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Article

On some new subclass of bi-univalent functions associated with the Opoola differential operator

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Abstract: By applying Opoola differential operator, in this article, two new subclasses $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(m,\psi,k,\tau)$ and $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(m,\xi,k,\tau)$ of bi-univalent functions class $\mathcal{H}$ defined in $\bigtriangledown$ are introduced and investigated. The estimates on the coefficients $|l_2|$ and $|l_3|$ for functions of the classes are also obtained.

Keywords: Univalent function, bi-univalent function, coefficient bounds, Opoola differential operator.

MSC: 30C45, 30C50.

1. Introduction

Let $\mathcal{J}$ denote the subclass of $\mathcal{G}$ which is of the form

$$\Im(z) = z + \sum_{k=2}^{\infty} l_k z^k \tag{1}$$

consisting of functionas which are holomorphic and univalent in the unit disk $\bigtriangledown$. Let $\Im^{-1}$ be inverse of the function $\Im(z)$, then we have

$$\Im^{-1}(\Im(z)) = z$$

and

$$\Im^{-1}(\Im(b)) = b, \quad |b| < r_0(\Im); r_0(\Im) \geq \frac{1}{4}$$

where

$$\Im^{-1}(\Im(b)) = b - l_2 b^2 + (2l_2^2 - l_3)b^3 - (5l_2^3 - 5l_2 l_3 + l_4)b^4 + \cdots . \tag{2}$$

A function $\Im(z) \in \mathcal{G}$ denoted by $\mathcal{H}$ is said to be bi-univalent in $\bigtriangledown$ if both $\Im(z)$ and $\Im^{-1}(z)$ are univalent in $\triangle$ [1]. Subclasses of $\mathcal{H}$, such as class of bi-convex and starlike functions and bi-strongly convex and starlike function similar to the well known subclasses $\mathcal{L}^*(\vartheta)$ and $\mathcal{K}(\vartheta)$ of starlike and convex functions of order $\vartheta(0 < \vartheta < 1)$ respectively [2].

Recently, numerous researchers [1,3,4] obtained the coefficient $|l_2|$ and $|l_3|$ of bi-univalent functions for the several subclasses of functions in the class $\mathcal{H}$. Motivated by the work of Darus and Singh [5], we introduce the subclasses $\mathcal{M}^{\mu,\beta}_{\mathcal{H}}(m,\psi,k,\tau)$ and $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(m,\xi,k,\tau)$ of the function class $\mathcal{H}$, which are associated with the Opoola differential operator and to obtain estimates on the coefficients $|l_2|$ and $|l_3|$ for functions in these new subclasses of the function class $\mathcal{H}$ applying the techniques used earlier by Darus and Singh [5], Frasin and Aouf [4] and Srivastava *et al.*, [1].

Lemma 1. *[6] Suppose $u(z) \in \mathcal{P}$ and $z \in \bigtriangledown$, then $|w_k| \leq 2$ for each k, where $\mathcal{P}$ is the family of all function u analytic in $\bigtriangledown$ for which $\Re(u(z)) > 0$,*

$$u(z) = 1 + w_1 z + w_2 z^2 + \cdots .$$

Definition 1. A function $\Im(z) \in \mathcal{G}$ is in the class $\mathcal{M}_{\mathcal{H},\sigma}^{\mu,\beta}(m,\psi,\tau)$ if the following condition are fulfilled:

$$\left|\arg\left[\frac{(1-\sigma)D_{\tau,\beta}^{m,\mu}\Im(z)+\sigma D_{\tau,\beta}^{m+1,\mu}\Im(z)}{z}\right]\right| < \frac{\psi\pi}{2}, \tag{3}$$

$$\left|\arg\left[\frac{(1-\sigma)D_{\tau,\beta}^{m,\mu}h(b)+\sigma D_{\tau,\beta}^{m+1,\mu}h(b)}{b}\right]\right| < \frac{\psi\pi}{2} \tag{4}$$

where $0<\psi\leq 1, \sigma\geq 1, \tau\geq 0, z\in\triangle, b\in\triangle, 0\leq\mu\leq\beta, m\in\mathcal{N}_0$ and

$$h(b) = b - l_2b^2 + (2l_2^2-l_3)b^3 - (5l_2^3 - 5l_2l_3 + l_4)b^4 + \cdots \tag{5}$$

and

$$D_{\tau,\beta}^{m,\mu}\Im(z) = z + \sum_{k=2}^{\infty}(1+(k+\mu-\beta-1)\tau)^m l_k z^k \tag{6}$$

where $0\leq\mu\leq\beta, \tau\geq 0$ and $m\in\mathbb{N}_0 = \{0,1,2,3\cdots\}$ is the generalized Al-oboudi derivative defined by Opoola [7].

