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Estimating Production Function with Economic Content Using Data Envelopment Analysis as a Complement to Marginal Analysis in Rice Production of Kwara State, Nigeria

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Authors' contributions

This work was carried out in collaboration between all authors. Author IAA designed the study, wrote the protocol and wrote the first draft of the manuscript. Author OMB managed the literature searches. Author JOA did analyses of the study. Author KUI managed the data collection. Author JOO did the coding and data computation. All authors read and approved the final manuscript.

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ABSTRACT

The consideration in the present study is mainly conceptual. The objective is to show how Data Envelopment Analysis (DEA) can be used to reveal the true input-output relations in rice production. In the estimation of a production function, it is assumed that all firms use the existing technology efficiently. However, in the real world the observed firms produce homogeneous outputs with differences in factor intensities and in managerial capacity. Hence, inefficiencies are hidden in the estimated production functions. In order to overcome this drawback of the parametric approach and to reveal the true nature of the input-output relations in production, given the available technology, the DEA approach is

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applied. In this study DEA is applied in order to select the farms that utilize efficiently the existing technology, allowing the estimation of a production function that reveals the true input-output relations in rice farming, using farm accounting data from a sample of 60 rice farms.

Keywords: Data envelopment analysis; production function; parametric approach.

1. INTRODUCTION

Production function has been used as an important tool of economic analysis in the neoclassical tradition. It is generally believed that [1] was the first economist to algebraically formulate the relationship between output and inputs as $P = f(x_1, x_2, ..., x_m)$ although there are some evidences suggesting that Johann von Thünen first formulated it in the 1840's [2].

It is relevant to note that among others there are two leading concepts of efficiency relating to a production system: the one often called the 'technical efficiency' and the other called the 'allocative efficiency' [3]. The formulation of production function assumes that the engineering and managerial problems of technical efficiency have already been addressed and solved, so that analysis can focus on the problems of allocative efficiency. That is why a production function is (correctly) defined as a relationship between the maximal technically feasible output and the inputs needed to produce that output [3,4].

However, in many theoretical and most empirical studies it is loosely defined as a technical relationship between output and inputs, and the assumption that such output is maximal (and inputs minimal) is often tacit. Further, although the relationship of output with inputs is fundamentally physical, production function often uses their monetary values. The production process uses several types of inputs that cannot be aggregated in physical units. It also produces several types of output (joint production) measured in different physical units. There is an extreme view that (in a sense) all production processes produce multiple outputs [3,5]. One of the ways to deal with the multiple output case is to aggregate different products by assigning price weights to them. In so doing, one abstracts away from essential and inherent aspects of physical production processes, including error, entropy or waste. Moreover, production functions do not ordinarily model the business processes, whereby ignoring the role of management, of sunk cost investments and the relation of fixed overhead to variable costs.

It has been noted that although the notion of production function generally assumes that technical efficiency has been achieved, this is not true in reality. Some economists and operations research workers [6,7,8,9,10,11] addressed this problem by what is known as the 'Data Envelopment Analysis' or DEA. The advantages of DEA are: first that here one need not specify a mathematical form for the production function explicitly; it is capable of handling multiple inputs and outputs and being used with any input/output measurement and efficiency at technical/managerial level is not presumed. It has been found useful for investigating into the hidden relationships and causes of inefficiency. Technically, it uses linear programming as a method of analysis.

In the estimation of a production function it is assumed that all firms use the existing technology efficiently. However, in the real world the observed firms produce homogeneous outputs with differences in factor intensities and in managerial capacity. Hence, inefficiencies are hidden in the estimated production functions.

In order to overcome this drawback of the parametric approach and to reveal the true nature of the input-output relations in production, given the available technology, the DEA approach is applied. The objective of this study is to use DEA in order to select the farms that utilize efficiently the existing technology and allowing the estimation of a production function that reveals the true input-output relations in rice farming of Kwara state. Hence, the main objective of this study is to estimate the production function with economic content using DEA as a complement to Marginal Analysis in rice agriculture. The result of this study will provide the basic information on the factors affecting rice production in Nigeria.

1.1 An Outline of the Methodologies

Data Envelopment Analysis (DEA) has its origins in the seminal work by [7] who reformulated [6] approach. In this study, they described DEA as a “mathematical programming model applied to the observational data that provides a new way of obtaining empirical estimates of extremely relations – such as the production functions and / or efficient production possibility surfaces that are a cornerstone of modern economics”.

