMA | 1569.1208 | 1.2616 | 1.6647 | 71.000 | 23.500 | 1.500 |
| MAKMR | 1554.3454 | 1.1653 | 1.7902 | 69.000 | 13.500 | 3.000 |
| MRKAM | 1431.1874 | .8950 | 4.1510 | 63.000 | 4.500 | 12.000 |
| MRKMD | 87.3085 | 17.2130 | 31.3791 | 4.000 | 45.500 | 25.000 |
| MRKLHM | 90.1304 | 19.4663 | 48.6852 | 5.000 | 53.500 | 29.000 |
| MDKAM | 671.2837 | .8950 | 77.6019 | 38.000 | 4.500 | 34.000 |
| GMKAM | 470.5319 | .8950 | 31.5650 | 34.000 | 4.500 | 26.000 |
| .999 | ASIMOTA | .7490 | .7490 | .7490 | 1.000 | 1.000 | 1.000 |
| GFA | 2.5600 | 2.5600 | 2.5600 | 2.000 | 29.000 | 12.000 |
| FAPR | 5.7000 | 5.7000 | 5.7000 | 3.000 | 31.000 | 17.000 |
| FASR | 71.7000 | 71.7000 | 71.7000 | 5.000 | 43.000 | 19.000 |
| FAMR | 17.2000 | 17.2000 | 17.2000 | 4.000 | 32.000 | 18.000 |
| MAK | 16600.0000 | 1.2600 | 1.1300 | 68.500 | 24.000 | 4.000 |
| MAKMN | 1360.0000 | 201.0000 | 1.1100 | 10.000 | 64.500 | 2.500 |
| MAKMA | 16600.0000 | 1.2600 | 1.1100 | 68.500 | 24.000 | 2.500 |
| MAKMR | 16600.0000 | 1.1600 | 1.1400 | 68.500 | 14.500 | 5.000 |

APPENDIX 11: MSE of the Ridge Parameters when P=4, n=30 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GHK | .2114 | .2114 | .2114 | 45.500 | 2.000 | 47.500 |
| GFA | .1845 | .1845 | .1845 | 28.000 | 1.000 | 29.000 |
| AMK | .1361 | 1.0435 | .1329 | 5.000 | 45.500 | 4.000 |
| AMKMD | .1333 | .7153 | .1320 | 4.000 | 28.500 | 3.000 |
| AMKGM | .1367 | .7702 | .1345 | 6.000 | 36.500 | 5.000 |
| AMKHM | .1281 | .5385 | .1275 | 3.000 | 20.500 | 2.000 |
| MAKMN | .1166 | .2406 | .5073 | 1.000 | 6.500 | 71.500 |
| MRKLHM | .1251 | .5385 | .1249 | 2.000 | 20.500 | 1.000 |
| .900 | GKMN | .3609 | .1304 | .3555 | 54.000 | 4.500 | 56.000 |
| AMKMN | .2250 | .1304 | .2026 | 27.000 | 4.500 | 27.000 |
| AMKMD | .1498 | .3539 | .1401 | 4.000 | 22.500 | 9.000 |
| AMKGM | .1529 | .4699 | .1367 | 5.000 | 34.500 | 7.000 |
| MNK | .4244 | .1304 | .4260 | 62.000 | 4.500 | 66.000 |
| MNKMN | .4410 | .1304 | .4402 | 66.000 | 4.500 | 69.000 |
| MAKMN | .1408 | .1304 | .4232 | 1.000 | 4.500 | 63.500 |
| MAKMD | .2018 | .3539 | .1276 | 18.000 | 22.500 | 2.000 |
| MAKHM | .1622 | .2597 | .1186 | 9.000 | 13.500 | 1.000 |
| MRKMN | .2084 | .1304 | .2725 | 20.000 | 4.500 | 38.000 |
| MRKMD | .1465 | .3539 | .1288 | 3.000 | 22.500 | 3.000 |
| MRKGM | .1621 | .4699 | .1298 | 8.000 | 34.500 | 4.000 |
| MRKLHM | .1433 | .2597 | .1330 | 2.000 | 13.500 | 5.000 |
| MDKMN | .2504 | .1304 | .2398 | 31.000 | 4.500 | 34.000 |
| GMKMN | .2740 | .1304 | .2591 | 38.000 | 4.500 | 36.000 |
| .950 | GKMN | .6486 | .1008 | .6375 | 56.000 | 4.500 | 60.000 |
| AMKMN | .3718 | .1008 | .3300 | 31.000 | 4.500 | 36.000 |
| AMKMD | .2135 | .1772 | .1956 | 4.000 | 20.500 | 13.000 |
| AMKGM | .2146 | .2689 | .1749 | 5.000 | 29.500 | 9.000 |
| MNK | .7053 | .1008 | .7074 | 66.000 | 4.500 | 68.000 |
| MNKMN | .7424 | .1008 | .7393 | 68.000 | 4.500 | 70.000 |
| MAK | .2836 | 1.2507 | .1420 | 16.000 | 67.500 | 4.000 |
| MAKMN | .2154 | .1008 | .3537 | 6.000 | 4.500 | 38.500 |
| MAKMD | .2542 | .1772 | .1282 | 14.000 | 20.500 | 1.000 |
| MAKGM | .3576 | .2689 | .1313 | 29.000 | 29.500 | 3.000 |
| MAKHM | .2079 | .1457 | .1311 | 3.000 | 12.500 | 2.000 |
| MRKMN | .3461 | .1008 | .2548 | 27.000 | 4.500 | 23.000 |
| MRKMD | .1991 | .1772 | .1682 | 1.000 | 20.500 | 6.000 |
| MRKGM | .2323 | .2689 | .1555 | 9.000 | 29.500 | 5.000 |
| MRKLHM | .2050 | .1457 | .1844 | 2.000 | 12.500 | 11.000 |
| MDKMN | .4089 | .1008 | .3889 | 35.000 | 4.500 | 40.000 |
| GMKMN | .4435 | .1008 | .4147 | 40.000 | 4.500 | 44.000 |
| .990 | ASIMOTA | .1860 | .1860 | .1860 | 1.000 | 36.000 | 1.000 |
| GFA | .5944 | .5944 | .5944 | 3.000 | 38.000 | 31.000 |
| GKGM | 2.4461 | .1164 | 2.4732 | 53.000 | 4.500 | 57.000 |
| AMKGM | .7590 | .1164 | .3888 | 16.000 | 4.500 | 18.000 |
| MNKGM | 2.7733 | .1164 | 2.8195 | 62.000 | 4.500 | 65.000 |
| MAK | .8548 | 1.2509 | .2477 | 21.000 | 64.500 | 5.000 |
| MAKAM | 2.8935 | .8631 | .2043 | 64.000 | 44.500 | 2.000 |
| MAKMR | 3.0124 | 1.1582 | .2391 | 67.000 | 54.500 | 4.000 |
| MAKGM | 1.5391 | .1164 | .2085 | 40.000 | 4.500 | 3.000 |
| MAKHM | .6052 | .1411 | .2965 | 4.000 | 31.500 | 10.000 |
| MRK | .5915 | 1.1582 | .4748 | 2.000 | 54.500 | 22.000 |
| MRKMD | .6092 | .1354 | .4921 | 5.000 | 23.500 | 23.000 |
| MRKGM | .9074 | .1164 | .3541 | 24.000 | 4.500 | 17.000 |
| MDKGM | .7485 | .1164 | .5575 | 15.000 | 4.500 | 29.000 |
| GMK | .8476 | .1164 | .8390 | 20.000 | 4.500 | 39.000 |
| GMKGM | .7015 | .1164 | .6486 | 10.000 | 4.500 | 33.000 |
| .999 | ASIMOTA | .1384 | .1384 | .1384 | 2.000 | 3.000 | 2.000 |
| GFA | .9645 | .9645 | .9645 | 5.000 | 49.000 | 24.000 |
| FAPR | .3213 | .3213 | .3213 | 4.000 | 15.000 | 4.000 |
| FASR | .1776 | .1776 | .1776 | 3.000 | 12.000 | 3.000 |
| FAMR | .1333 | .1333 | .1333 | 1.000 | 2.000 | 1.000 |
| MAKAM | 41.8677 | .8533 | .4271 | 69.000 | 44.500 | 5.000 |
| HMKGM | 11.5180 | .0843 | 11.6670 | 25.000 | 1.000 | 48.000 |

APPENDIX 12: MSE of the Ridge Parameters when P=4, n=30 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GKMN | 1.7934 | .3092 | 1.7644 | 62.000 | 4.500 | 61.000 |
| AMKMN | .9217 | .3092 | .8158 | 34.000 | 4.500 | 39.000 |
| AMKMD | .5383 | .7442 | .4743 | 5.000 | 21.500 | 15.000 |
| AMKGM | .5380 | .7963 | .4622 | 4.000 | 29.500 | 13.000 |
| MNK | 1.8657 | .3092 | 1.8607 | 65.000 | 4.500 | 65.000 |
| MNKMN | 1.9426 | .3092 | 1.9283 | 67.000 | 4.500 | 67.000 |
| MAK | .8247 | 1.2542 | .3837 | 31.000 | 60.500 | 4.000 |
| MAKMN | .5893 | .3092 | .6328 | 9.000 | 4.500 | 31.500 |
| MAKMD | .6895 | .7442 | .3573 | 24.000 | 21.500 | 2.000 |
| MAKGM | .7458 | .7963 | .3636 | 29.000 | 29.500 | 3.000 |
| MAKHM | .5423 | .5775 | .3526 | 6.000 | 13.500 | 1.000 |
| MRKAM | .6804 | 1.0580 | .4125 | 22.000 | 38.500 | 5.000 |
| MRKMN | .9255 | .3092 | .5133 | 35.000 | 4.500 | 20.000 |
| MRKMD | .5183 | .7442 | .4315 | 1.000 | 21.500 | 10.000 |
| MRKGM | .5284 | .7963 | .4228 | 2.000 | 29.500 | 6.000 |
| MRKLHM | .5359 | .5775 | .4808 | 3.000 | 13.500 | 16.000 |
| MDKMN | .9377 | .3092 | .8791 | 37.000 | 4.500 | 40.000 |
| GMKMN | 1.0795 | .3092 | .9951 | 44.000 | 4.500 | 44.000 |
| .900 | GFA | .6519 | .6519 | .6519 | 1.000 | 34.000 | 28.000 |
| GKMN | 3.0752 | .2381 | 3.0191 | 64.000 | 4.500 | 64.000 |
| AMKMN | 1.4322 | .2381 | 1.2389 | 36.000 | 4.500 | 39.000 |
| AMKMD | .7403 | .4183 | .6099 | 3.000 | 20.500 | 21.000 |
| MNK | 3.1886 | .2381 | 3.1706 | 67.000 | 4.500 | 67.000 |
| MNKMN | 3.3116 | .2381 | 3.2815 | 68.000 | 4.500 | 68.000 |
| MAK | 1.2943 | 1.2531 | .3644 | 29.000 | 60.500 | 3.000 |
| MAKAM | 2.1786 | .9680 | .3795 | 46.000 | 39.500 | 4.000 |
| MAKMN | .8865 | .2381 | .5486 | 16.000 | 4.500 | 18.500 |
| MAKMD | .9498 | .4183 | .3497 | 21.000 | 20.500 | 2.000 |
| MAKGM | 1.2115 | .5178 | .3373 | 28.000 | 29.500 | 1.000 |
| MAKHM | .7434 | .3401 | .3853 | 4.000 | 12.500 | 5.000 |
| MRKMN | 1.4942 | .2381 | .5028 | 37.000 | 4.500 | 14.000 |
| MRKMD | .7041 | .4183 | .5465 | 2.000 | 20.500 | 17.000 |
| MRKLHM | .7523 | .3401 | .6473 | 5.000 | 12.500 | 27.000 |
| MDKMN | 1.4050 | .2381 | 1.2869 | 34.000 | 4.500 | 40.000 |
| GMKMN | 1.7066 | .2381 | 1.5439 | 42.000 | 4.500 | 45.000 |
| .950 | GFA | .7691 | .7691 | .7691 | 1.000 | 36.000 | 23.000 |
| GKMN | 5.6802 | .2636 | 5.5726 | 65.000 | 4.500 | 65.000 |
| AMKMN | 2.4302 | .2636 | 2.0425 | 32.000 | 4.500 | 39.000 |
| AMKMD | 1.1493 | .3027 | .8635 | 3.000 | 20.500 | 27.000 |
| MNK | 5.8963 | .2636 | 5.8577 | 67.000 | 4.500 | 68.000 |
| MNKMN | 6.1141 | .2636 | 6.0534 | 69.000 | 4.500 | 69.000 |
| MAK | 2.9937 | 1.2525 | .3694 | 40.000 | 60.500 | 3.000 |
| MAKAM | 4.8623 | .9140 | .3585 | 53.000 | 41.500 | 2.000 |
| MAKMN | 1.4564 | .2636 | .4872 | 14.000 | 4.500 | 9.500 |
| MAKMR | 5.5431 | 1.1652 | .4112 | 63.000 | 50.500 | 5.000 |
| MAKMD | 1.5388 | .3027 | .4086 | 18.000 | 20.500 | 4.000 |
| MAKGM | 2.3999 | .3520 | .3571 | 31.000 | 29.500 | 1.000 |
| MAKHM | 1.1734 | .2904 | .4783 | 5.000 | 12.500 | 8.000 |
| MRKMN | 2.6149 | .2636 | .5008 | 35.000 | 4.500 | 13.000 |
| MRKMD | 1.0587 | .3027 | .7618 | 2.000 | 20.500 | 22.000 |
| MRKLHM | 1.1597 | .2904 | .9486 | 4.000 | 12.500 | 30.000 |
| MDKMN | 2.3243 | .2636 | 2.0662 | 26.000 | 4.500 | 40.000 |
| GMKMN | 2.9627 | .2636 | 2.6275 | 39.000 | 4.500 | 45.000 |
| .990 | ASIMOTA | 1.4342 | 1.4342 | 1.4342 | 2.000 | 65.000 | 21.000 |
| GFA | 1.0259 | 1.0259 | 1.0259 | 1.000 | 45.000 | 19.000 |
| FAPR | 3.9783 | 3.9783 | 3.9783 | 5.000 | 67.000 | 36.000 |
| MAK | 33.1222 | 1.2519 | .5658 | 68.000 | 60.500 | 5.000 |
| MAKAM | 35.9518 | .8656 | .5020 | 69.000 | 40.500 | 1.000 |
| MAKMN | 5.4995 | .6703 | .5449 | 11.000 | 24.500 | 3.500 |
| MAKMA | 37.8099 | 1.2519 | .5449 | 71.000 | 60.500 | 3.500 |
| MAKMR | 37.1416 | 1.1591 | .5155 | 70.000 | 50.500 | 2.000 |
| MRKMD | 3.5136 | .6345 | 1.7216 | 3.000 | 16.500 | 22.000 |
| MRKLHM | 3.8250 | .6932 | 2.4755 | 4.000 | 32.500 | 30.000 |
| HMKMD | 12.1142 | .2637 | 12.3230 | 32.000 | 1.000 | 49.000 |
| HMKGM | 10.8417 | .3344 | 11.0746 | 28.000 | 3.000 | 47.000 |
| HMKHM | 12.8651 | .3001 | 12.9439 | 34.500 | 2.000 | 51.000 |
| .999 | ASIMOTA | .1876 | .1876 | .1876 | 1.000 | 1.000 | 1.000 |
| GFA | 1.2425 | 1.2425 | 1.2425 | 2.000 | 30.000 | 12.000 |
| FAPR | 1.5473 | 1.5473 | 1.5473 | 3.000 | 40.000 | 13.000 |
| FASR | 2.6987 | 2.6987 | 2.6987 | 5.000 | 43.000 | 19.000 |
| FAMR | 2.0039 | 2.0039 | 2.0039 | 4.000 | 42.000 | 17.000 |
| MAKAM | 477.3075 | .8541 | .4153 | 68.000 | 6.500 | 5.000 |
| MAKMN | 48.5562 | 5.2797 | .4126 | 10.000 | 56.500 | 3.500 |
| MAKMA | 479.8913 | 1.2517 | .4126 | 71.000 | 35.500 | 3.500 |
| MAKMR | 479.0183 | 1.1576 | .4084 | 70.000 | 24.500 | 2.000 |
| HMKGM | 102.8317 | .2017 | 103.5541 | 26.000 | 2.000 | 48.000 |

