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A Comparative Study of Metaheuristics Techniques for Portfolio Selection Problem

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Abstract: Portfolio Selection Problem (PSP) is one of the major interesting research areas in finance which have drawn interest of several researchers over the years. Over time, the different approaches had been engaged in solving the PSP ranging from computational techniques to metaheuristics techniques with varying results. In this study, we engaged three different metaheuristics techniques under this same condition to solve extended Markowitz mean-variance portfolio selection model. The three metaheuristics techniques are Non-dominated Sorting Genetic Algorithm II (NSGAII), Speed-constrained Multi-objective Particle Swarm Optimization (SMPSO) and Generalized Differential Evolution 3 (GDE3). A comparative analysis was carried out with results obtained with existing benchmark data available in literature. The outcome of the findings reveals that SMPSO shows superior performance, followed by NSGAII in many different instances, however, the mean execution time of GDE3 was the fastest among the three techniques considered.

Key word: Portfolio selection, speed-constrained multi-objective particle swarm optimization, non-dominated sorting Genetic algorithm, generalized differential evolution 3, metaheuristic

INTRODUCTION

There are quite a number of methods researcher had engaged to tackle the portfolio selection problem with one short coming or the other. The following are the prevalent methods that have been applied to PSP in literature. Fuzzy set theory had been immensely engaged in portfolio selection, among the few works reported in literature are the research of Tanaka *et al*. (2000), Leon *et al*., (2002), Huang (2006), Gupta *et al*. (2008), Li *et al*. (2009) and Li and Xu (2013). Genetic algorithm (GA) has also been extensively used to solve PSP as reported in research of Loraschi *et al*. (1995), Lin and Wang (2002), Oh *et al*. (2006), Zhang *et al*. (2006) and Lin and Liu (2008). In the research of Golmakani and Fazel (2011) engaged a heuristic technique of Particle Swarm Optimization (PSO) to extend Markowitz mean variance portfolio selection problem. Their findings compared with GA revealed a superior performance over GA Model. Also in a similar research by Zhu *et al*. (2011), developed PSO Model for PSP and compared their results with GA Model. Their finding showed that PSO Model demonstrated high computational efficiency in building optimal risky portfolios. Others related works that engaged PSO for PSP are Pulido and Coello (2004), Xu *et al*. (2007) and Zheng *et al*. (2007). Few related works that engaged GDE for portfolio selection problem are as follows, Ardia *et al*. (2011) and Ma *et al*. (2012).

This study presents an empirical comparative study of three different metaheuristics techniques to portfolio selection model and relates the findings obtained to what has been reported in extant literature. The finding reveals that SMPSO shows superior performance, followed by NSGAII in many different instances, however, the mean execution time of GDE3 was the fastest among the three techniques considered.

Portfolio selection problem: This study describes the PSP Model used in this research as formulated in the research (Adebiyi and Ayo, 2015). The model is an extension of Markowitz's mean variance portfolio selection model in the research of Zhu *et al*. (2011). To explain the PSP Model the definition of following variables are of importance. Therefore:

N = The Number of available assets
M = The number of assets to be selected from Navailable assets
B = The total available budget
R = The investor's expected rate of return
F^2_p = The return variance of the portfolio
F_{ij} = The covariance of returns of asset i and j

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B_{lower} = The minimum amount of budget that can be invested in asset i
B_{upper} = The maximum amount of budget that can be invested in asset i
C_i = The minimum transaction lots for asset I
x_i = The number of C_i's that is purchased
w_i = The decision variable that represents the weight of the budget to be invested in asset i
w_j = The decision variable that represents the weight of budget to be invested in asset j
z_i = A binary variable {0,1} if 1 asset i is in the portfolio and otherwise 0
w_i = The expert opinion, a random variable of equal or greater than 0.5 if the asset i is selected and otherwise 0
I = The index of securities

Investors are always desire to minimize risk of investment and maximize possible return. The extended Markowitz model for the portfolio selection problem used in this study is as formulated as follows:

$$\min \quad \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

Where:

$$w_j = \frac{x_i c_i z_i}{\sum_{i=1}^{N} x_i c_i z_i},\ j = 1, ..., N \tag{2}$$

And:

$$\sum_{i=1}^{N} z_i = M \le N;\ M, N \in N \tag{3}$$

Subject to:

$$\sum_{i=1}^{N} x_i c_i z_i e_i r_i \ge BR \tag{4}$$

$$\sum_{i=1}^{N} x_i c_i z_i e_i \le B \tag{5}$$

$$0 \le B_{lower_i} \le x_i c_i \le B_{upper_i} \le B,\ i = 1, ..., N \tag{6}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{7}$$

$$w_i \ge 0,\ \forall_i \in \{1, 2, ..., N\} \tag{8}$$

$$e_i \in \{0, 1\} \tag{9}$$

Where:

$$z_i = \begin{cases} 1 \text{ if } e_i \ge 0.5 \\ 0 \text{ if } e_i \ge 0.5 \end{cases} \tag{10}$$