In general, DEA methodology uses a set of production units of a sample to construct an efficiency frontier consisting of all possible linear combinations of efficient production units. The frontier technology consists of convex input and output sets enveloping the data points with linear facets. Consequently, the efficient units lie by definition on that frontier while the inefficiency of units that are not on the frontier is indicated in direct proportion to their distance from the frontier. Individual units are considered as Decision Making Units (DMUs) and efficiency can be measured relative to the highest observed performance rather than against some average. The proposed measure of efficiency of any DMU is obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to the condition that the similar ratios for every DMU be less than or equal to unity.

Since DEA is now a well-established method it is not necessary to go into details about the theoretical background of the approach. The basic version of the DEA model, which is also known as the CCR model (it was initially proposed by Charnes, Cooper and Rhodes) can be found in [7]. The extensions that have been proposed can be found in [10].

For the purpose of this study out of the six measures of relative efficiency (overall cost-minimising efficiency, technical efficiency, allocative efficiency, pure technical efficiency, scale efficiency and efficiency due to input congestion) only technical efficiency is computed applying the input oriented model.

The main advantage of DEA is that it does not require specification of the functional form of the production function. DEA calculations focus on individual observations in contrast to population averages. It can simultaneously utilize multiple outputs and multiple inputs with each being stated in different units of measurement. DEA also focus on revealed best-practice frontiers rather than on central-tendency properties or frontier and it generates the set of “peer” units with which a unit is compared. However, several properties that represent strengths in one capacity may act as limitations in another. One of the main criticisms of DEA is that the method does not at first sight have any statistical foundation, i.e. that it is not possible to make inference about estimated DEA parameters, sensitivity, asymptotic properties etc. This poses a problem, seeing that uncertainty and measurement errors will often be present in observed data. Sometimes Stochastic Production Frontier (SPF) analysis may be preferred as a method that includes measurement errors and uncertainty [12].

The Cobb-Douglas model has been widely used in the agricultural economics. The use of single equation models for agricultural production functions has been justified by [13] and [14] who argue that because inputs in agriculture are largely predetermined because of a considerable lag in production and due to the fact that error is weather determined, simultaneous equation bias will be small for well specified production functions. The production environment in the present study does not seem to differ from the specification requirements postulated by the authors mentioned above.

The primary purpose of the estimation of a production function is to obtain estimates of regression coefficients and marginal factor productivities, which can be useful for the study of efficiency when they are compared with marginal factor costs.

Parametric approach requires more assumptions about the production function and also about the distribution of the errors, although it is possible to test for the validity of the assumptions and to determine whether particular variables are relevant. The main weaknesses of the regression approach is that it fits a function on the basis of average behavior; it requires pre-specification of the functional form; it does not take efficiency into consideration; it only gives residuals.

2. METHODOLOGY

The study was carried out in Kwara state, one of the six States in North Central region of Nigeria. The State has sixteen Local Government Areas which covers an area of 74,256sq km of the total area of Nigeria (923,768sq km, approximately one-twelfth). In the State, there are 247,975 farm families with 254,242 hectare of cropped area. The farm accounting data for this empirical application were collected through a farm management survey, of a sample of 60 rice farms, carried out during the 2011-2012 period.

All these farms have the required characteristics for the empirical application of DEA. Each rice farmers consumes varying amounts of inputs to produce different outputs. The application of DEA involves the identification and measurement of relevant inputs and outputs, which are common in all units. The relevant inputs that will be used in this empirical application are: (i) The cultivated land area for rice (ha) (ii) The sum of family labour (person days) (iii) sum of hired labour (person days) (iv) Quantity of seed planted (kg) (v) quantity of fertilizer used (kg) (vi) quantity of herbicides used (litres) (vii) age of farmers (years). The rice output that will be used is total farm output of rice (kg). It is of importance to state here that the relative efficiency score associated to rice farmers is not affected by the choice of a different unit of measure. The measure of efficiency is independent of the units of measurement used. This property is referred to as “unit’s invariance”.

The approach applied consists of three steps. In the first step the input oriented DEA model is applied in a sample of 60 farms of the rice sector. Only 24 of these farms are technically efficient. DEA is applied again using this time as initial sample these 24 farms. The results indicate that only 12 of the farms are relatively technically efficient. The same procedure is followed using the sample of 12 farms and the results indicate that all farms lie on the efficiency frontier. Thus, a sub- sample has been formulated where all DMU’s are relative technical efficient.

The aim of this procedure is not to estimate the efficiency score of the DMUs, but to end up with a sample where each of the farms is laid on the efficiency frontier. Through this non-parametric analysis three sub-samples have been formulated; the first one contains 60 farms, the second 24 farms, the third 12 farms.