APPENDIX 13: MSE of the Ridge Parameters when P=4, n=30 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .8552 | .8552 | .8552 | 1.000 | 33.000 | 19.000 |
| GKMN | 4.9114 | .4464 | 4.8239 | 64.000 | 4.500 | 63.000 |
| AMKMN | 2.2616 | .4464 | 1.9342 | 35.000 | 4.500 | 39.000 |
| AMKMD | 1.2202 | .8019 | .9386 | 5.000 | 20.500 | 22.000 |
| MNK | 5.0876 | .4464 | 5.0580 | 66.000 | 4.500 | 66.000 |
| MNKMN | 5.2783 | .4464 | 5.2296 | 68.000 | 4.500 | 67.000 |
| MAK | 2.8597 | 1.2640 | .6236 | 45.000 | 60.500 | 4.000 |
| MAKAM | 2.7555 | 1.0869 | .6011 | 44.000 | 38.500 | 3.000 |
| MAKMN | 1.4340 | .4464 | .7648 | 12.000 | 4.500 | 13.500 |
| MAKMD | 1.6392 | .8019 | .5813 | 24.000 | 20.500 | 2.000 |
| MAKGM | 1.7910 | .8485 | .5763 | 27.000 | 28.500 | 1.000 |
| MAKHM | 1.2439 | .6557 | .6315 | 7.000 | 12.500 | 5.000 |
| MRKMN | 2.3759 | .4464 | .7358 | 37.000 | 4.500 | 12.000 |
| MRKMD | 1.1408 | .8019 | .8391 | 2.000 | 20.500 | 18.000 |
| MRKGM | 1.1667 | .8485 | .8042 | 3.000 | 28.500 | 16.000 |
| MRKLHM | 1.2053 | .6557 | 1.0094 | 4.000 | 12.500 | 25.000 |
| MDKMN | 2.2127 | .4464 | 2.0036 | 32.000 | 4.500 | 40.000 |
| GMKMN | 2.7013 | .4464 | 2.4238 | 43.000 | 4.500 | 44.000 |
| .900 | GFA | .9505 | .9505 | .9505 | 1.000 | 35.000 | 18.000 |
| GKMN | 8.4938 | .4534 | 8.3329 | 64.000 | 4.500 | 64.000 |
| AMKMN | 3.6008 | .4534 | 2.9934 | 30.000 | 4.500 | 39.000 |
| AMKMD | 1.7803 | .5470 | 1.2246 | 4.000 | 20.500 | 25.000 |
| MNK | 8.8093 | .4534 | 8.7503 | 66.000 | 4.500 | 67.000 |
| MNKMN | 9.1275 | .4534 | 9.0370 | 67.000 | 4.500 | 68.000 |
| MAK | 6.1847 | 1.2588 | .6176 | 45.000 | 60.500 | 4.000 |
| MAKAM | 6.4438 | .9917 | .5706 | 46.000 | 39.500 | 1.000 |
| MAKMN | 2.1916 | .4534 | .6983 | 15.000 | 4.500 | 7.500 |
| MAKMR | 7.8942 | 1.1838 | .6151 | 60.000 | 50.500 | 3.000 |
| MAKMD | 2.5615 | .5470 | .6259 | 20.000 | 20.500 | 5.000 |
| MAKGM | 3.4036 | .6136 | .5815 | 25.000 | 29.500 | 2.000 |
| MRKMN | 3.8724 | .4534 | .7226 | 33.000 | 4.500 | 11.000 |
| MRKMD | 1.6197 | .5470 | 1.0684 | 2.000 | 20.500 | 21.000 |
| MRKGM | 1.8388 | .6136 | .9039 | 5.000 | 29.500 | 17.000 |
| MRKLHM | 1.7247 | .5007 | 1.3565 | 3.000 | 12.500 | 28.000 |
| MDKMN | 3.4581 | .4534 | 3.0507 | 26.000 | 4.500 | 40.000 |
| GMKMN | 4.4000 | .4534 | 3.8756 | 41.000 | 4.500 | 44.000 |
| .950 | GFA | 1.0613 | 1.0613 | 1.0613 | 1.000 | 45.000 | 19.000 |
| GKGM | 13.1041 | .5179 | 13.2685 | 49.000 | 4.500 | 55.000 |
| AMKMD | 2.8939 | .5533 | 1.7441 | 4.000 | 13.500 | 27.000 |
| AMKGM | 3.6465 | .5179 | 1.2631 | 12.000 | 4.500 | 20.000 |
| AMKHM | 2.9946 | .5794 | 2.2356 | 5.000 | 21.500 | 31.000 |
| MNKGM | 14.8057 | .5179 | 15.0160 | 58.000 | 4.500 | 63.000 |
| MAK | 15.3308 | 1.2563 | .6474 | 61.000 | 60.500 | 3.000 |
| MAKAM | 15.9092 | .9302 | .5976 | 64.000 | 40.500 | 1.000 |
| MAKMR | 18.1461 | 1.1715 | .6203 | 68.000 | 50.500 | 2.000 |
| MAKGM | 7.4757 | .5179 | .6666 | 31.000 | 4.500 | 4.000 |
| MRKMD | 2.5368 | .5533 | 1.4507 | 2.000 | 13.500 | 21.000 |
| MRKGM | 3.5552 | .5179 | 1.0248 | 9.000 | 4.500 | 17.000 |
| MRKLHM | 2.7060 | .5794 | 1.9502 | 3.000 | 21.500 | 29.000 |
| MDKGM | 4.3105 | .5179 | 2.1132 | 17.000 | 4.500 | 30.000 |
| GMK | 3.3452 | .5179 | 2.5098 | 8.000 | 4.500 | 32.000 |
| GMKGM | 3.2401 | .5179 | 2.6102 | 7.000 | 4.500 | 34.000 |
| .990 | ASIMOTA | 3.9044 | 3.9044 | 3.9044 | 2.000 | 66.000 | 24.000 |
| GFA | 1.2746 | 1.2746 | 1.2746 | 1.000 | 39.000 | 12.000 |
| FAPR | 9.2530 | 9.2530 | 9.2530 | 3.000 | 67.000 | 35.000 |
| MAK | 116.7216 | 1.2542 | .8701 | 69.000 | 34.500 | 5.000 |
| MAKAM | 115.3495 | .8706 | .8280 | 68.000 | 15.500 | 4.000 |
| MAKMN | 14.6047 | 1.7455 | .8036 | 10.000 | 52.500 | 2.500 |
| MAKMA | 121.3811 | 1.2542 | .8036 | 71.000 | 34.500 | 2.500 |
| MAKMR | 119.2593 | 1.1609 | .7936 | 70.000 | 24.500 | 1.000 |
| MRKMD | 9.5561 | 1.6322 | 3.6578 | 4.000 | 44.500 | 21.000 |
| MRKLHM | 9.8969 | 1.7969 | 5.5839 | 5.000 | 60.500 | 27.000 |
| HMKMD | 33.3800 | .5240 | 33.8900 | 29.000 | 2.000 | 48.000 |
| HMKGM | 29.8932 | .4076 | 30.4377 | 27.000 | 1.000 | 47.000 |
| HMKHM | 35.4611 | .6597 | 35.6141 | 32.500 | 3.000 | 49.000 |
| .999 | ASIMOTA | .2851 | .2851 | .2851 | 1.000 | 1.000 | 1.000 |
| GFA | 1.4269 | 1.4269 | 1.4269 | 2.000 | 30.000 | 12.000 |
| FAPR | 3.3661 | 3.3661 | 3.3661 | 3.000 | 40.000 | 17.000 |
| FASR | 15.0139 | 15.0139 | 15.0139 | 5.000 | 60.000 | 21.000 |
| FAMR | 8.5889 | 8.5889 | 8.5889 | 4.000 | 42.000 | 18.000 |
| MAKAM | 1360.2493 | .8556 | .5054 | 68.000 | 6.500 | 3.000 |
| MAKMN | 134.0344 | 14.5506 | .5078 | 10.000 | 55.500 | 4.500 |
| MAKMA | 1367.6369 | 1.2536 | .5078 | 71.000 | 25.500 | 4.500 |
| MAKMR | 1365.1515 | 1.1584 | .4985 | 69.000 | 15.500 | 2.000 |
| HMKGM | 285.4804 | .4364 | 287.3061 | 26.000 | 2.000 | 48.000 |