Remark 1. .

1. $\mathcal{M}_{\mathcal{H},1}^{\mu,\beta}(0,\psi,\tau)$=$\mathcal{M}_{\mathcal{H}}(\psi)$ which Srivastava *et al.*, [1] presented and studied.
2. $\mathcal{M}_{\mathcal{H},\sigma}^{\mu,\beta}(0,\psi,\tau)$=$\mathcal{M}_{\mathcal{H},\sigma}(\psi)$ which Frasin and Aouf [4] presented and studied.
3. $\mathcal{M}_{\mathcal{H},\sigma}^{1,1}(m,\psi,1)$=$\mathcal{M}_{\mathcal{H},\sigma}(m,\psi)$ which Porwal and Darus [8] presented and studied.
4. $\mathcal{M}_{\mathcal{H},\sigma}^{1,1}(m,\psi,\tau)$=$\mathcal{M}_{\mathcal{H},\sigma}(m,\psi,\tau)$ which Darus and Singh [5] presented and studied.

2. Coefficient Bounds For The Function Class $\mathcal{M}_{\mathcal{H}}^{\mu,\beta}(m,\psi,k,\tau)$

Theorem 1. *Let* $\Im(z)\in\mathcal{G}$ *be in the class* $\mathcal{M}_{\mathcal{H}}^{\mu,\beta}(m,\psi,k,\tau)$, $0<\psi\leq 1$, $\sigma\geq 1$, $\tau\geq 0$, $z\in\triangle$, $b\in\triangle$, $0\leq\mu\leq\beta$, $m\in\mathcal{N}_0$ *, then*

$$|l_2| \leq \frac{2\psi}{\sqrt{\begin{array}{c}2\psi[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]-\\ \psi(\psi-1)[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2\end{array}}} \tag{7}$$

and

$$\begin{aligned}|l_3| \leq\ & \frac{2\psi}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}\\ &+\frac{4\psi^2}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}.\end{aligned} \tag{8}$$

Proof. It follows from (3) and (4) that

$$\frac{(1-\sigma)D_{\tau,\beta}^{m,\mu}\Im(z)+\sigma D_{\tau,\beta}^{m+1,\mu}\Im(z)}{z} = (q(z))^{\psi}, \tag{9}$$

and

$$\frac{(1-\sigma)D_{\tau,\beta}^{m,\mu}h(b)+\sigma D_{\tau,\beta}^{m+1,\mu}h(b)}{b} = (t(b))^{\psi}, \tag{10}$$

where $q(z) = 1+q_1z+q_2z^2+q_3z^3+\cdots$ and $t(b) = 1+t_1b+t_2b^2+t_3b^3\cdots$ are in $\mathcal{P}$. Equating the coefficient in (9) and (10), we have

$$[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]l_2 = \psi q_1, \tag{11}$$

$$[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3=\psi q_2+\frac{\psi(\psi-1)}{2}q_1^2, \tag{12}$$

$$-[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]l_2=\psi t_1, \tag{13}$$

$$[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}](2l_2^2-l_3)=\psi t_2+\frac{\psi(\psi-1)}{2}t_1^2. \tag{14}$$

From (11) and (13), we get

$$q_1=-t_1, \tag{15}$$

and

$$2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2=\psi^2(q_1^2+t_1^2). \tag{16}$$

From (12),(14) and (16), we get

$$\begin{aligned}&2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2\\&-[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_3=\psi t_2+\frac{\psi(\psi-1)}{2}t_1^2\end{aligned}$$

implies

$$\begin{aligned}&2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2\\&=[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_3+\psi t_2+\frac{\psi(\psi-1)}{2}t_1^2.\end{aligned}$$

Then from (12), we have

$$2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2 = \psi q_2+\frac{\psi(\psi-1)}{2}q_1^2+\psi t_2+\frac{\psi(\psi-1)}{2}t_1^2,$$

implies

$$2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2 = \psi(q_2+t_2)+\frac{\psi(\psi-1)}{2}(q_1^2+t_1^2).$$