Based on these, parametric analysis using Least Squares was applied in order to estimate the production function parameters. An implicit assumption of production functions is that they assume that there are no different endowments of fixed factors of production and no management bias; in other words all farms are technically efficient. Nevertheless, the production frontier indicates the maximum potential output for a given set of inputs. From the production frontier it is possible to measure the relative efficiency of certain groups or set of practices from the relationship between observed production and some ideal or potential production [4]. This ascertainment was the elementary guide for the study. The basic concept was to investigate through DEA how the production function estimators are affected by the aforementioned drawback of the parametric method.

In order to examine this case thoroughly and to reveal this particular aspect of the problem, the specified production function will be estimated for each sample formulated with the assistance of DEA. Production functions are based on the assumption that in a given system or enterprise type, levels of output can be predicted by a given set of inputs, the mix of which basically describes the conversion of inputs into outputs [7]. An understanding of the technology of production is central to the development of realistic theories and to the formulation of a wide range of policies [9].

The most commonly used production function forms are: linear, quadratic, log-linear (Cobb-Douglas, C-D), Constant Elasticity of Substitution (CES) and translog [15]. The linear functional form is commonly used in linear programming models; the quadratic describes a parabolic function that is familiar to biologists [16]. Economists however prefer using the C-D as well as CES models. The C-D model has unity elasticity of substitution whereas the CES permits the empirical data to determine the degree of substitutability among inputs. CES is however difficult to apply when more than two inputs are used; therefore the C-D model is mostly preferred by economists [17].

In this study, the C-D production function was used. It has evolved since its development early in the 1900s and has been widely used in both theoretical and empirical production analyses. There are however some criticisms to the use of this model. These are as follows: it cannot handle a large number of inputs; the function is based on restrictive assumptions of perfect competition in the factor and product markets; it assumes constant returns to scale; serial correlation and heteroscedasticity are common problems that beset this function too; labour and capital, are correlated and the estimates are bound to be biased; unitary elasticity of substitution is unrealistic; it is inflexible in form; single equation estimates are bound to be inconsistent and it cannot measure technical efficiency levels and growth very effectively [18].

The translog function which is considered an alternative, especially in addressing the inability of C-D function in handling unitary elasticity of substitution between inputs, was not used in this study. The C-D was chosen for the following advantages [16,18].

- The partial elasticities of production, which measures the responsiveness of output to unit increase of input, are identical to the production coefficients (β_i). Therefore, a percentage change in output that results from a given percentage change in output use can be easily identified.

- The sum of partial elasticities of production ($\sum\beta_i$) can be interpreted as a measure of economies of scale, i.e., the percentage change of output relative to the percentage change in all inputs used. If $\sum\beta_i$ >1, for example, positive economies of scale exist. This implies that a doubling of the use of all inputs will result in more than a doubling of output.
- Estimation is simple because input and output data can readily be used without aggregation as they are in the CES function.
- Unlike the linear and quadratic forms that preordain the shapes of production surfaces, the unconstrained C-D function can describe a production surface that demonstrates increasing, unitary or decreasing returns to scale depending upon the data.
- Unlike the quadratic function that requires more degrees of freedom because of interaction terms, the C-D function requires only one degree of freedom per explanatory variable.
- Various econometric estimation problems, such as serial correlation, heteroscedasticity and Multicolinearity can be handled adequately and easily.
- It facilitates computations and has the properties of explicit representability, uniformity, parsimony and flexibility.
- Even the problem of simultaneity can be accounted for through the use of stochastic C-D production function.

The C-D function was expressed as follows:

$$\ln Y = \beta_o + \sum_{i=1}^{7} \beta_i \ln X_i + \sum_{i=1}^{11}\sum_{j=1}^{11} \beta_{ij} \ln X_i \ln X_j \tag{1}$$

Where the dependent variable *y,* which was measures of total farm output of rice (kg), X_1, X_2, X_3, X_4, X_5, X_6 X_7, X_8, X_9, X_{10} *and,* X_{11} are “independent” variables representing some measure of the inputs, and the B_j *(j = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11)* are unobserved population parameters.

In this case Y = total farm output of rice (kg)
X_1 = cultivated land area for rice (ha)
X_2 = sum of family labour (person days)
X_3 = sum of hired labour (person days)
X_4 = quantity of seed planted (kg)
X_5 = quantity of fertilizer used (kg)
X_6 = quantity of herbicides used (litres)
X_7 = age of farmers
X_8 = education level (in years)
X_9 = farming experience (in years)
X_{10} = family size (in number)
X_{11} = extension contact (number of visit in a season)

Based on the DEA approach that was used, three models (function of the same sample) was used, in order to obtain the estimates of the regressions coefficients. Hence, in total OLS was applied three times and three different elasticities was calculated for each input [19].