APPENDIX 14: MSE of the Ridge Parameters when P=4, n=30 and v=49

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.1095 | 1.1095 | 1.1095 | 1.000 | 34.000 | 15.000 |
| GKMN | 9.5786 | .6523 | 9.3964 | 64.000 | 4.500 | 63.000 |
| AMKMN | 4.2259 | .6523 | 3.5417 | 32.000 | 4.500 | 39.000 |
| AMKMD | 2.2143 | .8886 | 1.5455 | 5.000 | 20.500 | 22.000 |
| MNK | 9.9328 | .6523 | 9.8665 | 67.000 | 4.500 | 67.000 |
| MNKMN | 10.2941 | .6523 | 10.1936 | 68.000 | 4.500 | 68.000 |
| MAK | 6.6771 | 1.2786 | .8889 | 47.000 | 59.500 | 5.000 |
| MAKAM | 5.4013 | 1.1302 | .8155 | 44.000 | 39.500 | 1.000 |
| MAKMN | 2.6490 | .6523 | .9172 | 14.000 | 4.500 | 6.500 |
| MAKMR | 6.8906 | 1.2344 | .8411 | 48.000 | 49.500 | 2.000 |
| MAKMD | 3.1178 | .8886 | .8672 | 23.000 | 20.500 | 4.000 |
| MAKGM | 3.4304 | .9268 | .8468 | 26.000 | 28.500 | 3.000 |
| MRKMN | 4.4951 | .6523 | .9932 | 37.000 | 4.500 | 11.000 |
| MRKMD | 2.0259 | .8886 | 1.3551 | 2.000 | 20.500 | 19.000 |
| MRKGM | 2.0851 | .9268 | 1.2821 | 3.000 | 28.500 | 18.000 |
| MRKLHM | 2.1405 | .7729 | 1.7045 | 4.000 | 12.500 | 26.000 |
| MDKMN | 4.0989 | .6523 | 3.6454 | 30.000 | 4.500 | 40.000 |
| GMKMN | 5.1058 | .6523 | 4.5200 | 43.000 | 4.500 | 44.000 |
| .900 | GFA | 1.2081 | 1.2081 | 1.2081 | 1.000 | 55.000 | 15.000 |
| GKMD | 14.2303 | .7399 | 14.3643 | 51.000 | 4.500 | 55.000 |
| AMKMD | 3.3348 | .7399 | 2.0413 | 4.000 | 4.500 | 24.000 |
| AMKHM | 3.4166 | .7415 | 2.5946 | 5.000 | 12.500 | 30.000 |
| MNKMD | 15.9620 | .7399 | 16.1367 | 61.000 | 4.500 | 63.000 |
| MAK | 15.4425 | 1.2673 | .9047 | 59.000 | 60.500 | 5.000 |
| MAKAM | 13.5768 | 1.0272 | .8361 | 49.000 | 38.500 | 1.000 |
| MAKMN | 4.0855 | .7762 | .8830 | 14.000 | 29.500 | 3.500 |
| MAKMA | 19.4672 | 1.2673 | .8830 | 68.000 | 60.500 | 3.500 |
| MAKMR | 16.6842 | 1.2005 | .8372 | 64.000 | 49.500 | 2.000 |
| MAKMD | 5.1544 | .7399 | 1.0009 | 20.000 | 4.500 | 9.000 |
| MRKMD | 2.9750 | .7399 | 1.7326 | 2.000 | 4.500 | 21.000 |
| MRKLHM | 3.1094 | .7415 | 2.2884 | 3.000 | 12.500 | 28.000 |
| MDK | 4.8887 | .7399 | 3.1865 | 19.000 | 4.500 | 35.000 |
| MDKMD | 4.1424 | .7399 | 2.8788 | 15.000 | 4.500 | 32.000 |
| GMKMD | 3.9990 | .7399 | 3.6222 | 11.000 | 4.500 | 37.000 |
| .950 | GFA | 1.3162 | 1.3162 | 1.3162 | 1.000 | 64.000 | 15.000 |
| AMKMD | 5.5296 | .9292 | 2.9590 | 4.000 | 15.500 | 24.000 |
| AMKHM | 5.6091 | 1.0129 | 3.9164 | 5.000 | 32.500 | 30.000 |
| MAK | 36.9701 | 1.2619 | .9666 | 67.000 | 59.500 | 5.000 |
| MAKAM | 34.1077 | .9545 | .9231 | 66.000 | 24.500 | 4.000 |
| MAKMN | 6.8759 | 1.0766 | .8991 | 10.000 | 40.500 | 2.500 |
| MAKMA | 42.3169 | 1.2619 | .8991 | 69.000 | 59.500 | 2.500 |
| MAKMR | 38.9803 | 1.1808 | .8801 | 68.000 | 49.500 | 1.000 |
| MRKMD | 4.7830 | .9292 | 2.3644 | 2.000 | 15.500 | 21.000 |
| MRKLHM | 4.9654 | 1.0129 | 3.3077 | 3.000 | 32.500 | 29.000 |
| HMKMD | 14.1146 | .7635 | 14.3483 | 29.000 | 2.000 | 46.000 |
| HMKHM | 15.0632 | .7052 | 15.1427 | 30.500 | 1.000 | 47.000 |
| .990 | ASIMOTA | 7.6081 | 7.6081 | 7.6081 | 2.000 | 66.000 | 25.000 |
| GFA | 1.5120 | 1.5120 | 1.5120 | 1.000 | 39.000 | 8.000 |
| FAPR | 16.1251 | 16.1251 | 16.1251 | 3.000 | 67.000 | 35.000 |
| MAK | 244.9087 | 1.2577 | 1.1442 | 69.000 | 26.500 | 4.000 |
| MAKAM | 237.3063 | .8780 | 1.1596 | 68.000 | 5.500 | 5.000 |
| MAKMN | 28.2335 | 3.3580 | 1.0406 | 9.000 | 52.500 | 1.500 |
| MAKMA | 249.7960 | 1.2577 | 1.0406 | 71.000 | 26.500 | 1.500 |
| MAKMR | 245.4269 | 1.1638 | 1.0577 | 70.000 | 15.500 | 3.000 |
| MRKMD | 18.7409 | 3.1286 | 6.5391 | 4.000 | 44.500 | 23.000 |
| MRKLHM | 19.0223 | 3.4521 | 10.1861 | 5.000 | 60.500 | 28.000 |
| HMKGM | 58.4746 | .5172 | 59.4777 | 27.000 | 1.000 | 47.000 |
| .999 | ASIMOTA | .4308 | .4308 | .4308 | 1.000 | 1.000 | 1.000 |
| GFA | 1.6449 | 1.6449 | 1.6449 | 2.000 | 30.000 | 12.000 |
| FAPR | 5.6550 | 5.6550 | 5.6550 | 3.000 | 39.000 | 17.000 |
| FASR | 50.2198 | 50.2198 | 50.2198 | 5.000 | 69.000 | 28.000 |
| FAMR | 22.5139 | 22.5139 | 22.5139 | 4.000 | 42.000 | 19.000 |
| MAKAM | 2685.6776 | .8577 | .6730 | 68.000 | 6.500 | 5.000 |
| MAKMN | 262.2420 | 28.4562 | .6671 | 10.000 | 55.500 | 3.500 |
| MAKMA | 2700.2526 | 1.2566 | .6671 | 71.000 | 25.500 | 3.500 |
| MAKMR | 2695.3570 | 1.1597 | .6572 | 69.000 | 15.500 | 2.000 |
| HMKGM | 559.4617 | .7882 | 562.9347 | 25.000 | 2.000 | 49.000 |

APPENDIX 15: MSE of the Ridge Parameters when P=4, n=30 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | 1.4350 | 1.4350 | 1.4350 | 1.000 | 64.000 | 7.000 |
| GKHM | 17.2022 | 1.0222 | 17.2553 | 54.000 | 4.500 | 55.000 |
| AMKMD | 4.3105 | 1.0728 | 2.7849 | 5.000 | 12.500 | 22.000 |
| AMKHM | 4.4606 | 1.0222 | 3.5090 | 7.000 | 4.500 | 29.000 |
| MNKHM | 19.1734 | 1.0222 | 19.3033 | 64.000 | 4.500 | 65.000 |
| MAK | 15.2013 | 1.3094 | 1.3817 | 48.000 | 57.500 | 5.000 |
| MAKAM | 11.1571 | 1.2220 | 1.2493 | 44.000 | 39.500 | 4.000 |
| MAKMN | 5.2033 | 1.0900 | 1.2154 | 14.000 | 20.500 | 2.500 |
| MAKMA | 18.8005 | 1.3094 | 1.2154 | 62.000 | 57.500 | 2.500 |
| MAKMR | 14.2929 | 1.2948 | 1.2149 | 47.000 | 49.500 | 1.000 |
| MAKHM | 4.5523 | 1.0222 | 1.7304 | 8.000 | 4.500 | 13.000 |
| MRKMD | 3.8843 | 1.0728 | 2.3954 | 2.000 | 12.500 | 20.000 |
| MRKGM | 4.0214 | 1.0931 | 2.2424 | 3.000 | 28.500 | 18.000 |
| MRKLHM | 4.0909 | 1.0222 | 3.1235 | 4.000 | 4.500 | 26.000 |
| MDKHM | 5.2414 | 1.0222 | 4.3907 | 15.000 | 4.500 | 35.000 |
| GMKHM | 5.8994 | 1.0222 | 5.4805 | 20.000 | 4.500 | 38.000 |
| HMK | 10.0549 | 1.0222 | 10.1357 | 40.500 | 4.500 | 48.000 |
| .900 | GFA | 1.5589 | 1.5589 | 1.5589 | 1.000 | 64.000 | 8.000 |
| AMKMD | 6.6388 | 1.1500 | 3.7222 | 4.000 | 24.500 | 24.000 |
| AMKHM | 6.7033 | 1.2531 | 4.8525 | 5.000 | 42.500 | 30.000 |
| MAK | 36.1969 | 1.2853 | 1.4173 | 66.000 | 51.500 | 5.000 |
| MAKAM | 29.1827 | 1.1027 | 1.3746 | 51.000 | 14.500 | 4.000 |
| MAKMN | 8.0801 | 1.4620 | 1.2224 | 12.000 | 59.500 | 1.500 |
| MAKMA | 41.9265 | 1.2853 | 1.2224 | 68.000 | 51.500 | 1.500 |
| MAKMR | 35.9496 | 1.2359 | 1.2700 | 65.000 | 34.500 | 3.000 |
| MRKMD | 5.8619 | 1.1500 | 3.0883 | 2.000 | 24.500 | 21.000 |
| MRKLHM | 6.0235 | 1.2531 | 4.2041 | 3.000 | 42.500 | 28.000 |
| HMKHM | 16.8040 | 1.0590 | 16.8822 | 34.500 | 1.000 | 47.000 |
| .950 | GFA | 1.6839 | 1.6839 | 1.6839 | 1.000 | 40.000 | 8.000 |
| AMKMD | 11.1444 | 1.7276 | 5.4921 | 4.000 | 44.500 | 24.000 |
| AMKHM | 11.1581 | 1.9337 | 7.4400 | 5.000 | 52.500 | 31.000 |
| MAK | 84.5189 | 1.2739 | 1.5092 | 68.000 | 26.500 | 4.000 |
| MAKAM | 73.7245 | 1.0062 | 1.5733 | 65.000 | 7.500 | 5.000 |
| MAKMN | 13.7057 | 2.1128 | 1.2885 | 10.000 | 60.500 | 1.500 |
| MAKMA | 91.5930 | 1.2739 | 1.2885 | 69.000 | 26.500 | 1.500 |
| MAKMR | 84.3748 | 1.2006 | 1.3688 | 67.000 | 16.500 | 3.000 |
| MRKMD | 9.5965 | 1.7276 | 4.2633 | 2.000 | 44.500 | 21.000 |
| MRKLHM | 9.7536 | 1.9337 | 6.1333 | 3.000 | 52.500 | 28.000 |
| HMKMD | 28.6461 | .9043 | 29.0879 | 29.000 | 1.000 | 46.000 |
| HMKGM | 26.7270 | 1.0002 | 27.2754 | 28.000 | 3.000 | 45.000 |
| HMKHM | 30.5800 | .9432 | 30.7098 | 31.500 | 2.000 | 48.000 |
| .990 | ASIMOTA | 15.4773 | 15.4773 | 15.4773 | 2.000 | 66.000 | 25.000 |
| GFA | 1.9068 | 1.9068 | 1.9068 | 1.000 | 30.000 | 6.000 |
| FAPR | 29.0200 | 29.0200 | 29.0200 | 3.000 | 67.000 | 34.000 |
| MAK | 518.3198 | 1.2652 | 1.6055 | 70.000 | 24.500 | 4.000 |
| MAKAM | 497.8694 | .8936 | 1.7707 | 68.000 | 5.500 | 5.000 |
| MAKMN | 57.1821 | 6.7843 | 1.4364 | 9.000 | 52.500 | 1.500 |
| MAKMA | 524.1440 | 1.2652 | 1.4364 | 71.000 | 24.500 | 1.500 |
| MAKMR | 514.9848 | 1.1699 | 1.5194 | 69.000 | 14.500 | 3.000 |
| MRKMD | 38.3364 | 6.3081 | 12.6606 | 4.000 | 44.500 | 24.000 |
| MRKLHM | 38.4318 | 6.9692 | 19.9448 | 5.000 | 60.500 | 28.000 |
| HMKGM | 119.2142 | .7499 | 121.1868 | 26.000 | 1.000 | 47.000 |
| .999 | ASIMOTA | .7399 | .7399 | .7399 | 1.000 | 1.000 | 1.000 |
| GFA | 2.0655 | 2.0655 | 2.0655 | 2.000 | 30.000 | 12.000 |
| FAPR | 9.8399 | 9.8399 | 9.8399 | 3.000 | 31.000 | 17.000 |
| FASR | 155.1674 | 155.1674 | 155.1674 | 5.000 | 69.000 | 29.000 |
| FAMR | 62.3640 | 62.3640 | 62.3640 | 4.000 | 67.000 | 23.000 |
| MAK | 5530.2622 | 1.2632 | .9677 | 70.000 | 24.500 | 5.000 |
| MAKMN | 534.6806 | 58.0048 | .9450 | 10.000 | 54.500 | 2.500 |
| MAKMA | 5532.3212 | 1.2632 | .9450 | 71.000 | 24.500 | 2.500 |
| MAKMR | 5522.3293 | 1.1626 | .9531 | 69.000 | 14.500 | 4.000 |