Then from (16), we get

$$\begin{aligned}&2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2l_2^2\\&=\psi(q_2+t_2)+\frac{\psi(\psi-1)}{2}\frac{2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2}{\psi^2}l_2^2,\end{aligned}$$

implies

$$l_2^2=\frac{\psi^2(q_2+t_2)}{\begin{array}{c}2\psi[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]-\\ \psi(\psi-1)[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2\end{array}}. \tag{17}$$

Applying Lemma 1 for (17), we get

$$|l_2|\leq\frac{2\psi}{\sqrt{\begin{array}{c}2\psi[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]-\\ \psi(\psi-1)[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2\end{array}}}$$

which gives the desired estimate on $|l_2|$ in (7). Hence in order to find the bound on $|l_3|$,

$$\begin{aligned}&[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3-[(1-\sigma)(1+\tau(2+\mu-\beta))^m\\&+\sigma(1+\tau(2+\mu-\beta))^{m+1}](2l_2^2-l_3)=\psi q_2+\frac{\psi(\psi-1)}{2}q_1^2-[\psi t_2+\frac{\psi(\psi-1)}{2}t_1^2],\end{aligned}$$

implies

$$2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3$$
$$=[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]2l_2^2+\psi(q_2-t_2)+\frac{\psi(\psi-1)}{2}(q_1^2-t_1^2).$$

Since $(q_1)^2=(-t_1)^2 \implies q_1^2=t_1^2$, then we have

$$2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3$$
$$=\psi(q_2-t_2)+[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]2l_2^2$$

$$l_3 = \frac{\psi(q_2-t_2)}{2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]} + \frac{[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}{2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}2l_2^2.$$

From (16), we have

$$l_3 = \frac{\psi(q_2-t_2)}{2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]} + \frac{\psi^2(q_1^2+t_1^2)}{2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2}.$$

Applying Lemma 1 for coefficient q_1, q_2, t_1 and t_2, we have

$$|l_3| \leq \frac{2\psi}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]} + \frac{4\psi^2}{[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2}.$$

□

3. Coefficient bounds for the function class $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(m,\xi,k,\tau)$

Definition 2. A function $\Im(z) \in \mathcal{G}$ is said to be in the class $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(m,\xi,k,\tau)$ if the following condition are fulfilled:

$$\Re\left[\frac{(1-\sigma)D^{m,\mu}_{\tau,\beta}\Im(z)+\sigma D^{m+1,\mu}_{\tau,\beta}\Im(z)}{z}\right] > \xi, \tag{18}$$

$$\Re\left[\frac{(1-\sigma)D^{m,\mu}_{\tau,\beta}h(b)+\sigma D^{m+1,\mu}_{\tau,\beta}h(b)}{b}\right] > \xi, \tag{19}$$

where $\Im(z) \in \mathcal{H}, 0 \leq \xi < 1, \sigma \geq 1, \tau \geq 0, z \in \triangle, b \in \triangle, 0 \leq \mu \leq \beta, m \in \mathcal{N}_0$, and

$$h(b) = b - l_2b^2 + (2l_2^2 - l_3)b^3 - (5l_2^3 - 5l_2l_3 + l_4)b^4 + \cdots, \tag{20}$$

and

$$D^{m,\mu}_{\tau,\beta}\Im(z) = z + \sum_{k=2}^{\infty}(1+(k+\mu-\beta-1)\tau)^m l_k z^k, \tag{21}$$

where $0 \leq \mu \leq \beta, \tau \geq 0$ and $m \in \mathbb{N}_0 = \{0,1,2,3\cdots\}$ is the generalized Al-oboudi derivative defined by Opoola [7].

Remark 2. .

1. $\mathcal{M}^{\mu,\beta}_{\mathcal{H},1}(0,\xi,\tau)$=$\mathcal{M}_{\mathcal{H}}(\xi)$ which Srivastava *et al.*, [1] presented and studied.

2. $\mathcal{M}^{\mu,\beta}_{\mathcal{H},\sigma}(0,\xi,\tau)$=$\mathcal{M}_{\mathcal{H},\sigma}(\xi)$ which Frasin and Aouf [4] presented and studied.
3. $\mathcal{M}^{1,1}_{\mathcal{H},\sigma}(m,\xi,1)$=$\mathcal{M}_{\mathcal{H},\sigma}(m,\xi)$ which Porwal and Darus [8] presented and studied.
4. $\mathcal{M}^{1,1}_{\mathcal{H},\sigma}(m,\xi,\tau)$=$\mathcal{M}_{\mathcal{H},\sigma}(m,\xi,\tau)$ which Darus and Singh [5] presented and studied.