3. RESULTS AND DISCUSSION

3.1 Interpretation of the DEA Results

The initial sample consists of 60 farms. DEA was applied on this sample and the average technical efficiency for this group was 94.3 percent. 36 of the farms included in the initial sample resulted to be technically inefficient (this means a percent below 100). By excluding these 36 farms from the sample a new sub- sample was constructed, which consisted of 24 farms. The same input oriented DEA model was applied to this sub-sample and the average technical efficiency for this group was 97.9 percent. This time 12 farms were technically inefficient. By excluding these 12 farms from the sample a second sub-sample was formulated, which included 12 farms. Again the same procedure was applied and all farms resulted to be technically efficient [3]. This means that all 12 farms lie on the frontier. The results from the application of DEA in each case are presented in Table 1.

Table 1. DEA results

	Total Samples (Number of farms)	Number of inefficient farms	Technical efficiency (Mean)
Model I	60	36	0.9430
Model II	24	12	0.9790
Model III	12	-	1.0000

3.2 Descriptive Statistics of the Variables Included in the Cobb-Douglas Production Function

Descriptive statistics of the variables included in the production function analysis are presented in Table 2. The average production of rice was approximately 6.5tonne per household, which translates, to a mean yield of about 3.1 tonnes per hectare. Rice production was highly variable ranging from 62 kilograms to a maximum of 31.1 tonnes per household.

The table revealed that the minimum farm size in the Kwara state of Nigeria is 0.2 hectare, and the maximum is 3 hectare which translates to give 1.29 hectare as the mean farm size of the rice farmers. It further shows the average family labour used was approximately 74.9 man-day per hectare while the average hired labour used was approximately 74.7 man-day per hectare. It further shows the mean fertilizer used was 646kilogram per hectare, which was equivalent to approximately 287 kilogram per household, which is in congruent with [20]. The seed planted was measured in kilograms and it shows that 51.7kg of rice is the mean rice planted per hectare in the state while the minimum and maximum rice planted per hectare is 4.9kg and 120.5kg respectively.

It is shown from the Table 2 below that the herbicides used to controlled weed in rice production ranges from 13.4 litres to 130.2 litres per hectare with a mean value of 59.6litres per hectare supported by [21].

The ages of rice farmers varies 33 years to 81 years and almost 80 percent of the household members were adults while majority of them are secondary school leavers with a wide range of farming experience in rice production. It also shown from the table that most of them enjoyed extension services which show 8 physical appearances of extension agents on their farm per year. Their family size ranges from 0 to 7 with a mean family size of 3.68 which implies that on the average, majority of the rice farmers in the zone has nothing less than 4 members per farm families and this could be as a result of family labour need in rice production, the result is inline with the study of [22].

Table 2. Summary statistic of variables of the efficient rice farms

Variables	Minimum	Maximum	Mean	Std Deviation
Rice Farm size (ha)	0.2	3.0	1.27	0.61
Family labour (man-days)	20.0	140.0	74.9	36.6
Hired labour (man-days)	16.0	140.0	74.7	30.7
Seed planted (kg)	4.9	120.5	51.7	34.3
Fertilizer (kg)	144.0	1313.0	646.6	341.3
Herbicides (litres)	13.4	130.2	59.6	36.9
Age (years)	33.0	89.0	52.2	12.7
Education level (years)	0.0	16.0	9.16	4.89
Farming experience (years)	0.0	51.0	21.2	10.6
Family size (number)	0.0	7.0	3.68	2.49
Extension contact (number)	0.0	8.0	1.64	7.77

3.2 Factor Elasticities of Cobb-Douglas Production Estimates

Table 3 below presented the results of Cobb-Douglas production function estimates for MODEL 1, MODEL II and MODEL III respectively.

It is of interest to extensionists which inputs are significant to the production process and of those inputs, which have a greater per-unit effect on the total production relative to the other inputs [19]. One can interpret the positive production coefficients of the respective inputs as implying that an increase in output can be accomplished by increasing the intensity of input use. On the other hand, negative coefficients suggest that use of that particular input should be reduced.