APPENDIX 16: MSE of the Ridge Parameters when P=4, n=50 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | HKMA | .1464 | .1464 | .1464 | 25.000 | 3.000 | 24.000 |
| ASIMOTA | .1524 | .1524 | .1524 | 30.000 | 4.000 | 31.000 |
| GFA | .1281 | .1281 | .1281 | 1.000 | 1.000 | 1.000 |
| FAPR | .1449 | .1449 | .1449 | 15.000 | 2.000 | 15.000 |
| FAMR | .1569 | .1569 | .1569 | 35.000 | 5.000 | 35.000 |
| GMKMN | .1391 | .7206 | .1397 | 2.000 | 12.500 | 2.000 |
| HMK | .1402 | .8795 | .1402 | 4.500 | 28.500 | 6.000 |
| HMKMD | .1402 | 1.1842 | .1402 | 3.000 | 61.000 | 4.000 |
| HMKGM | .1402 | 1.1932 | .1402 | 7.000 | 71.000 | 5.000 |
| HMKHM | .1402 | 1.1885 | .1402 | 4.500 | 62.000 | 3.000 |
| .900 | GHK | .2562 | .2562 | .2562 | 44.500 | 4.000 | 45.500 |
| HKMA | .2505 | .2505 | .2505 | 40.000 | 3.000 | 42.000 |
| ASIMOTA | .2581 | .2581 | .2581 | 46.000 | 5.000 | 47.000 |
| GFA | .1961 | .1961 | .1961 | 1.000 | 1.000 | 1.000 |
| FAPR | .2419 | .2419 | .2419 | 31.000 | 2.000 | 30.000 |
| AMKMN | .2056 | .5533 | .2082 | 3.000 | 12.500 | 2.000 |
| MRKMN | .2054 | .5533 | .3965 | 2.000 | 12.500 | 67.000 |
| GMK | .2125 | .7358 | .2120 | 4.000 | 36.500 | 3.000 |
| GMKMD | .2125 | .6191 | .2120 | 5.000 | 20.500 | 4.000 |
| GMKHM | .2126 | .6355 | .2121 | 6.000 | 28.500 | 5.000 |
| .950 | GHK | .4073 | .4073 | .4073 | 42.500 | 4.000 | 45.500 |
| HKMA | .4340 | .4340 | .4340 | 58.000 | 5.000 | 62.000 |
| ASIMOTA | .4056 | .4056 | .4056 | 41.000 | 3.000 | 44.000 |
| GFA | .2899 | .2899 | .2899 | 2.000 | 1.000 | 1.000 |
| FAPR | .3894 | .3894 | .3894 | 38.000 | 2.000 | 40.000 |
| AMK | .2964 | .9555 | .2940 | 6.000 | 47.500 | 5.000 |
| AMKMN | .3002 | .5020 | .2939 | 8.000 | 11.500 | 4.000 |
| AMKMD | .2938 | .5246 | .2923 | 3.000 | 19.500 | 2.000 |
| AMKHM | .2950 | .5295 | .2937 | 5.000 | 27.500 | 3.000 |
| MRKMN | .2874 | .5020 | .4380 | 1.000 | 11.500 | 65.000 |
| MRKMD | .2943 | .5246 | .2975 | 4.000 | 19.500 | 6.000 |
| .990 | ASIMOTA | .4887 | .4887 | .4887 | 1.000 | 12.000 | 6.000 |
| MAK | .4961 | 1.1675 | .4190 | 2.000 | 64.500 | 1.000 |
| MAKAM | 2.0493 | .9128 | .4740 | 69.000 | 43.500 | 5.000 |
| MAKMN | .5063 | .5136 | .5190 | 4.000 | 24.500 | 9.500 |
| MAKMD | .5030 | .5176 | .4522 | 3.000 | 32.500 | 2.000 |
| MAKGM | 1.0953 | .4578 | .4737 | 42.000 | 6.500 | 4.000 |
| MAKHM | .5306 | .5133 | .4531 | 5.000 | 16.500 | 3.000 |
| HMKMD | 1.0563 | .4536 | 1.0575 | 41.000 | 2.000 | 51.000 |
| HMKHM | 1.0341 | .4536 | 1.0404 | 38.500 | 1.000 | 50.000 |
| .999 | ASIMOTA | .3858 | .3858 | .3858 | 1.000 | 1.000 | 1.000 |
| FAPR | .6240 | .6240 | .6240 | 4.000 | 16.000 | 6.000 |
| FASR | .5727 | .5727 | .5727 | 3.000 | 14.000 | 5.000 |
| FAMR | .4983 | .4983 | .4983 | 2.000 | 12.000 | 4.000 |
| MAKMD | .8460 | .9011 | .4943 | 5.000 | 37.500 | 2.000 |
| MAKHM | 1.0920 | .8785 | .4976 | 7.000 | 29.500 | 3.000 |
| HMKGM | 6.1757 | .3992 | 6.4759 | 33.000 | 2.000 | 49.000 |

APPENDIX 17: MSE of the Ridge Parameters when P=4, n=50 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .4467 | .4467 | .4467 | 1.000 | 1.000 | 1.000 |
| MAKMN | .4546 | .7469 | .7472 | 2.000 | 5.500 | 48.500 |
| MRK | .5126 | 1.1749 | .4887 | 9.000 | 51.500 | 5.000 |
| MRKMD | .4859 | .8776 | .4809 | 4.000 | 13.500 | 3.000 |
| MRKGM | .4926 | .9825 | .4880 | 5.000 | 31.500 | 4.000 |
| MRKLHM | .4836 | .9000 | .4807 | 3.000 | 22.500 | 2.000 |
| .900 | GFA | .5443 | .5443 | .5443 | 2.000 | 1.000 | 5.000 |
| MAK | .6945 | 1.1796 | .4997 | 16.000 | 60.500 | 2.000 |
| MAKMN | .5343 | .6074 | .7042 | 1.000 | 5.500 | 31.500 |
| MAKMD | .5468 | .6680 | .4984 | 3.000 | 13.500 | 1.000 |
| MAKGM | .7727 | .7654 | .5297 | 26.000 | 29.500 | 4.000 |
| MAKHM | .5711 | .6816 | .5020 | 4.000 | 21.500 | 3.000 |
| MRKGM | .5835 | .7654 | .5553 | 5.000 | 29.500 | 6.000 |
| .950 | GFA | .6584 | .6584 | .6584 | 3.000 | 34.000 | 16.000 |
| GKMN | 2.7793 | .5964 | 2.7569 | 59.000 | 4.500 | 58.000 |
| AMKMN | 1.2468 | .5964 | 1.1008 | 29.000 | 4.500 | 37.000 |
| MNK | 3.0120 | .5964 | 3.0099 | 66.000 | 4.500 | 67.000 |
| MNKMN | 3.0833 | .5964 | 3.0684 | 67.000 | 4.500 | 68.000 |
| MAK | 1.2594 | 1.1731 | .5233 | 30.000 | 60.500 | 1.000 |
| MAKAM | 2.5525 | .9604 | .5869 | 51.000 | 40.500 | 5.000 |
| MAKMN | .6713 | .5964 | .6693 | 4.000 | 4.500 | 17.500 |
| MAKMD | .6003 | .6181 | .5298 | 1.000 | 12.500 | 3.000 |
| MAKGM | 1.1200 | .6339 | .5454 | 23.000 | 28.500 | 4.000 |
| MAKHM | .6319 | .6187 | .5266 | 2.000 | 20.500 | 2.000 |
| MRKMN | 1.2468 | .5964 | .6518 | 28.000 | 4.500 | 15.000 |
| MRKGM | .7069 | .6339 | .6275 | 5.000 | 28.500 | 9.000 |
| MDKMN | 1.2861 | .5964 | 1.2111 | 32.000 | 4.500 | 40.000 |
| GMKMN | 1.5512 | .5964 | 1.4338 | 37.000 | 4.500 | 41.000 |
| .990 | GFA | .8820 | .8820 | .8820 | 2.000 | 29.000 | 20.000 |
| AMK | 5.4793 | .9142 | .7146 | 30.000 | 41.500 | 4.000 |
| AMKMA | 11.3648 | 1.1683 | .7387 | 48.000 | 60.500 | 5.000 |
| MAKMN | 1.3139 | .8703 | .7673 | 4.000 | 16.500 | 14.500 |
| MAKMD | .8753 | .8960 | .5519 | 1.000 | 33.500 | 2.000 |
| MAKHM | 1.0830 | .8762 | .5507 | 3.000 | 24.500 | 1.000 |
| MRKGM | 2.2099 | .5971 | .6386 | 10.000 | 7.500 | 3.000 |
| MRKLHM | 1.5854 | .8762 | 1.2328 | 5.000 | 24.500 | 21.000 |
| HMKMD | 7.3067 | .5589 | 7.3043 | 38.000 | 2.000 | 50.000 |
| HMKGM | 5.8357 | .5708 | 6.1254 | 31.000 | 3.000 | 47.000 |
| HMKHM | 7.0540 | .5444 | 7.1173 | 35.500 | 1.000 | 48.000 |
| .999 | ASIMOTA | .4387 | .4387 | .4387 | 1.000 | 1.000 | 1.000 |
| GFA | 1.0422 | 1.0422 | 1.0422 | 2.000 | 19.000 | 12.000 |
| FAPR | 2.0491 | 2.0491 | 2.0491 | 3.000 | 41.000 | 19.000 |
| FAMR | 4.5712 | 4.5712 | 4.5712 | 4.000 | 67.000 | 27.000 |
| MAKAM | 249.5822 | .9034 | .6170 | 69.000 | 6.500 | 5.000 |
| MAKMN | 7.2895 | 4.1091 | .6141 | 7.000 | 46.500 | 3.500 |
| MAKMA | 251.1422 | 1.1673 | .6141 | 71.000 | 34.500 | 3.500 |
| MAKMR | 250.6151 | 1.1049 | .6117 | 70.000 | 24.500 | 2.000 |
| MAKMD | 5.3430 | 4.3628 | 1.2180 | 5.000 | 62.500 | 13.000 |
| HMKGM | 52.8057 | .4669 | 55.4618 | 27.000 | 2.000 | 49.000 |

APPENDIX 18: MSE of the Ridge Parameters when P=4, n=50 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .6543 | .6543 | .6543 | 1.000 | 1.000 | 3.000 |
| MAK | 1.2624 | 1.2003 | .6787 | 37.000 | 58.500 | 5.000 |
| MAKMN | .7082 | .7992 | .8620 | 3.000 | 5.500 | 24.500 |
| MAKMD | .7028 | .9206 | .6415 | 2.000 | 13.500 | 1.000 |
| MAKGM | .8732 | 1.0142 | .6698 | 12.000 | 29.500 | 4.000 |
| MAKHM | .7269 | .9406 | .6453 | 4.000 | 21.500 | 2.000 |
| MRKGM | .7481 | 1.0142 | .7267 | 5.000 | 29.500 | 8.000 |
| .900 | GFA | .7465 | .7465 | .7465 | 1.000 | 9.000 | 6.000 |
| GKMN | 3.9510 | .7157 | 3.9189 | 60.000 | 4.500 | 58.000 |
| AMKMN | 1.6914 | .7157 | 1.4611 | 29.000 | 4.500 | 37.000 |
| MNK | 4.2833 | .7157 | 4.2802 | 65.000 | 4.500 | 66.000 |
| MNKMN | 4.3884 | .7157 | 4.3664 | 66.000 | 4.500 | 67.000 |
| MAK | 2.5017 | 1.1829 | .6888 | 44.000 | 60.500 | 4.000 |
| MAKAM | 2.7450 | 1.0279 | .7208 | 45.000 | 39.500 | 5.000 |
| MAKMN | .8593 | .7157 | .8110 | 5.000 | 4.500 | 16.500 |
| MAKMD | .7501 | .7656 | .6481 | 2.000 | 13.500 | 2.000 |
| MAKGM | 1.2532 | .8245 | .6636 | 14.000 | 29.500 | 3.000 |
| MAKHM | .7914 | .7736 | .6444 | 3.000 | 21.500 | 1.000 |
| MRKMN | 1.6915 | .7157 | .7979 | 30.000 | 4.500 | 14.000 |
| MRKGM | .8412 | .8245 | .7841 | 4.000 | 29.500 | 10.000 |
| MDKMN | 1.7561 | .7157 | 1.6369 | 33.000 | 4.500 | 40.000 |
| GMKMN | 2.1463 | .7157 | 1.9616 | 37.000 | 4.500 | 41.000 |
| .950 | GFA | .8520 | .8520 | .8520 | 2.000 | 36.000 | 18.000 |
| MAK | 6.5202 | 1.1754 | .7476 | 48.000 | 60.500 | 4.000 |
| MAKAM | 7.0043 | .9701 | .7516 | 52.000 | 40.500 | 5.000 |
| MAKMN | 1.1080 | .7852 | .8035 | 5.000 | 14.500 | 13.500 |
| MAKMD | .8452 | .8048 | .6615 | 1.000 | 31.500 | 2.000 |
| MAKGM | 2.4451 | .7368 | .7117 | 20.000 | 5.500 | 3.000 |
| MAKHM | .9354 | .7969 | .6529 | 3.000 | 22.500 | 1.000 |
| MRKGM | 1.0221 | .7368 | .7962 | 4.000 | 5.500 | 11.000 |
| HMKMN | 5.0170 | .7343 | 4.8394 | 42.000 | 1.000 | 49.000 |
| .990 | GFA | 1.0379 | 1.0379 | 1.0379 | 1.000 | 21.000 | 8.000 |
| MAKMN | 2.6560 | 1.5836 | .9194 | 4.000 | 44.500 | 4.500 |
| MAKMA | 61.2880 | 1.1699 | .9194 | 71.000 | 36.500 | 4.500 |
| MAKMR | 60.0070 | 1.1092 | .9165 | 70.000 | 26.500 | 3.000 |
| MAKMD | 1.8086 | 1.6520 | .7560 | 2.000 | 60.500 | 1.000 |
| MAKHM | 2.5773 | 1.6013 | .8106 | 3.000 | 52.500 | 2.000 |
| MRKLHM | 3.0932 | 1.6013 | 2.0192 | 5.000 | 52.500 | 21.000 |
| HMKMD | 19.7508 | .7692 | 19.7186 | 34.000 | 3.000 | 50.000 |
| HMKGM | 15.5968 | .6126 | 16.4023 | 30.000 | 1.000 | 47.000 |
| HMKHM | 19.0351 | .7256 | 19.1914 | 32.500 | 2.000 | 48.000 |
| .999 | ASIMOTA | .5440 | .5440 | .5440 | 1.000 | 1.000 | 1.000 |
| GFA | 1.1538 | 1.1538 | 1.1538 | 2.000 | 21.000 | 12.000 |
| FAPR | 4.1358 | 4.1358 | 4.1358 | 3.000 | 41.000 | 19.000 |
| FAMR | 18.6911 | 18.6911 | 18.6911 | 5.000 | 68.000 | 29.000 |
| MAKAM | 713.7591 | .9042 | .6543 | 68.000 | 6.500 | 5.000 |
| MAKMN | 19.1494 | 10.5801 | .6543 | 6.000 | 46.500 | 3.500 |
| MAKMA | 718.2106 | 1.1687 | .6543 | 71.000 | 26.500 | 3.500 |
| MAKMR | 716.7139 | 1.1056 | .6491 | 69.000 | 15.500 | 2.000 |
| MAKMD | 14.5896 | 11.2837 | 2.7662 | 4.000 | 62.500 | 17.000 |
| HMKGM | 146.0590 | .6021 | 153.4045 | 27.000 | 2.000 | 49.000 |