Theorem 2. *Let $\Im(z) \in \mathcal{G}$ be in the class $\mathcal{M}^{\mu,\beta}_{\mathcal{H}}(m,\xi,k,\tau)$, $0 \le \xi < 1$, $\sigma \ge 1$, $\tau \ge 0$, $z \in \triangle$, $b \in \triangle$, $0 \le \mu \le \beta$, $m \in \mathcal{N}_0$, then*

$$|l_2| \le \sqrt{\frac{2(1-\xi)}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}]}}, \tag{22}$$

and

$$|l_3| \le \frac{4(1-\xi)^2}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}]^2} + \frac{2(1-\xi)}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}]}. \tag{23}$$

Proof. From (18) and (19), where $q(z), t(z) \in \mathcal{P}$,

$$\frac{(1-\sigma)D^{m,\mu}_{\tau,\beta}\Im(z) + \sigma D^{m+1,\mu}_{\tau,\beta}\Im(z)}{z} = \xi + (1-\xi)q(z), \tag{24}$$

and

$$\frac{(1-\sigma)D^{m,\mu}_{\tau,\beta}h(b) + \sigma D^{m+1,\mu}_{\tau,\beta}h(b)}{b} = \xi + (1-\xi)t(b), \tag{25}$$

where $q(z) = 1 + q_1 z + q_2 z^2 + q_3 z^3 + \cdots$ and $t(b) = 1 + t_1 b + t_2 b^2 + t_3 b^3 \cdots$. Now on equating the coefficient in (24) and (25), we have

$$[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]l_2 = (1-\xi)q_1, \tag{26}$$

$$[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3 = (1-\xi)q_2, \tag{27}$$

$$-[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]l_2 = (1-\xi)t_1, \tag{28}$$

$$[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}](2l_2^2 - l_3) = (1-\xi)t_2. \tag{29}$$

From (26) and (28), we have

$$q_1 = -t_1, \tag{30}$$

and

$$2[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]^2 l_2^2 = (1-\xi)^2(q_1^2 + t_1^2). \tag{31}$$

From (27) and (29), we have

$$2[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]l_2^2 = (1-\xi)(q_2 + t_2), \tag{32}$$

or we have

$$l_2^2 = \frac{(1-\xi)(q_2+t_2)}{2[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]}$$

implies

$$|l_2^2| \le \frac{2(1-\xi)}{[(1-\sigma)(1+\tau(1+\mu-\beta))^m + \sigma(1+\tau(1+\mu-\beta))^{m+1}]}$$

which is the bound on $|l_2|$ as given in (22). Hence in order to find the bound on $|l_3|$, we subtract (27) and (29) and get

$$[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3$$
$$-[(1-\sigma)(1+\tau(2+\mu-\beta))^m + \sigma(1+\tau(2+\mu-\beta))^{m+1}](2l_2^2 - l_3) = (1-\xi)q_2 - [(1-\xi)t_2],$$

implies

$$2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]l_3$$
$$=[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]2l_2^2+(1-\xi)(q_2-t_2),$$

implies

$$l_3=l_2^2+\frac{(1-\xi)(q_2-t_2)}{2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}.$$

Then from (31), we have

$$l_3 = \frac{(1-\xi)^2(q_1^2+t_1^2)}{2[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2} + \frac{(1-\xi)(q_2-t_2)}{2[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]}.$$

Applying Lemma 1 for the coefficient q_1, q_2, t_1 and t_2, we get

$$|l_3| \leq \frac{4(1-\xi)^2}{[(1-\sigma)(1+\tau(1+\mu-\beta))^m+\sigma(1+\tau(1+\mu-\beta))^{m+1}]^2} + \frac{2(1-\xi)}{[(1-\sigma)(1+\tau(2+\mu-\beta))^m+\sigma(1+\tau(2+\mu-\beta))^{m+1}]},$$

which is the bond on $|l_3|$ in (23). □

4. Conclusion

In this present paper, two new subclasses of bi-univalent functions associated with Opoola differential operator $D^{m,\mu}_{\tau,\beta}$ were introduced and worked on. Furthermore, the coefficient bounds for $|l_2|$ and $|l_3|$ of functions in these classes are obtained.

Conflicts of Interest: "The author declares no conflict of interest."

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