$x_i c_i$ represents the number of units of asset i in the selected portfolio. Z_i is the decision variable in which it is equal to 1 if the asset i is upheld in the portfolio and otherwise 0. The inequality in Eq. 3 denotes cardinality constraint. Equation 5 represents the budget constraint. Equation 6 indicates the bounds on holdings constraint. Equation 7 and 8 ensure that the total budgets are invested in the portfolio. Equation 9 and 10 represent the expert opinion constraint. The expert opinion constraint is a practicable and useful constraint in a real life scenario of portfolio selection because the expert has detailed information about sector capitalization where each asset i to be selected in the portfolio belong in order to minimize investment risk. Beyond sector capitalization the expert or financial analyst can access other information regarding each asset i to be selected in the portfolio such as price/annual earning, management calibre, dividend rate, book value and so on. An in-depth analysis of these information can guide the expert upon which an opinion is formed whether asset i should be included in the portfolio or not. This study considered four different set of important constraints to the portfolio selection problem. This extended Markowitz Model was solved with three efficient Metaheuristics to find the optimum solution and compared the results with one another.

MATERIALS AND METHODS

This study describes data set used and experimental details. The extended Markowitz Model used in this researach was implemented with efficient Metaheuristics method of NSGAII, SMPSO and GDE3 with each set of data of 31 and 85 stocks from the stock markets of Hong Kong Hang Seng and the German DAX 100, respectively. The data were obtained from test data from OR-Library, 2104. Each data set contains the number of assets (N). The mean return and standard deviation of return for each asset i and correlation between asset i and j for all possible pairs of assets. In order to evaluate the performance of the algorithm on the portfolio model used. It was run on a PC with Intel Pentium 4.3 GHz with 2GB RAM. The parameter settings for each of the data set is as follows: expert opinion was set to >0.5 if the asset is selected in the portfolio, the value of the budget was set to 2800, expected rate of returns was set to 0.004, 0.005 and 0.006, respectively. A predetermined upper and lower bound was set for each of the selected assets. The size of portfolio was set to 15, 20, 25 for each of the data set.

Five criteria were used to compare the performance of the results obtained by the NSGAII, SMPSO and GDE3 algorithms used for the portfolio model. The criteria are as follows:

- Best variance, depict lowest risk from algorithm runs, showing the best solution found
- Mean variance, the average of the objective function found by the algorithm
- Worst variance, depicts the highest risk from algorithm runs, showing the worst solution
- Standard deviation of variance, depicts how close the solution found by the algorithms are close to each other and
- Mean execution time, depicts the amount of time needed to arrive to a solution

RESULTS AND DISCUSSION

The results of GDE3, NSGAII and SMPSO algorithms for data set of 31 stocks are tabulated in Table 1 over 50 independent iterations. Similarly, the results obtained for data set of 85 stocks with GDE3, NSGAII, SMPSO are contained in Table 2 accordingly.

The computational experiment as indicated in Table 1 when the size of data set is 31 shows that SMPSO have the best results in all the instances when the size of the portfolio is 15, 20 and 25, respectively. This is followed by the results of NSGAII. However, it was observed that computational time of GDE3 is lesser than NSGAII and SMPSO metaheuristics.

Similarly, to further evaluate the performance of the extended Markowitz portfolio model in a complex scenario of larger dataset of 85 stocks. Table 2 shows the results obtained with 85 stock data set and comparison with metaheuristics used depicts similar trend as SMPSO gave superior performance over the other

Table 1: Results of applying GDE3, NSGAII and SMPSO algorithms to 31 stocks data set across 50 independent executions