APPENDIX 19: MSE of the Ridge Parameters when P=4, n=50 and v=49

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .8106 | .8106 | .8106 | 1.000 | 1.000 | 4.000 |
| MAKAM | 1.8589 | 1.1496 | .8548 | 32.000 | 39.500 | 5.000 |
| MAKMN | 1.0195 | .8776 | .9662 | 4.000 | 5.500 | 13.500 |
| MAKMD | .9117 | .9849 | .7984 | 2.000 | 13.500 | 2.000 |
| MAKGM | 1.1784 | 1.0618 | .8070 | 9.000 | 29.500 | 3.000 |
| MAKHM | .9440 | 1.0015 | .7959 | 3.000 | 21.500 | 1.000 |
| MRKGM | 1.0327 | 1.0618 | .9829 | 5.000 | 29.500 | 17.000 |
| .900 | GFA | .9037 | .9037 | .9037 | 1.000 | 9.000 | 7.000 |
| GKMN | 7.6033 | .8781 | 7.5335 | 60.000 | 4.500 | 58.000 |
| AMKMN | 2.9364 | .8781 | 2.4275 | 28.000 | 4.500 | 37.000 |
| MNK | 8.2658 | .8781 | 8.2584 | 65.000 | 4.500 | 66.000 |
| MNKMN | 8.4730 | .8781 | 8.4276 | 66.000 | 4.500 | 67.000 |
| MAK | 6.1630 | 1.1879 | .8806 | 47.000 | 60.500 | 4.000 |
| MAKAM | 5.1480 | 1.0504 | .8846 | 44.000 | 39.500 | 5.000 |
| MAKMN | 1.2680 | .8781 | .9280 | 5.000 | 4.500 | 8.500 |
| MAKMD | 1.0119 | .9118 | .8145 | 2.000 | 21.500 | 2.000 |
| MAKGM | 2.0371 | .9129 | .8313 | 15.000 | 29.500 | 3.000 |
| MAKHM | 1.0922 | .9114 | .8050 | 3.000 | 13.500 | 1.000 |
| MRKMN | 2.9533 | .8781 | .9636 | 29.000 | 4.500 | 13.000 |
| MRKGM | 1.1320 | .9129 | 1.0157 | 4.000 | 29.500 | 17.000 |
| MDKMN | 3.0662 | .8781 | 2.7989 | 30.000 | 4.500 | 40.000 |
| GMKMN | 3.8858 | .8781 | 3.4781 | 36.000 | 4.500 | 41.000 |
| .950 | GFA | 1.0075 | 1.0075 | 1.0075 | 1.000 | 21.000 | 13.000 |
| MAKMN | 1.6868 | 1.0683 | .9565 | 5.000 | 33.500 | 4.500 |
| MAKMA | 19.6516 | 1.1787 | .9565 | 68.000 | 60.500 | 4.500 |
| MAKMR | 17.6935 | 1.1311 | .9457 | 67.000 | 50.500 | 3.000 |
| MAKMD | 1.2158 | 1.0847 | .8306 | 2.000 | 41.500 | 2.000 |
| MAKHM | 1.4251 | 1.0642 | .8245 | 3.000 | 25.500 | 1.000 |
| MRKGM | 1.5108 | .8910 | .9863 | 4.000 | 7.500 | 11.000 |
| HMKMN | 9.5806 | .8525 | 9.2114 | 41.000 | 3.000 | 49.000 |
| HMKMD | 8.2130 | .8397 | 8.2124 | 38.000 | 1.000 | 48.000 |
| HMKHM | 7.9614 | .8504 | 8.0261 | 36.500 | 2.000 | 46.000 |
| .990 | GFA | 1.0075 | 1.0075 | 1.0075 | 1.000 | 21.000 | 13.000 |
| MAKMN | 1.6868 | 1.0683 | .9565 | 5.000 | 33.500 | 4.500 |
| MAKMA | 19.6516 | 1.1787 | .9565 | 68.000 | 60.500 | 4.500 |
| MAKMR | 17.6935 | 1.1311 | .9457 | 67.000 | 50.500 | 3.000 |
| MAKMD | 1.2158 | 1.0847 | .8306 | 2.000 | 41.500 | 2.000 |
| MAKHM | 1.4251 | 1.0642 | .8245 | 3.000 | 25.500 | 1.000 |
| MRKGM | 1.5108 | .8910 | .9863 | 4.000 | 7.500 | 11.000 |
| HMKMN | 9.5806 | .8525 | 9.2114 | 41.000 | 3.000 | 49.000 |
| HMKMD | 8.2130 | .8397 | 8.2124 | 38.000 | 1.000 | 48.000 |
| HMKHM | 7.9614 | .8504 | 8.0261 | 36.500 | 2.000 | 46.000 |
| .999 | ASIMOTA | .7017 | .7017 | .7017 | 1.000 | 1.000 | 1.000 |
| GFA | 1.2858 | 1.2858 | 1.2858 | 2.000 | 30.000 | 12.000 |
| FAPR | 6.7445 | 6.7445 | 6.7445 | 3.000 | 41.000 | 18.000 |
| MAKAM | 1410.7928 | .9053 | .7490 | 68.000 | 6.500 | 5.000 |
| MAKMN | 36.9325 | 20.2860 | .7447 | 5.000 | 46.500 | 3.500 |
| MAKMA | 1419.5723 | 1.1708 | .7447 | 71.000 | 25.500 | 3.500 |
| MAKMR | 1416.6254 | 1.1066 | .7389 | 69.000 | 15.500 | 2.000 |
| MAKMD | 28.4859 | 21.6638 | 5.0978 | 4.000 | 62.500 | 17.000 |
| HMKGM | 285.9381 | .8047 | 300.3157 | 27.000 | 2.000 | 49.000 |

APPENDIX 20: MSE of the Ridge Parameters when P=4, n=50 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .9943 | .9943 | .9943 | 1.000 | 1.000 | 1.000 |
| MAKAM | 3.1516 | 1.2113 | 1.0921 | 29.000 | 40.500 | 5.000 |
| MAKMD | 1.3212 | 1.1213 | 1.0915 | 2.000 | 14.500 | 4.000 |
| MAKGM | 1.8096 | 1.1627 | 1.0620 | 8.000 | 31.500 | 2.000 |
| MAKHM | 1.3735 | 1.1307 | 1.0766 | 3.000 | 22.500 | 3.000 |
| MRKAM | 1.6066 | 1.2113 | 1.2771 | 5.000 | 40.500 | 13.000 |
| MRKGM | 1.5768 | 1.1627 | 1.4625 | 4.000 | 31.500 | 17.000 |
| .900 | GFA | 1.1063 | 1.1063 | 1.1063 | 1.000 | 21.000 | 1.000 |
| AMKGM | 2.0737 | 1.1005 | 1.7822 | 5.000 | 15.500 | 20.000 |
| MAKMN | 2.0812 | 1.2230 | 1.1382 | 6.000 | 60.500 | 4.500 |
| MAKMA | 17.9650 | 1.1984 | 1.1382 | 67.000 | 36.500 | 4.500 |
| MAKMD | 1.5454 | 1.2225 | 1.1311 | 2.000 | 52.500 | 3.000 |
| MAKHM | 1.7208 | 1.2042 | 1.1139 | 4.000 | 44.500 | 2.000 |
| MRKGM | 1.6905 | 1.1005 | 1.4305 | 3.000 | 15.500 | 17.000 |
| HMKMN | 10.2753 | 1.0599 | 9.8913 | 44.000 | 1.000 | 49.000 |
| HMKMA | 6.7268 | 1.0898 | 6.9899 | 32.000 | 2.000 | 42.000 |
| HMKMD | 8.8474 | 1.0921 | 8.8453 | 41.000 | 3.000 | 48.000 |
| .950 | GFA | 1.2246 | 1.2246 | 1.2246 | 1.000 | 40.000 | 5.000 |
| MAKMN | 2.8796 | 1.6700 | 1.2134 | 5.000 | 52.500 | 3.500 |
| MAKMA | 42.6321 | 1.1858 | 1.2134 | 68.000 | 27.500 | 3.500 |
| MAKMD | 2.0187 | 1.6795 | 1.1831 | 2.000 | 60.500 | 1.000 |
| MAKHM | 2.5021 | 1.6318 | 1.1886 | 3.000 | 44.500 | 2.000 |
| MRKGM | 2.6002 | 1.2184 | 1.3789 | 4.000 | 35.500 | 10.000 |
| HMKMD | 16.4418 | .9666 | 16.4303 | 37.000 | 2.000 | 48.000 |
| HMKHM | 15.9208 | .9555 | 16.0455 | 34.500 | 1.000 | 46.000 |
| .990 | GFA | 1.4203 | 1.4203 | 1.4203 | 1.000 | 29.000 | 5.000 |
| MAK | 263.5825 | 1.1770 | 1.4020 | 70.000 | 24.500 | 4.000 |
| MAKMN | 8.8542 | 4.9263 | 1.2738 | 3.000 | 44.500 | 1.500 |
| MAKMA | 267.7132 | 1.1770 | 1.2738 | 71.000 | 24.500 | 1.500 |
| MAKMR | 262.1953 | 1.1147 | 1.3302 | 69.000 | 14.500 | 3.000 |
| MAKMD | 6.4171 | 5.1945 | 1.8054 | 2.000 | 60.500 | 7.000 |
| MAKHM | 9.9590 | 4.9986 | 2.1478 | 4.000 | 52.500 | 12.000 |
| MRKLHM | 10.0022 | 4.9986 | 5.5467 | 5.000 | 52.500 | 21.000 |
| HMKGM | 61.3396 | .8084 | 64.5459 | 29.000 | 1.000 | 47.000 |
| .999 | ASIMOTA | 1.0367 | 1.0367 | 1.0367 | 1.000 | 9.000 | 6.000 |
| GFA | 1.5413 | 1.5413 | 1.5413 | 2.000 | 30.000 | 12.000 |
| FAPR | 11.4839 | 11.4839 | 11.4839 | 3.000 | 41.000 | 18.000 |
| GKAM | 1210.6169 | .9078 | 1210.9919 | 43.000 | 4.500 | 56.000 |
| AMK | 2810.3756 | .9078 | 2.2274 | 62.000 | 4.500 | 15.000 |
| AMKAM | 2761.6990 | .9078 | 2.5501 | 60.000 | 4.500 | 16.000 |
| MNKAM | 1385.9141 | .9078 | 1412.7979 | 51.000 | 4.500 | 64.000 |
| MAK | 2908.6450 | 1.1752 | .9239 | 70.000 | 24.500 | 4.000 |
| MAKAM | 2892.2264 | .9078 | .9456 | 68.000 | 4.500 | 5.000 |
| MAKMN | 74.7238 | 40.9104 | .9092 | 5.000 | 46.500 | 1.500 |
| MAKMA | 2910.1742 | 1.1752 | .9092 | 71.000 | 24.500 | 1.500 |
| MAKMR | 2904.1592 | 1.1085 | .9126 | 69.000 | 14.500 | 3.000 |
| MAKMD | 58.0303 | 43.7201 | 10.0573 | 4.000 | 62.500 | 17.000 |
| MRKAM | 2842.5937 | .9078 | 1.3994 | 64.000 | 4.500 | 11.000 |
| MDKAM | 1123.4283 | .9078 | 163.1930 | 37.000 | 4.500 | 35.000 |
| GMKAM | 990.8168 | .9078 | 33.4395 | 35.000 | 4.500 | 24.000 |

APPENDIX 21: MSE of the Ridge Parameters when P=4, n=100 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | OLS | .0753 | .0753 | .0753 | 39.000 | 5.000 | 37.000 |
| GHK | .0652 | .0652 | .0652 | 4.500 | 2.000 | 4.500 |
| HKMA | .0709 | .0709 | .0709 | 34.000 | 3.000 | 34.000 |
| ASIMOTA | .0729 | .0729 | .0729 | 36.000 | 4.000 | 36.000 |
| GFA | .0635 | .0635 | .0635 | 1.000 | 1.000 | 1.000 |
| GK | .0652 | 1.2669 | .0652 | 4.500 | 72.000 | 4.500 |
| GKMN | .0648 | .7254 | .0649 | 2.000 | 12.500 | 2.000 |
| GKMD | .0651 | .8192 | .0651 | 3.000 | 20.500 | 3.000 |
| .900 | GHK | .1015 | .1015 | .1015 | 10.500 | 2.000 | 7.500 |
| HKMA | .1169 | .1169 | .1169 | 45.000 | 4.000 | 40.000 |
| ASIMOTA | .1228 | .1228 | .1228 | 49.000 | 5.000 | 44.000 |
| GFA | .0978 | .0978 | .0978 | 1.000 | 1.000 | 1.000 |
| FAPR | .1163 | .1163 | .1163 | 37.000 | 3.000 | 32.000 |
| GKMN | .1007 | .5225 | .1009 | 2.000 | 12.500 | 3.000 |
| GKMD | .1008 | .5725 | .1009 | 3.000 | 20.500 | 2.000 |
| GKGM | .1014 | .7396 | .1011 | 8.000 | 36.500 | 5.000 |
| GKHM | .1009 | .6188 | .1009 | 4.000 | 28.500 | 4.000 |
| GMK | .1012 | .7396 | .1015 | 5.000 | 36.500 | 9.000 |
| .950 | GHK | .1717 | .1717 | .1717 | 29.500 | 2.000 | 29.500 |
| HKMA | .1998 | .1998 | .1998 | 50.000 | 4.000 | 50.000 |
| ASIMOTA | .2138 | .2138 | .2138 | 54.000 | 5.000 | 53.000 |
| GFA | .1513 | .1513 | .1513 | 6.000 | 1.000 | 1.000 |
| FAPR | .1966 | .1966 | .1966 | 42.000 | 3.000 | 41.000 |
| AMK | .1485 | .9603 | .1535 | 4.000 | 47.500 | 3.000 |
| AMKMN | .1485 | .4406 | .1527 | 3.000 | 12.500 | 2.000 |
| AMKMD | .1508 | .4611 | .1571 | 5.000 | 20.500 | 4.000 |
| MRK | .1468 | 1.1359 | .1599 | 2.000 | 57.500 | 10.000 |
| MRKMN | .1452 | .4406 | .3476 | 1.000 | 12.500 | 68.000 |
| GMKGM | .1568 | .5666 | .1576 | 9.000 | 36.500 | 5.000 |
| .990 | GFA | .3439 | .3439 | .3439 | 10.000 | 1.000 | 11.000 |
| MAK | .2698 | 1.1831 | .2813 | 1.000 | 67.500 | 1.000 |
| MAKMN | .2742 | .4191 | .4302 | 2.000 | 16.500 | 38.500 |
| MAKMD | .2875 | .4242 | .2939 | 3.000 | 32.500 | 2.000 |
| MAKHM | .3177 | .4238 | .3058 | 5.000 | 24.500 | 3.000 |
| MRK | .3234 | 1.1183 | .3165 | 7.000 | 56.500 | 5.000 |
| MRKLHM | .3075 | .4238 | .3130 | 4.000 | 24.500 | 4.000 |
| HMKMN | .5233 | .4033 | .5202 | 40.000 | 2.000 | 50.000 |
| .999 | ASIMOTA | .3192 | .3192 | .3192 | 1.000 | 1.000 | 1.000 |
| AMK | .3972 | .8947 | .4062 | 3.000 | 45.500 | 9.000 |
| MAKMN | .4984 | .5692 | .4965 | 5.000 | 25.500 | 19.500 |
| MAKMD | .3713 | .5754 | .3773 | 2.000 | 33.500 | 4.000 |
| MAKHM | .3998 | .5630 | .3468 | 4.000 | 17.500 | 3.000 |
| MRK | .8560 | 1.1141 | .3204 | 10.000 | 55.500 | 2.000 |
| MRKAM | 4.0642 | .8947 | .3887 | 49.000 | 45.500 | 5.000 |
| HMKGM | 2.4438 | .3447 | 2.5850 | 38.000 | 2.000 | 50.000 |