The inputs specified in model 1 were rice farm size, family labour, hired labour, seed planted, fertilizer, herbicides while some socio-economic variable were also added which were age, educational level, farming experience, family size and extension contact respectively. Econometric estimation indicated that rice farm size was statistically significant at 1% level of significance. This implies that more attention should be given to this input. It is also shown that family labour significant at 1%. Out of the two input that was significant at 1%, family labour has the highest coefficient which is 0.923 and this implies that a 10% increases in farm size and family labour will increase rice production by 8.3% and 9.2% respectively.

It is further should that hired labour and seed planted is significant at 5% and 10% level of significant respectively. The factor elasticity for hired labour and seed planted were 0.232 and 0.173 respectively which indicted that a 10% increase in hired labour and seed planted will increase rice yield by 2.3% and 1.7% respectively.

On the other hand, only age and extension contact were also significant at 10% with a positive sign which indicated 10% increase in age and the extension contact will increase rice productivity by 1.3% and 0.6% respectively. The factor elasticity for educational level was 0.082 which is statistically significant at 5% which implies that, if the educational level of the farmers is been increase by 10%, rice productivity will also increase by 0.8%. Similar finding were reported by [23,24,25].

The value of F test estimation indicated that the model is significant at 1%. The value of adjusted R^2 is 0.76 which reveals that the model has explained 76% of total variation in rice production dues variation in the explanatory variable. According to [23], the coefficient of determination (adjusted R^2) is a summary measure that tells how well the sample regression line fits the data. The fit of the model is said to be better the closer is R^2 to 1.

From Model II, all the input were statistically significant at 10%, level of significance except seed planted which is not significant at all. Factor elasticities for rice farm size, family labour, hired labour, and fertilizer applied were 0.183, 0.639, 0.964 and 0.989 respectively. Fertilizer applied has the highest coefficient out of those variables which implies the most important inputs out of all of them. The implication drawn from their elasticities is that 10% increase in those inputs will increase rice yield by 1.8%, 6.3%m 9.6% and 9.8% respectively.

In addition, age, farming experience and extension contact were also significant at 1% and 10% level of significance with a positive value of elasticities of 0.79, 1.83 and 0.24 respectively which implies a 10% increase in age, farming experience and extension contact will surely increase rice production by 7.9%, 18.3% and 2.4%. The value of F-Test in model II indicated that the model is also significant at 1%. The value of the model II adjusted R^2 is 0.841 which is better than model 1 according to [26].

From the model III, all inputs variables were significant at difference level of significance except hired labour which was not significant at all. The value of F-test in the estimation indicated that the model is significant at 1%. The Value of adjusted R^2 is 0.97 which reveals that the model has explained 97% of total variation in rice production due to the variation in the explanatory variables therefore, in this model III, 97% variation in rice production has defined by independent variables included in the model III. The intercept is significant at 1% level which implies that the level of output when the value of all independent variable is zero.

The coefficient of rice farm size is positive and significant at 5% level which implies that, other factors keeping constant, 5% increase in area would result in 0.72% increase in rice production. The signs of the regression coefficients were also in consonance with a priori expectations. Similarly, the coefficient of family labour, also significant at 10%, seed planted and herbicides used also significant at 5% while fertilizer applied was also significant at 1% level of significance. The elasticities of herbicides used has the highest value of 3.373 which implies that one percent increase in herbicides, fertilizer and seed planted would result into 3.37%, 2.17 % and 1.89% increase in rice production from the use of respective variables.

Similarly, age of the farmers, Education level are also significant at 10% level, which reveals that other factors keeping constant, when one percent age of the farmers increased, the rice production would be increase by 0.92 and 1.01% respectively. The family size and extension contact is significant at 1% level and has a positive value which indicates the under utilization of family size and extension contact.

In summary, according to [26], the coefficient of determination (adjusted R^2) is a summary measure that tells how well the simple regression line fits the data. The fit of the model is said to be better the closer is R^2 and 1. Therefore, from the 3 models, the model III appeared to have better results and obtained regression coefficients of production function that are free of technical inefficiency.

Table 3. Cobb-Douglas production function estimation for rice farms in kwara State of Nigeria