Variance/Size of portfolio/ Expected rate of return/Algorithms	0.004			0.005			0.006		
	GDE3	NSGAII	SMPSO	GDE3	NSGAII	SMPSO	GDE3	NSGAII	SMPSO
15									
Best	0.45123666	0.35203899	0.15107619	0.45057284	0.34717306	0.214028489	0.54322836	0.35378268	0.25157078
Mean	0.83230867	0.67218507	0.61424974	0.79284597	0.64333504	0.566691078	0.71944886	0.68509601	0.60348172
Worst	1.10275477	0.98431799	0.77090457	1.09839860	0.95442150	0.735944859	0.96206750	1.07633423	0.69708422
SD	0.34063198	0.13738563	0.13578541	0.22195648	0.15035748	0.129776277	0.19958594	0.17494801	0.14831623
Mean exe. time (sec)	22.81358	34.74636	38.32675	24.20438	29.6409	31.0872	32.41838	28.78512	35.6251
20									
Best	0.73169554	0.52981593	0.32947806	0.90755939	0.51186707	0.373316492	0.536085601	0.315823093	0.287602147
Mean	1.46968870	0.93358625	0.55694377	1.49795920	0.91360268	0.800158982	1.472028375	0.908991852	0.705545178
Worst	2.31710848	1.56117885	0.95614994	2.07168664	1.38676291	1.234916183	2.403294162	1.448040121	0.943375093
SD	0.35321293	0.21277851	0.17553432	0.31429033	0.19925772	0.114052924	0.356558781	0.232814076	0.188732521
Mean exe. time (sec)	37.39024	43.38638	45.93512	28.52338	30.10292	32.5496	36.85026	33.25678	39.98751
25									
Best	0.88679723	0.65977877	0.45408498	1.14151308	0.63744981	0.510079919	0.845219905	0.635221067	0.434594241
Mean	1.65110704	1.08509956	0.68236685	1.81419318	1.10107861	0.946176642	1.703029315	1.058870543	0.721922588
Worst	2.57558983	2.09389079	0.86684947	2.55817477	1.71801513	1.368720214	2.705876324	1.723429881	1.239178664
SD	0.39776689	0.28499070	0.16394986	0.35767066	0.26451288	0.214568917	0.383476778	0.223461266	0.172342988
Mean exe. time (sec)	25.19022	36.53728	40.18954	21.07522	25.48038	29.50243	25.05926	20.6254	28.9627

Table 2: Results of applying GDE3, NSGAII and SMPSO algorithms to 85 stock data set across 50 independent executions

Variance/Size of portfolio/ Expected rate of return/Algorithms	0.004			0.005			0.006		
	GDE3	NSGAII	SMPSO	GDE3	NSGAII	SMPSO	GDE3	NSGAII	SMPSO
15									
Best	0.22518776	0.14043814	0.13438406	0.17278134	0.09654804	0.079737446	0.235832895	0.198249716	0.075848570
Mean	0.60096916	0.29943517	0.26523937	0.54470193	0.28107119	0.254470528	0.547518040	0.322653323	0.278438959
Worst	0.96524521	0.55895829	0.48650518	0.94764824	0.43885621	0.394134793	1.023240679	0.598769274	0.356018608
SD	0.16767265	0.08599229	0.07007661	0.18088869	0.07421649	0.067178084	0.168622109	0.083048395	0.064038095
Mean exe. time (sec)	33.55548	39.26028	42.63432	37.81104	45.05456	47.11035	42.08742	44.13692	46.98563
20									
Best	0.35715149	0.20805088	0.17834219	0.41142129	0.24077569	0.158127063	0.432725211	0.217592437	0.137872202
Mean	0.81708831	0.41541530	0.35715149	0.78676979	0.40527862	0.326921371	0.832714277	0.427922810	0.283161073
Worst	1.23085633	0.70692483	0.59088845	1.35634723	0.70969466	0.527231052	1.351143703	0.743734672	0.487801885
SD	0.19587954	0.10504707	0.09101289	0.19614638	0.11433733	0.083268498	0.230234543	0.124465355	0.074373463
Mean exe. time (sec)	37.95714	42.44526	43.56093	32.60874	38.62946	41.78632	35.80182	37.14916	39.9572
25									
Best	0.39090568	0.27617457	0.19016071	0.4885082	0.30835802	0.232138784	0.356562971	0.325238055	0.304761661
Mean	0.83595081	0.52020104	0.35529302	0.85784000	0.52260675	0.488508212	0.877025262	0.529653841	0.410432784
Worst	1.37399055	0.78384814	0.68452987	1.13836791	0.95231780	0.720440502	1.336416098	0.789133804	0.633696336
SD	0.21105804	0.11005574	0.08231027	0.17048585	0.13102051	1.028592817	0.211977567	0.113633008	0.10946686
Mean exe. time (sec)	38.01702	44.0845	45.94935	36.69042	41.6266	44.889324	42.45796	50.19838	53.33126

metaheuristics of NSGAII and GDE3. However, GDE3 metaheuristics have less computation time to generate solutions with the portfolio model in comparison to other metaheuristics.

The results in this study corroborate others finding in extant literature that SMPSO metaheuristics provide alternative promising method in solving portfolio selection problem. It can be used as a guide to investors to minimize their risks of investment.

CONCLUSION

In this research work, a comparative study of three metaheuristics algorithms are engaged to solve portfolio selection problem. The three metaheuristics techniques are Non-dominated Sorting Genetic Algorithm II (NSGAII), Speed-constrained Multi-objective Particle Swarm Optimization (SMPSO) and Generalized Differential Evolution 3 (GDE3). The outcome of the findings reveals that SMPSO shows superior performance, followed by NSGAII in many different instances, however, the computational time of GDE3 was the fastest among the three techniques considered.

RECOMMENDATION

The future studies are to engage hybrid of swarm intelligence techniques to solve PSP model for optimum performance.

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