APPENDIX 22: MSE of the Ridge Parameters when P=4, n=100 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GHK | .4332 | .4332 | .4332 | 52.500 | 2.000 | 51.500 |
| HKMA | .4838 | .4838 | .4838 | 62.000 | 3.000 | 63.000 |
| GFA | .2530 | .2530 | .2530 | 1.000 | 1.000 | 1.000 |
| FAPR | .6019 | .6019 | .6019 | 66.000 | 4.000 | 66.000 |
| FAMR | .6737 | .6737 | .6737 | 67.000 | 5.000 | 69.000 |
| AMK | .2906 | 1.1051 | .2898 | 12.000 | 45.500 | 4.000 |
| MAKMN | .2750 | .7371 | .6334 | 5.000 | 12.500 | 67.500 |
| MRK | .2720 | 1.1927 | .2794 | 2.000 | 55.500 | 2.000 |
| MRKMD | .2732 | .8297 | .2845 | 3.000 | 20.500 | 3.000 |
| MRKLHM | .2746 | .8886 | .2908 | 4.000 | 28.500 | 5.000 |
| .900 | GFA | .3132 | .3132 | .3132 | 3.000 | 1.000 | 2.000 |
| MAK | .2971 | 1.1996 | .3114 | 1.000 | 67.500 | 1.000 |
| MAKMN | .2990 | .5464 | .5942 | 2.000 | 5.500 | 48.500 |
| MAKMD | .3152 | .5948 | .3260 | 4.000 | 13.500 | 3.000 |
| MAKHM | .3432 | .6388 | .3402 | 8.000 | 21.500 | 5.000 |
| MRK | .3457 | 1.1578 | .3402 | 9.000 | 56.500 | 4.000 |
| MRKLHM | .3341 | .6388 | .3403 | 5.000 | 21.500 | 6.000 |
| .950 | GFA | .4011 | .4011 | .4011 | 5.000 | 1.000 | 5.000 |
| MAK | .3248 | 1.1907 | .3286 | 1.000 | 61.500 | 1.000 |
| MAKMN | .3537 | .4830 | .5566 | 3.000 | 5.500 | 33.500 |
| MAKMD | .3495 | .5025 | .3570 | 2.000 | 13.500 | 2.000 |
| MAKGM | .5560 | .5894 | .3904 | 23.000 | 29.500 | 4.000 |
| MAKHM | .3727 | .5195 | .3618 | 4.000 | 21.500 | 3.000 |
| .990 | GFA | .6221 | .6221 | .6221 | 5.000 | 37.000 | 20.000 |
| AMK | .5438 | .9088 | .4886 | 4.000 | 41.500 | 12.000 |
| MAKMN | .5124 | .5750 | .5530 | 3.000 | 24.500 | 17.500 |
| MAKMD | .3982 | .5815 | .4042 | 1.000 | 32.500 | 3.000 |
| MAKGM | 1.8977 | .4644 | .4468 | 29.000 | 7.500 | 5.000 |
| MAKHM | .4253 | .5710 | .3768 | 2.000 | 16.500 | 1.000 |
| MRK | .9444 | 1.1187 | .4041 | 9.000 | 50.500 | 2.000 |
| MRKAM | 3.2478 | .9088 | .4446 | 44.000 | 41.500 | 4.000 |
| HMKMN | 3.2349 | .4634 | 3.1641 | 43.000 | 3.000 | 49.000 |
| HMKMD | 3.0267 | .4525 | 3.0123 | 41.000 | 2.000 | 48.000 |
| HMKHM | 2.8430 | .4488 | 2.8771 | 38.500 | 1.000 | 46.000 |
| .999 | ASIMOTA | .5534 | .5534 | .5534 | 1.000 | 2.000 | 2.000 |
| GFA | .8131 | .8131 | .8131 | 3.000 | 13.000 | 9.000 |
| MAKAM | 86.3826 | .8949 | .6549 | 69.000 | 17.500 | 5.000 |
| MAKMN | 1.0892 | 1.9292 | .6575 | 4.000 | 53.500 | 6.500 |
| MAKMR | 86.8620 | 1.1142 | .6526 | 70.000 | 27.500 | 4.000 |
| MAKMD | .6671 | 1.9527 | .4015 | 2.000 | 61.500 | 1.000 |
| MAKHM | 2.1769 | 1.8445 | .5973 | 5.000 | 45.500 | 3.000 |
| HMKGM | 19.7707 | .3751 | 20.9218 | 32.000 | 1.000 | 49.000 |

APPENDIX 23: MSE of the Ridge Parameters when P=4, n=100 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .3988 | .3988 | .3988 | 2.000 | 1.000 | 1.000 |
| MAK | .4505 | 1.2205 | .4190 | 6.000 | 60.500 | 2.000 |
| MAKMN | .3913 | .7601 | .7121 | 1.000 | 5.500 | 40.500 |
| MAKMD | .4019 | .8503 | .4233 | 3.000 | 13.500 | 3.000 |
| MAKHM | .4274 | .9072 | .4389 | 4.000 | 21.500 | 4.000 |
| MRKGM | .4409 | .9932 | .4566 | 5.000 | 29.500 | 5.000 |
| .900 | GFA | .4535 | .4535 | .4535 | 4.000 | 1.000 | 5.000 |
| MAK | .5816 | 1.2015 | .4183 | 9.000 | 60.500 | 1.000 |
| MAKMN | .4362 | .5938 | .6597 | 2.000 | 5.500 | 29.500 |
| MAKMD | .4160 | .6389 | .4249 | 1.000 | 13.500 | 2.000 |
| MAKGM | .5866 | .7793 | .4519 | 10.000 | 29.500 | 4.000 |
| MAKHM | .4370 | .6780 | .4269 | 3.000 | 21.500 | 3.000 |
| MRKGM | .4967 | .7793 | .5088 | 5.000 | 29.500 | 6.000 |
| .950 | GFA | .5412 | .5412 | .5412 | 4.000 | 1.000 | 7.000 |
| MAK | 1.1754 | 1.1920 | .4400 | 29.000 | 60.500 | 2.000 |
| MAKAM | 2.2790 | .9690 | .5257 | 47.000 | 40.500 | 5.000 |
| MAKMN | .5052 | .5673 | .6257 | 3.000 | 5.500 | 18.500 |
| MAKMD | .4431 | .5845 | .4493 | 1.000 | 13.500 | 3.000 |
| MAKGM | .8638 | .6343 | .4584 | 14.000 | 29.500 | 4.000 |
| MAKHM | .4626 | .5947 | .4362 | 2.000 | 21.500 | 1.000 |
| MRKGM | .5582 | .6343 | .5597 | 5.000 | 29.500 | 10.000 |
| .990 | GFA | .7436 | .7436 | .7436 | 4.000 | 13.000 | 15.000 |
| AMK | 5.3267 | .9105 | .6114 | 25.000 | 41.500 | 4.000 |
| AMKGM | 1.4607 | .5857 | .6423 | 6.000 | 8.500 | 5.000 |
| MAKMN | .6944 | .8855 | .7640 | 2.000 | 25.500 | 17.500 |
| MAKMD | .4686 | .8946 | .4352 | 1.000 | 33.500 | 2.000 |
| MAKHM | .7123 | .8639 | .4286 | 3.000 | 17.500 | 1.000 |
| MRKGM | 1.5767 | .5857 | .5196 | 7.000 | 8.500 | 3.000 |
| MRKLHM | 1.4542 | .8639 | 1.1357 | 5.000 | 17.500 | 21.000 |
| HMKMN | 8.5651 | .5826 | 8.3321 | 38.000 | 3.000 | 49.000 |
| HMKMD | 7.9630 | .5460 | 7.8966 | 37.000 | 2.000 | 48.000 |
| HMKGM | 6.1285 | .5828 | 6.4749 | 30.000 | 4.000 | 45.000 |
| HMKHM | 7.4329 | .5201 | 7.5098 | 35.500 | 1.000 | 46.000 |
| .999 | ASIMOTA | 1.0209 | 1.0209 | 1.0209 | 2.000 | 19.000 | 13.000 |
| GFA | .8899 | .8899 | .8899 | 1.000 | 2.000 | 7.000 |
| FAPR | 6.3746 | 6.3746 | 6.3746 | 5.000 | 67.000 | 27.000 |
| MAK | 259.4017 | 1.1825 | .6043 | 69.000 | 35.500 | 5.000 |
| MAKMN | 2.2666 | 4.6447 | .6033 | 4.000 | 53.500 | 3.500 |
| MAKMA | 261.0597 | 1.1825 | .6033 | 71.000 | 35.500 | 3.500 |
| MAKMR | 260.3307 | 1.1146 | .6001 | 70.000 | 24.500 | 1.000 |
| MAKMD | 1.5090 | 4.7025 | .6023 | 3.000 | 61.500 | 2.000 |
| HMKGM | 54.4163 | .4355 | 57.5538 | 29.000 | 1.000 | 49.000 |

APPENDIX 24: MSE of the Ridge Parameters when P=4, n=100 and v=49

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .5137 | .5137 | .5137 | 2.000 | 1.000 | 1.000 |
| MAK | .8426 | 1.2254 | .5402 | 24.000 | 58.500 | 4.000 |
| MAKMN | .5216 | .7944 | .7808 | 3.000 | 5.500 | 25.500 |
| MAKMD | .5023 | .8811 | .5160 | 1.000 | 13.500 | 2.000 |
| MAKGM | .6156 | 1.0158 | .5491 | 8.000 | 29.500 | 5.000 |
| MAKHM | .5259 | .9350 | .5238 | 4.000 | 21.500 | 3.000 |
| MRKAM | .5966 | 1.1304 | .5975 | 5.000 | 38.500 | 6.000 |
| .900 | GFA | .5620 | .5620 | .5620 | 3.000 | 1.000 | 5.000 |
| MAK | 1.5548 | 1.2043 | .5488 | 35.000 | 60.500 | 4.000 |
| MAKMN | .5814 | .6646 | .7356 | 4.000 | 5.500 | 18.500 |
| MAKMD | .5193 | .7047 | .5217 | 1.000 | 13.500 | 2.000 |
| MAKGM | .8191 | .8185 | .5346 | 8.000 | 29.500 | 3.000 |
| MAKHM | .5457 | .7367 | .5132 | 2.000 | 21.500 | 1.000 |
| MRKGM | .6432 | .8185 | .6559 | 5.000 | 29.500 | 11.000 |
| .950 | GFA | .6473 | .6473 | .6473 | 3.000 | 1.000 | 7.000 |
| MAK | 3.9886 | 1.1938 | .6062 | 45.000 | 60.500 | 4.000 |
| MAKMN | .6554 | .6933 | .7294 | 4.000 | 5.500 | 17.500 |
| MAKMD | .5355 | .7073 | .5313 | 1.000 | 29.500 | 2.000 |
| MAKGM | 1.4972 | .7013 | .5615 | 17.000 | 13.500 | 3.000 |
| MAKHM | .5904 | .7071 | .5103 | 2.000 | 21.500 | 1.000 |
| MRKAM | 2.3761 | .9774 | .6413 | 32.000 | 40.500 | 5.000 |
| MRKGM | .6834 | .7013 | .6624 | 5.000 | 13.500 | 9.000 |
| .990 | GFA | .8384 | .8384 | .8384 | 2.000 | 13.000 | 5.000 |
| AMKGM | 2.8905 | .7671 | .7950 | 7.000 | 8.500 | 4.000 |
| MAKMN | .9481 | 1.3504 | .8912 | 3.000 | 52.500 | 7.500 |
| MAKMD | .6227 | 1.3636 | .5063 | 1.000 | 60.500 | 1.000 |
| MAKHM | 1.2961 | 1.3025 | .5601 | 4.000 | 44.500 | 2.000 |
| MRKGM | 3.5547 | .7671 | .7528 | 9.000 | 8.500 | 3.000 |
| MRKLHM | 2.1993 | 1.3025 | 1.5301 | 5.000 | 44.500 | 21.000 |
| HMKMN | 16.5544 | .7610 | 16.0706 | 38.000 | 4.000 | 49.000 |
| HMKMD | 15.3610 | .6859 | 15.2090 | 34.000 | 3.000 | 48.000 |
| HMKGM | 11.7380 | .6081 | 12.4042 | 29.000 | 1.000 | 45.000 |
| HMKHM | 14.3110 | .6267 | 14.4446 | 31.500 | 2.000 | 46.000 |
| .999 | ASIMOTA | 1.7215 | 1.7215 | 1.7215 | 2.000 | 38.000 | 15.000 |
| GFA | .9649 | .9649 | .9649 | 1.000 | 10.000 | 10.000 |
| FAPR | 10.6698 | 10.6698 | 10.6698 | 5.000 | 67.000 | 27.000 |
| MAK | 522.0777 | 1.1835 | .6307 | 70.000 | 25.500 | 2.000 |
| MAKAM | 519.1856 | .8960 | .6361 | 68.000 | 5.500 | 5.000 |
| MAKMN | 4.0338 | 8.7158 | .6331 | 4.000 | 53.500 | 3.500 |
| MAKMA | 523.4963 | 1.1835 | .6331 | 71.000 | 25.500 | 3.500 |
| MAKMR | 522.0440 | 1.1150 | .6279 | 69.000 | 15.500 | 1.000 |
| MAKMD | 2.8036 | 8.8248 | .9217 | 3.000 | 61.500 | 6.000 |
| HMKGM | 106.3840 | .5259 | 112.4948 | 28.000 | 1.000 | 48.000 |