Variables	Model I			Model II			Model III		
	Coefficient	**Std. Error**	**T-ratio**	**Coefficient**	**Std. Error**	**T-ratio**	**Coefficient**	**Std. Error**	**T-ratio**
Constant	4.049	0.113	35.63***	1.762	0.570	3.09 ***	2.710	0.718	3.77 ***
Rice farm size	0.833	0.045	17.64 ***	0.183	0.070	2.66 ***	0.274	0.131	2.09 **
Family Labour	0.923	0.043	21.17 ***	0.689	0.183	3.75 ***	0.687	0.363	1.89 *
Hired labour	0.232	0.111	2.09 **	0.964	0.384	2.51 ***	1.222	0.906	1.35
Seed planted	0.173	0.901	1.93 *	0.002	0.277	0.01	1.894	0.945	2.00 **
Fertilizer	0.156	0.101	1.54	0.989	0.402	2.46 **	2.171	0.856	2.53 ***
Herbicides	0.115	0.101	1.14	0.189	0.511	0.37	3.373	1.511	2.23 **
Age	0.130	0.078	1.55 *	0.790	0.292	2.71 ***	0.921	0.515	1.79 *
Education level	-0.082	0.036	-2.25 **	0.016	0.046	0.34	1.011	0.518	1.95 *
Farming experience	0.048	0.031	1.52	1.833	0.490	3.74 ***	0.062	0.188	0.33
Family size	-0.023	0.033	-0.70	0.049	0.113	0.43	0.773	0.214	3.61 ***
Extension contact	0.067	0.034	1.97 *	0.241	0.132	1.82 *	1.521	0.210	7.21 ***
Sum Squared Resid			7.680	Sum Squared Resid		5.728	Sum Squared Resid		3.961
R-squared			0.773	R-squared		0.873	R-squared		0.970
Adjusted R-squared			0.762	Adjusted R-squared		0.841	Adjusted R-squared		0.860
F (11, 228)			70.69	F (11, 44)		27.39	F (11, 13)		7.92
P-value (F)			0.0000	P-value (F)		0.0000	P-value (F)		0.0000
Number of Observation			60	Number of Observation		24	Number of Observation		12

** Indicates significance at the 10% level, ** Indicates significance at the 5% level, *** Indicates significance at the 1% level*

3.3 Variance Inflation Factor Analysis for Multicolinearity in Rice Farms (Model III)

Assumption 10 of the classical linear regression model (CLRM) is that there is no Multicolinearity among the regression included in the regression model. According to [26], the term Multicolinearity is due to Ragnar Frisch. Originally it meant the existence of a perfect or exact linear relationship among some or all explanatory variables of a regression model.

The speed with which variance and covariance's increases can be seen with the variance inflating factor (VIF) which is defined as

$$VIF = \frac{1}{\left(1 - r_{23}^2\right)}$$

VIF show how the variance of an estimator is inflated by the presence of Multicolinearity. As r_{23}^2 approaches 1, the VIF approaches infinity. That is as the extent of colinearity increases the variance of an estimator increases and in the limit it can become infinite. VIF as a measure of collinearity is not free of criticism, a high VIF can be counter balanced by a low σ^2 or a high $\sum x^2 j$; and to put it differently a high VIF is either necessary or sufficient to get high variance and high standard errors. Therefore, high Multicolinearity as measured by a high VIF, may not necessarily cause high standard errors.

One of the important assumption of the classical linear regression model is that the variance of each disturbance term U_i, conditional on the chosen values of the explanatory variables, is some constant number equal to σ^2. This is the assumption of homoscedasticity, or equal (homo) spread (scedasticity), that is, equal variance.

Heteroscedasticity likely to be encountered in cross-sectional analysis. Heteroscedasticity can also arise as a result of the presence of outliers. An outlying observation, or outlier is an observation that is much different (either very small or very large) in relation to the observations in the sample. More precisely, an outlier is an observation from a different population to that generating the remaining sample observation. The inclusion or exclusion of such an observation, especially if the sample size is small can substantially alter the results of the regression analysis. Another source of heteroscedasticity arises from violating the assumption 9 of CLRM namely that the regression model is correctly specified.

For this study, White's test was used to test for the presence of heteroscedasticity while VIF (variance inflation factor) was used to test for the presence of Multicolinearity. The Table 4 below shows that there is no problem of multi-colinearity for the preferred model III, VIF gave a value lower than 10 for all the explanatory variable considered in the model.

Any value of VIF that is greater than 10.0 strongly indicates the presence of multi-colinearity for such variable that is having VIF value greater than 10 the results of white's test for the northwest rice farmers shown that there is no problem of heteroscedasticity based on the fact the chi-square value obtained does not exceed the critical chi-square value.

The 5% critical chi-square value for 11 df is 10.0705, the 10% critical value is 8.1363 and the 25% critical value is 6.52468. For all practical purposes, one can conclude, on the basis of white test, that there is no problem of heteroscedasticity.