APPENDIX 25: MSE of the Ridge Parameters when P=4, n=100 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .6510 | .6510 | .6510 | 1.000 | 1.000 | 1.000 |
| MAK | 1.8428 | 1.2358 | .7181 | 38.000 | 57.500 | 5.000 |
| MAKMN | .7558 | .8671 | .8832 | 4.000 | 5.500 | 16.500 |
| MAKMD | .6791 | .9461 | .6713 | 2.000 | 13.500 | 3.000 |
| MAKGM | .8556 | 1.0637 | .6787 | 6.000 | 29.500 | 4.000 |
| MAKHM | .7052 | .9936 | .6649 | 3.000 | 21.500 | 2.000 |
| MRKAM | .8202 | 1.1623 | .7939 | 5.000 | 39.500 | 7.000 |
| .900 | GFA | .6984 | .6984 | .6984 | 1.000 | 1.000 | 4.000 |
| MAK | 4.3970 | 1.2102 | .7551 | 47.000 | 60.500 | 5.000 |
| MAKMN | .8343 | .8148 | .8616 | 4.000 | 5.500 | 14.500 |
| MAKMD | .7023 | .8442 | .6828 | 2.000 | 13.500 | 2.000 |
| MAKGM | 1.3766 | .9015 | .6912 | 9.000 | 29.500 | 3.000 |
| MAKHM | .7641 | .8609 | .6617 | 3.000 | 21.500 | 1.000 |
| MRKGM | .8763 | .9015 | .8788 | 5.000 | 29.500 | 17.000 |
| .950 | GFA | .7861 | .7861 | .7861 | 2.000 | 1.000 | 3.000 |
| MAKMN | .9317 | .9608 | .8969 | 4.000 | 24.500 | 14.500 |
| MAKMD | .7220 | .9677 | .6833 | 1.000 | 32.500 | 2.000 |
| MAKGM | 3.0836 | .8434 | .7988 | 18.000 | 6.500 | 4.000 |
| MAKHM | .8982 | .9456 | .6623 | 3.000 | 16.500 | 1.000 |
| MRKGM | .9469 | .8434 | .8337 | 5.000 | 6.500 | 5.000 |
| HMKMN | 7.5677 | .8271 | 7.3692 | 42.000 | 2.000 | 49.000 |
| .990 | GFA | .9781 | .9781 | .9781 | 1.000 | 11.000 | 3.000 |
| MAKMN | 1.4850 | 2.3376 | 1.0454 | 3.000 | 52.500 | 4.500 |
| MAKMA | 95.8439 | 1.1878 | 1.0454 | 69.000 | 36.500 | 4.500 |
| MAKMD | .9881 | 2.3592 | .6794 | 2.000 | 60.500 | 1.000 |
| MAKHM | 2.6370 | 2.2335 | .8742 | 4.000 | 44.500 | 2.000 |
| MRKLHM | 3.7458 | 2.2335 | 2.3366 | 5.000 | 44.500 | 21.000 |
| HMKGM | 23.6564 | .6616 | 24.9967 | 29.000 | 1.000 | 45.000 |
| HMKHM | 28.9244 | .8527 | 29.1740 | 30.500 | 2.000 | 46.000 |
| .999 | ASIMOTA | 3.2103 | 3.2103 | 3.2103 | 2.000 | 38.000 | 17.000 |
| GFA | 1.0989 | 1.0989 | 1.0989 | 1.000 | 11.000 | 10.000 |
| FAPR | 18.5082 | 18.5082 | 18.5082 | 5.000 | 66.000 | 27.000 |
| MAK | 1080.4747 | 1.1856 | .7267 | 70.000 | 25.500 | 4.000 |
| MAKAM | 1072.9627 | .8974 | .7336 | 68.000 | 5.500 | 5.000 |
| MAKMN | 7.7898 | 17.3642 | .7254 | 4.000 | 53.500 | 2.500 |
| MAKMA | 1081.8439 | 1.1856 | .7254 | 71.000 | 25.500 | 2.500 |
| MAKMR | 1078.8587 | 1.1160 | .7196 | 69.000 | 15.500 | 1.000 |
| MAKMD | 5.5694 | 17.5817 | 1.6086 | 3.000 | 61.500 | 12.000 |
| HMKGM | 216.8151 | .7175 | 229.2408 | 27.000 | 1.000 | 47.000 |

APPENDIX 26: MSE of the Ridge Parameters when P=4, n=200 and v=1

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GHK | .0399 | .0399 | .0399 | 4.500 | 1.000 | 5.500 |
| HKMA | .0461 | .0461 | .0461 | 34.000 | 4.000 | 34.000 |
| ASIMOTA | .0467 | .0467 | .0467 | 36.000 | 5.000 | 36.000 |
| GFA | .0422 | .0422 | .0422 | 11.000 | 3.000 | 11.000 |
| FAPR | .0401 | .0401 | .0401 | 7.000 | 2.000 | 8.000 |
| GK | .0399 | 1.2665 | .0399 | 4.500 | 72.000 | 5.500 |
| GKMN | .0397 | .5391 | .0398 | 1.000 | 12.500 | 1.000 |
| GKMD | .0398 | .6017 | .0398 | 2.000 | 20.500 | 2.000 |
| GKGM | .0400 | .8047 | .0399 | 6.000 | 36.500 | 4.000 |
| GKHM | .0398 | .6751 | .0398 | 3.000 | 28.500 | 3.000 |
| .900 | GHK | .0779 | .0779 | .0779 | 15.500 | 2.000 | 13.500 |
| HKMA | .0850 | .0850 | .0850 | 45.000 | 4.000 | 42.000 |
| ASIMOTA | .0874 | .0874 | .0874 | 48.000 | 5.000 | 45.000 |
| GFA | .0725 | .0725 | .0725 | 1.000 | 1.000 | 1.000 |
| FAPR | .0841 | .0841 | .0841 | 37.000 | 3.000 | 33.000 |
| GMK | .0739 | .5506 | .0740 | 2.000 | 36.500 | 2.000 |
| GMKMN | .0739 | .3671 | .0740 | 4.000 | 12.500 | 3.000 |
| GMKMD | .0739 | .3912 | .0741 | 3.000 | 20.500 | 4.000 |
| GMKHM | .0740 | .4283 | .0742 | 5.000 | 28.500 | 5.000 |
| .950 | GHK | .1399 | .1399 | .1399 | 34.500 | 2.000 | 33.500 |
| HKMA | .1560 | .1560 | .1560 | 52.000 | 4.000 | 51.000 |
| ASIMOTA | .1640 | .1640 | .1640 | 57.000 | 5.000 | 56.000 |
| GFA | .1203 | .1203 | .1203 | 8.000 | 1.000 | 4.000 |
| FAPR | .1531 | .1531 | .1531 | 44.000 | 3.000 | 43.000 |
| AMK | .1140 | .9211 | .1165 | 3.000 | 48.500 | 2.000 |
| AMKMN | .1140 | .3140 | .1163 | 4.000 | 12.500 | 1.000 |
| AMKMD | .1150 | .3216 | .1181 | 5.000 | 20.500 | 3.000 |
| MRK | .1130 | 1.1400 | .1211 | 2.000 | 57.500 | 6.000 |
| MRKMN | .1125 | .3140 | .2479 | 1.000 | 12.500 | 68.000 |
| GMKGM | .1210 | .3990 | .1210 | 9.000 | 36.500 | 5.000 |
| .990 | MAK | .2203 | 1.2054 | .2232 | 2.000 | 67.500 | 1.000 |
| MAKMN | .2190 | .3061 | .3323 | 1.000 | 24.500 | 32.500 |
| MAKMD | .2260 | .3072 | .2268 | 3.000 | 32.500 | 2.000 |
| MAKHM | .2516 | .3050 | .2356 | 5.000 | 16.500 | 3.000 |
| MRK | .2590 | 1.1306 | .2507 | 7.000 | 57.500 | 5.000 |
| MRKLHM | .2430 | .3050 | .2444 | 4.000 | 16.500 | 4.000 |
| HMKMD | .4227 | .2819 | .4218 | 38.000 | 1.000 | 48.000 |
| .999 | ASIMOTA | .2439 | .2439 | .2439 | 1.000 | 1.000 | 1.000 |
| AMK | .3505 | .8802 | .4631 | 5.000 | 44.500 | 20.000 |
| MAK | 1.3067 | 1.2048 | .2822 | 27.000 | 64.500 | 2.000 |
| MAKMD | .3415 | .4320 | .3468 | 4.000 | 33.500 | 14.000 |
| MAKHM | .3328 | .4194 | .3017 | 3.000 | 17.500 | 4.000 |
| MRK | .2860 | 1.1285 | .2971 | 2.000 | 54.500 | 3.000 |
| MRKAM | 3.2217 | .8802 | .3100 | 48.000 | 44.500 | 5.000 |
| HMKGM | 1.9571 | .2450 | 2.0773 | 38.000 | 2.000 | 49.000 |

APPENDIX 27: MSE of the Ridge Parameters when P=4, n=200 and v=9

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GHK | .2758 | .2758 | .2758 | 48.500 | 2.000 | 48.500 |
| HKMA | .3285 | .3285 | .3285 | 60.000 | 3.000 | 60.000 |
| GFA | .1704 | .1704 | .1704 | 1.000 | 1.000 | 1.000 |
| FAPR | .3949 | .3949 | .3949 | 64.000 | 4.000 | 65.000 |
| FAMR | .4289 | .4289 | .4289 | 65.000 | 5.000 | 66.000 |
| AMK | .2006 | 1.0399 | .2002 | 9.000 | 44.500 | 4.000 |
| AMKMD | .2003 | .6099 | .2011 | 8.000 | 20.500 | 5.000 |
| MRK | .1872 | 1.1773 | .1936 | 2.000 | 56.500 | 2.000 |
| MRKMN | .1896 | .5480 | .3836 | 4.000 | 12.500 | 64.000 |
| MRKMD | .1882 | .6099 | .1964 | 3.000 | 20.500 | 3.000 |
| MRKLHM | .1913 | .6824 | .2031 | 5.000 | 28.500 | 8.000 |
| .900 | GFA | .2295 | .2295 | .2295 | 3.000 | 1.000 | 1.000 |
| MAK | .2263 | 1.2127 | .2335 | 2.000 | 67.500 | 2.000 |
| MAKMN | .2240 | .3845 | .4931 | 1.000 | 5.500 | 57.500 |
| MAKMD | .2342 | .4081 | .2390 | 4.000 | 13.500 | 3.000 |
| MAKHM | .2587 | .4436 | .2504 | 10.000 | 21.500 | 5.000 |
| MRKLHM | .2454 | .4436 | .2489 | 5.000 | 21.500 | 4.000 |
| .950 | GFA | .3133 | .3133 | .3133 | 5.000 | 1.000 | 5.000 |
| MAK | .2596 | 1.2085 | .2664 | 1.000 | 62.500 | 1.000 |
| MAKMN | .2770 | .3447 | .4642 | 3.000 | 5.500 | 32.500 |
| MAKMD | .2759 | .3521 | .2785 | 2.000 | 13.500 | 2.000 |
| MAKGM | .4725 | .4149 | .3032 | 24.000 | 29.500 | 4.000 |
| MAKHM | .2962 | .3616 | .2818 | 4.000 | 21.500 | 3.000 |
| .990 | AMK | .4200 | .8882 | .5068 | 4.000 | 41.500 | 19.000 |
| MAK | 1.0738 | 1.2055 | .2802 | 15.000 | 60.500 | 1.000 |
| MAKAM | 3.8639 | .8882 | .3453 | 59.000 | 41.500 | 4.000 |
| MAKMN | .4202 | .4272 | .4005 | 5.000 | 24.500 | 12.500 |
| MAKMD | .3531 | .4288 | .3586 | 2.000 | 32.500 | 6.000 |
| MAKGM | 1.5525 | .3218 | .3305 | 30.000 | 6.500 | 3.000 |
| MAKHM | .3468 | .4181 | .3188 | 1.000 | 16.500 | 2.000 |
| MRK | .3631 | 1.1306 | .3732 | 3.000 | 50.500 | 9.000 |
| MRKAM | 2.6866 | .8882 | .3541 | 44.000 | 41.500 | 5.000 |
| HMKMD | 2.4309 | .3185 | 2.4144 | 41.000 | 2.000 | 48.000 |
| HMKHM | 2.2625 | .3114 | 2.2903 | 38.500 | 1.000 | 47.000 |
| .999 | ASIMOTA | .6008 | .6008 | .6008 | 3.000 | 12.000 | 7.000 |
| GFA | .6910 | .6910 | .6910 | 4.000 | 14.000 | 9.000 |
| MAKAM | 66.2608 | .8804 | .5892 | 69.000 | 18.500 | 3.000 |
| MAKMN | .5697 | 1.5402 | .5987 | 2.000 | 53.500 | 5.500 |
| MAKMR | 66.4924 | 1.1285 | .5918 | 70.000 | 27.500 | 4.000 |
| MAKMD | .3265 | 1.5467 | .2771 | 1.000 | 61.500 | 1.000 |
| MAKHM | 1.4222 | 1.4499 | .4024 | 5.000 | 45.500 | 2.000 |
| HMKGM | 15.7034 | .2634 | 16.6317 | 32.000 | 1.000 | 49.000 |