Table 4. Variance inflation factor analysis for multicolinearity in rice farms (Model III)

Variables	VIF
Rice Farm size (ha)	2.14
Family labour (man-days)	1.15
Hired labour (man-days)	2.95
Seed planted (kg)	1.97
Fertilizer (kg)	1.38
Herbicides (litres)	4.85
Age (years)	4.53
Education level (years)	3.15
Farming experience (years)	1.22
Family size (number)	1.41
Extension contact (Number)	3.22

Variance inflation factor for Multicolinearity. Minimum possible value = 1.0 Value > 10.0 indicate a colinearity problem

3.4 Marginal Physical Productivity of Rice Farms of Nigeria

Having estimated the elasticity of output with respect to the physical inputs, it become necessary to evaluate the resource-use productivities. This is done by estimating the marginal and average physical productivities and their respective values.

The Table 5 shows the marginal productivities of the physical inputs. Specifically, the marginal physical productivities of labour (Family and hired), seed planted, fertilizer and herbicides respectively. By looking at the geometric means of the independent variables, which are computed for naira per hectare output, so that a comparison between the difference cases can take place. It is absolutely clear that farms that compose the sample in the third model are utilizing inputs in a more productive sense. All inputs are decreasing in order to produce the same level of output. The marginal products of considered inputs change in the expected way i.e. for decreasing inputs the marginal products increase and vice versa. The marginal product of family labour computed at the mean of input and output is 6.78₦/ha in the model I, 6.29₦/ha in the model II and 6.90₦/ha in the model III. It is decreasing but at a very slow rate. This fulfilled the requirement of monotonicity, as they are non-decreasing and greater than zero.

Productivities of seed planted is estimated to be 1.69 and therefore the MPP estimates shows that if seed planted is increased by 1 unit, rice output would increase by 1.69 units showing a more than proportionate increment.

The MPP of fertilizer is 27.6 and therefore the MPP estimated shows that if fertilizer applied is increased by 1 unit rice output would also increase by 27.6 units. Herbicidies shows a marginal physical product of 83.5 which further revealed that if 1 unit of herbicides is been added it would surely generate 89.5 units of rice output.

Table 5. Marginal value products of factors used by the rice farms

Number of Farms	**60**	**24**	**12**
Sample Means			
Rice Yield (₦/ha)	225,000.00	243,000.00	261,000.00
Family labour (₦/ha)	30,644.68	26,588.95	25,940.00
Hired labour (₦/ha)	30,428.34	27,672.70	27,242.00
Seed Planted (₦/ha)	6190.97	3195.88	2913.50
Fertilizer (₦/ha)	36,263.12	23,027.71	20,510.83
Herbicides (₦/ha)	10,659.77	12,004.75	10,541.75
Marginal Products			
Family labour (₦/ha)	6.78	6.29	6.90
Hired labour (₦/ha)	1.72	8.47	11.7
Seed Planted (₦/ha)	0.63	0.15	1.69
Fertilizer (₦/ha)	0.96	10.4	27.6
Herbicides (₦/ha)	2.42	3.83	83.5

3.5 Returns-to-Scale and Resource Use Efficiency of the Rice Farms

The law of production describes the technical possibility in increasing total output by changing all factor of production in the long run horizon by the same proportion [27]. Thus returns to scale is technical properties of the production function with respect to input level changed by the same proportion. The question of interest is whether the resulting output increased by the same proportion (constant return to scale), more than proportionally (increasing return to scale), or less than proportionally (decreasing return to scale). Recalling our production function, Y = f(Xn)

$$Y = f(x_1, x_2, x_3 \ldots\ldots\ldots x_n)$$

Suppose to increase all factors of the function by the same proportion K, the new output level of Y* such that

$$Y^* = f(kx_1, kx_2, kx_3 \ldots\ldots\ldots kx_n)$$

After factor outing the K, the Y* can be expressed as a function of K (to any power),

$$Y^* = K^{\beta} f(X_1, X_2, X_3 \ldots\ldots X_n)$$

In Cobb-Douglas estimation function, adding up of the coefficient of independent variable yields the β mathematically and interpreted. The function β called homogenous and power β of k is called the degree of homogeneity of the production function and is measure of returns to scale. The value of β, greater than 1, less than 1 or equal to one represents the increasing, decreasing or constant return to scale respectively. In our production technology estimation, the return-to-scale values were found greater than one which employing increase return to scale.

Table 6 below showed Input Elasticities of Production and Returns-to-scale which are purely technical and do not include monitory aspects. The Table 6 revealed that rice farm size and family labour has input elasticities of production that less than one which indicated that rice farm size and family labour has no responsiveness to rice yield. The profit maximization objective oriented rice farm should keep on trying to stay in MVP equivalent with MIC. The tangent of the iso-cost line to the isoquant (production frontier) maintain long run profit maximization in factor-product relation. It is shown in the Table 7 (model III) that all the factors were under utilized as the MVP was higher than MIC. This results is similar to the study of [28,29], they stated that most of physical resources are underutilized due to cost of obtaining them. However in this research both cost and poor access to appropriate inputs were responsible for underutilization of herbicides and organic fertilizer [30].