APPENDIX 28: MSE of the Ridge Parameters when P=4, n=200 and v=25

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .2692 | .2692 | .2692 | 1.000 | 1.000 | 1.000 |
| MAK | .2788 | 1.2230 | .2933 | 2.000 | 67.500 | 2.000 |
| MAKMN | .2790 | .5650 | .6053 | 3.000 | 5.500 | 48.500 |
| MAKMD | .2879 | .6257 | .3023 | 4.000 | 13.500 | 3.000 |
| MAKHM | .3121 | .6962 | .3174 | 5.000 | 22.500 | 4.000 |
| MRKLHM | .3204 | .6962 | .3238 | 7.000 | 22.500 | 5.000 |
| .900 | GFA | .3267 | .3267 | .3267 | 3.000 | 1.000 | 4.000 |
| MAK | .3034 | 1.2133 | .3020 | 1.000 | 61.500 | 1.000 |
| MAKMN | .3294 | .4185 | .5482 | 4.000 | 5.500 | 30.500 |
| MAKMD | .3193 | .4410 | .3244 | 2.000 | 13.500 | 2.000 |
| MAKGM | .4748 | .5788 | .3436 | 13.000 | 29.500 | 5.000 |
| MAKHM | .3350 | .4731 | .3245 | 5.000 | 21.500 | 3.000 |
| .950 | GFA | .4140 | .4140 | .4140 | 4.000 | 17.000 | 6.000 |
| GKMN | 2.2324 | .4049 | 2.2269 | 58.000 | 4.500 | 58.000 |
| AMKMN | .9132 | .4049 | .8231 | 29.000 | 4.500 | 35.000 |
| MNK | 2.4761 | .4049 | 2.4766 | 65.000 | 4.500 | 65.000 |
| MNKMN | 2.5050 | .4049 | 2.5006 | 67.000 | 4.500 | 67.000 |
| MAK | .4315 | 1.2089 | .2990 | 5.000 | 60.500 | 1.000 |
| MAKAM | 1.8776 | .9245 | .3939 | 47.000 | 41.500 | 5.000 |
| MAKMN | .4013 | .4049 | .5004 | 3.000 | 4.500 | 17.500 |
| MAKMD | .3640 | .4117 | .3683 | 1.000 | 12.500 | 4.000 |
| MAKGM | .7156 | .4457 | .3532 | 17.000 | 29.500 | 3.000 |
| MAKHM | .3686 | .4166 | .3502 | 2.000 | 21.500 | 2.000 |
| MRKMN | .8801 | .4049 | .4912 | 25.000 | 4.500 | 15.000 |
| MDKMN | .9835 | .4049 | .9541 | 31.000 | 4.500 | 38.000 |
| GMKMN | 1.1882 | .4049 | 1.1189 | 37.000 | 4.500 | 41.000 |
| .990 | GFA | .6098 | .6098 | .6098 | 4.000 | 13.000 | 19.000 |
| AMK | 2.0404 | .8889 | .3946 | 11.000 | 41.500 | 3.000 |
| AMKMR | 7.4436 | 1.1308 | .4684 | 41.000 | 50.500 | 5.000 |
| AMKGM | 1.2081 | .4003 | .5262 | 5.000 | 6.500 | 13.000 |
| MAKMN | .4916 | .6672 | .6033 | 3.000 | 25.500 | 17.500 |
| MAKMD | .3469 | .6699 | .3492 | 1.000 | 33.500 | 2.000 |
| MAKHM | .4610 | .6421 | .3091 | 2.000 | 17.500 | 1.000 |
| MRKGM | 1.3222 | .4003 | .3973 | 7.000 | 6.500 | 4.000 |
| HMKMD | 6.3480 | .3906 | 6.2808 | 38.000 | 2.000 | 48.000 |
| HMKHM | 5.8623 | .3644 | 5.9268 | 36.500 | 1.000 | 46.000 |
| .999 | ASIMOTA | 1.3134 | 1.3134 | 1.3134 | 4.000 | 40.000 | 14.000 |
| GFA | .7418 | .7418 | .7418 | 2.000 | 10.000 | 7.000 |
| MAKAM | 204.3963 | .8807 | .4954 | 69.000 | 14.500 | 3.000 |
| MAKMN | .9243 | 3.7521 | .4979 | 3.000 | 53.500 | 4.500 |
| MAKMA | 205.4550 | 1.2050 | .4979 | 71.000 | 35.500 | 4.500 |
| MAKMR | 205.0949 | 1.1285 | .4933 | 70.000 | 24.500 | 2.000 |
| MAKMD | .5348 | 3.7690 | .3216 | 1.000 | 61.500 | 1.000 |
| MAKHM | 4.4139 | 3.5042 | .9317 | 5.000 | 45.500 | 11.000 |
| HMKGM | 43.1924 | .2994 | 45.7011 | 27.000 | 1.000 | 47.000 |

APPENDIX 29: MSE of the Ridge Parameters when P=4, n=200 and v=49

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .3489 | .3489 | .3489 | 1.000 | 1.000 | 1.000 |
| MAK | .4034 | 1.2248 | .3630 | 5.000 | 60.500 | 2.000 |
| MAKMN | .3658 | .5902 | .6532 | 3.000 | 5.500 | 34.500 |
| MAKMD | .3580 | .6489 | .3708 | 2.000 | 13.500 | 3.000 |
| MAKGM | .4600 | .8351 | .4040 | 8.000 | 29.500 | 5.000 |
| MAKHM | .3765 | .7165 | .3781 | 4.000 | 21.500 | 4.000 |
| .900 | GFA | .3998 | .3998 | .3998 | 2.000 | 1.000 | 5.000 |
| MAK | .6154 | 1.2143 | .3629 | 7.000 | 60.500 | 1.000 |
| MAKMN | .4297 | .4690 | .5907 | 4.000 | 5.500 | 18.500 |
| MAKMD | .3916 | .4898 | .3966 | 1.000 | 13.500 | 4.000 |
| MAKGM | .6282 | .6062 | .3936 | 9.000 | 29.500 | 3.000 |
| MAKHM | .4000 | .5169 | .3833 | 3.000 | 21.500 | 2.000 |
| MRKGM | .4934 | .6062 | .5030 | 5.000 | 29.500 | 9.000 |
| .950 | GFA | .4858 | .4858 | .4858 | 3.000 | 1.000 | 8.000 |
| MAK | 1.8208 | 1.2096 | .3902 | 31.000 | 60.500 | 2.000 |
| MAKAM | 3.5939 | .9277 | .4558 | 47.000 | 41.500 | 5.000 |
| MAKMN | .4881 | .4946 | .5581 | 4.000 | 13.500 | 17.500 |
| MAKMD | .4088 | .5007 | .4137 | 1.000 | 29.500 | 4.000 |
| MAKGM | 1.1534 | .4914 | .3990 | 16.000 | 5.500 | 3.000 |
| MAKHM | .4178 | .4987 | .3800 | 2.000 | 21.500 | 1.000 |
| MRKGM | .5527 | .4914 | .5249 | 5.000 | 5.500 | 15.000 |
| .990 | GFA | .6721 | .6721 | .6721 | 3.000 | 13.000 | 5.000 |
| AMKGM | 2.4999 | .5173 | .6008 | 8.000 | 7.500 | 4.000 |
| MAKMN | .5826 | 1.0261 | .7237 | 2.000 | 33.500 | 8.500 |
| MAKMD | .3771 | 1.0304 | .3608 | 1.000 | 41.500 | 1.000 |
| MAKHM | .8139 | .9770 | .3691 | 4.000 | 25.500 | 2.000 |
| MRKGM | 3.1169 | .5173 | .5526 | 10.000 | 7.500 | 3.000 |
| MRKLHM | 1.7914 | .9770 | 1.2824 | 5.000 | 25.500 | 21.000 |
| HMKMD | 12.2174 | .4980 | 12.0654 | 37.000 | 3.000 | 48.000 |
| HMKGM | 9.0997 | .4312 | 9.6446 | 29.000 | 1.000 | 45.000 |
| HMKHM | 11.2555 | .4435 | 11.3666 | 32.500 | 2.000 | 46.000 |
| .999 | ASIMOTA | 2.3831 | 2.3831 | 2.3831 | 4.000 | 40.000 | 18.000 |
| GFA | .7854 | .7854 | .7854 | 1.000 | 2.000 | 7.000 |
| MAK | 414.2757 | 1.2053 | .4693 | 69.000 | 33.500 | 2.000 |
| MAKAM | 413.7958 | .8810 | .4735 | 68.000 | 6.500 | 4.000 |
| MAKMN | 1.4665 | 7.0663 | .4747 | 3.000 | 53.500 | 5.500 |
| MAKMR | 415.2046 | 1.1287 | .4701 | 70.000 | 23.500 | 3.000 |
| MAKMD | .8852 | 7.0991 | .4182 | 2.000 | 61.500 | 1.000 |
| HMKGM | 84.4241 | .3531 | 89.2959 | 26.000 | 1.000 | 47.000 |

APPENDIX 30: MSE of the Ridge Parameters when P=4, n=200 and v=100

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Estimators | kl | Mkl1 | Mkl2 | Rkl | Rmkl1 | Rmkl2 |
| .800 | GFA | .4499 | .4499 | .4499 | 1.000 | 1.000 | 1.000 |
| MAK | .9349 | 1.2285 | .4846 | 24.000 | 60.500 | 4.000 |
| MAKMN | .5045 | .6432 | .7259 | 4.000 | 5.500 | 20.500 |
| MAKMD | .4664 | .6978 | .4739 | 2.000 | 13.500 | 3.000 |
| MAKGM | .6167 | .8664 | .4867 | 7.000 | 29.500 | 5.000 |
| MAKHM | .4814 | .7592 | .4689 | 3.000 | 21.500 | 2.000 |
| MRKGM | .5886 | .8664 | .5991 | 5.000 | 29.500 | 11.000 |
| .900 | GFA | .4926 | .4926 | .4926 | 2.000 | 1.000 | 4.000 |
| MAK | 2.2846 | 1.2164 | .5023 | 38.000 | 60.500 | 5.000 |
| MAKMN | .5729 | .5757 | .6767 | 4.000 | 5.500 | 16.500 |
| MAKMD | .4899 | .5928 | .4917 | 1.000 | 13.500 | 3.000 |
| MAKGM | 1.0233 | .6639 | .4817 | 9.000 | 29.500 | 2.000 |
| MAKHM | .5103 | .6094 | .4640 | 3.000 | 21.500 | 1.000 |
| MRKGM | .6295 | .6639 | .6397 | 5.000 | 29.500 | 13.000 |
| .950 | GFA | .5774 | .5774 | .5774 | 3.000 | 1.000 | 4.000 |
| MAKMN | .6229 | .6844 | .6922 | 4.000 | 24.500 | 18.500 |
| MAKMD | .4842 | .6889 | .4835 | 1.000 | 32.500 | 2.000 |
| MAKGM | 2.4242 | .5881 | .5458 | 20.000 | 5.500 | 3.000 |
| MAKHM | .5657 | .6722 | .4447 | 2.000 | 16.500 | 1.000 |
| MRKGM | .7096 | .5881 | .5944 | 5.000 | 5.500 | 5.000 |
| .990 | GFA | .7576 | .7576 | .7576 | 2.000 | 4.000 | 3.000 |
| MAKMN | .7819 | 1.7873 | .8259 | 3.000 | 52.500 | 5.500 |
| MAKMR | 72.6783 | 1.1319 | .8187 | 68.000 | 26.500 | 4.000 |
| MAKMD | .4902 | 1.7950 | .4219 | 1.000 | 60.500 | 1.000 |
| MAKHM | 1.7232 | 1.6874 | .5604 | 4.000 | 44.500 | 2.000 |
| MRKLHM | 2.8897 | 1.6874 | 1.8183 | 5.000 | 44.500 | 21.000 |
| HMKMD | 24.6877 | .7257 | 24.3494 | 32.000 | 3.000 | 48.000 |
| HMKGM | 18.3105 | .4704 | 19.3989 | 29.000 | 1.000 | 45.000 |
| HMKHM | 22.7137 | .6108 | 22.9176 | 30.500 | 2.000 | 46.000 |
| .999 | ASIMOTA | 4.6574 | 4.6574 | 4.6574 | 4.000 | 41.000 | 18.000 |
| GFA | .8587 | .8587 | .8587 | 1.000 | 2.000 | 12.000 |
| MAK | 862.7810 | 1.2061 | .5086 | 70.000 | 25.500 | 2.000 |
| MAKAM | 859.7947 | .8818 | .5088 | 68.000 | 6.500 | 3.000 |
| MAKMN | 2.6223 | 14.1047 | .5148 | 3.000 | 53.500 | 4.500 |
| MAKMA | 864.1987 | 1.2061 | .5148 | 71.000 | 25.500 | 4.500 |
| MAKMR | 862.7184 | 1.1290 | .5071 | 69.000 | 15.500 | 1.000 |
| MAKMD | 1.6458 | 14.1712 | .6364 | 2.000 | 61.500 | 6.000 |
| MAKHM | 18.8563 | 13.1196 | 3.5603 | 5.000 | 45.500 | 17.000 |
| HMKGM | 172.0391 | .4668 | 181.9291 | 26.000 | 1.000 | 47.000 |