Table 6. Input elasticities of production and returns to scale (RTS)

Variables	**Kwara State (Model III)**
Rice farm size	0.274
Family labour	0.687
Hired labour	1.222
Seed planted	1.894
Fertilizer	2.171
Herbicides	3.373
RTS	9.621

Table 7. Marginal value product and marginal input cost of the rice farms

Variables	**Family Labour**	**Hired Labour**	**Seed Planted**	**Fertilizer**	**Herbicides**
MODEL I (60)					
MVP	610.2	154.8	56.7	86.4	217.8
MIC	120.0	120.0	100.0	50.0	900.0
Efficiency Ratio	5.08	1.29	0.56	1.72	0.244
Input Use	Increase	Increase	Decrease	Increase	Decrease
MODEL II (24)					
MVP	566.1	762.3	13.5	93.6	344.7
MIC	120.0	120.0	100.0	50.0	900.0
Efficiency Ratio	4.71	6.35	0.13	1.87	0.38
Input Use	Increase	Increase	Decrease	Increase	Decrease
MODEL III (12)					
MVP	621.0	1053.0	152.1	2484.0	7515.0
MIC	120.0	120.0	100.0	50.0	900.0
Efficiency Ratio	5.17	8.77	1.52	49.6	8.35
Input Use	Increase	Increase	Increase	Increase	Increase

MVP = P_YMPP; P_Y is the price of the output, P_X = Input Price (MIC), While MPP = (Input elasticity X Mean yield) / Mean of input used

4. CONCLUSION AND RECOMMENDATION

Model III were farmers that is technical sound which form the basis for the estimation production function. In the estimation of a production function it is assumed that all firms use the existing technology efficiently. In choosing the various functional forms, great attention has been paid to the stochastic disturbance term *ui.* From a purely statistical viewpoint, the estimated regression line fits the data quite well when chosen CD production function.

Findings from this study conclude that rice farmers in Kwara State of Nigeria were technically inefficient in the use of farm resources. The inefficiency of the rice farmers may be directly or indirectly linked to the high cost of fertilizer and at time improved seed.

Production function obtained from Cobb-Douglas functional form (Model III) yielded the greater significance of fertilizer and farm size, and they were found under-utilized, which implies to make effective rearrangement of the available inputs basket to enhance the technical efficiency of rice farmers in Nigeria to some extent.

The findings of this study brought to fore a number of issues that need to be addressed. Therefore, the following are some of the desired actions required to ensure proportionately higher rice yield can be obtained through the use of more inputs, which is intensifying production methods. Giving the empirical, the proposed recommendations are:

1. It is affirmatively recommended that when working on the estimation of production function, the first thing to do is to thoroughly remove the inefficient firms with the help of a non-parametric approach, known as Data Envelopment Analysis which helps to select the farms that utilize efficiently the existing technology, allowing the estimation of a production function that reveals the true input-output relationship.
2. More so, the extension activities of the extension agents should be revived. So that farmers will make better technical decision and also help in allocating their production input effectively, this will make our local rice a good substitute for imported ones for better consumer patronage.
3. Rice production exhibited increasing returns to scale. Proportionately higher rice yield can be obtained through the use of more inputs, that is, intensifying production methods. It is therefore recommended that government, through the Extension Officers in the six regions, should train more farmers in rice production. Because of the viable nature of rice business in these regions, government should assist farmers to overcome the problems of high operating capital. Appropriate short-term credit schemes and practical research and effective extension should be made available.
4. Rice output shows a high responsiveness to farm size, hired labour, fertilizer and herbicides. Government should therefore continue to increase the subsidy on fertilizer, herbicides and even seed and try as much as possible to make those inputs available in smaller bag. Government owned rural fertilizer market could also be put in place in each local government of the country in order to offset the exortionary effects of merchant who make the demand for fertilizer somehow difficult for the farmers. The importation of fertilizer into the country should also be discouraged in order to reduce costs at local markets and encourage domestic production
5. Finally, the study recommends that more of the productive resources should be employed by the rice farmers for increase paddy rice production since all inputs are underutilized. It is recommended that the farmers be advised to use their up to the point the values of the marginal products (MVPs) equates their factor prices (i.e. MVPs = P_Xs).

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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