Application of Optimization Principle in Landmark University Project Selection under Multi-Period Capital Rationing Using Linear and Integer Programming

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Abstract
The current structure of Landmark University (LU) was induced by raising a generation of solution providers through a qualitative and life-applicable training system that focuses on values and creative knowledge by making it more responsive and relevant to the modern-day demands of demonstration, industrialization and development. The challenge facing Landmark University is the question of which of its numerous projects they should invest to give maximum output with minimum input. In this paper, we maximize the Net Present Value (NPV) and maintain the net discount cash overflow of each project per period as contained and extracted as the secondary data of cash inflows of the Landmark University (LU) monthly financial statement and annual reports from 2012 to 2017 of which the documents have been re-grouped as small and large scale projects as many enterprises make more use of the trial-and-error method and as such firms have been finding it difficult in allocating scarce resources in a manner that will ensure profit maximization and/or cost minimization with a simple and accurate decision making by the company through an optimization principle in selecting LU project under multi-period capital rationing using linear programming (LP) and integer programming (IP). The annual net cash flow which is the difference between the cash inflows and cash outflows during each period for the project was estimated and recorded. The discount factors were estimated at cost of capital of 10% for each cash flow per period with the corresponding NPV at 10% which revealed that the optimal decision achieves maximum returns of $110 × 10^2 and this assisted the project manager to select a large number of the variable projects that can maximize the profit which is far better than relying on an ad-hoc judgmental approach to project investment that could have cost
$160 \times 10^2$ for the same project. Sensitivity analysis on the project parameters are also carried out to test the extent to which project selection is sensitive to changes in the parameters of the system revealed that a little reduction and or addition of reduced cost by certain amount or percentages to its corresponding coefficient in the objective function effect no changes in the shadow prices with solution values for variables $(x_1), (x_4), (x_5)$ and the optimal objective function.

**Keywords**

Optimization, Linear, Integer, Programming, Sensitivity, Investment, Maximize, Net Present Value

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1. Introduction

According to [1], the problem optimizing a factor such as net profit value (NPV) in where resources are limited and funds available over the periods are considered will be recognized as a situation where linear programming and integer programming could be used to solve the problem of which both LP and IP have been used successfully in solving multi-period capital rationing problems. The first Mathematical programming formulation of the multi-period capital rationing (MRC) problem was provided by [2]. In his work, he maximized the net discount cash inflows for the project and maintained the cash inflow and availability of resources in each period and provided a framework using a deterministic linear programming approach. He used Net Present Value (NPV) in the model as an objective function. The values associated with the timing of a part cash flow as adjusted by an appropriate discount rate as opined by [3].

Moreover, [4] examined the application of Optimization principles to optimized parking slot using linear programming in Tamale/Bolgatanga main lorry station at the Tamale Metropolis in the Northern region of Ghana where the maximum parking capacity of the Terminal is examined and fully optimized to avoid traffic congestion in the metropolis and determined the best parking slot allocation to be distributed among different types of vehicle on limited parking space. [5] examined optimization of Landmark Poultry farm products using Simple Linear Programming whereby they investigated and examined the cost invested and as well as cost of producing each poultry farm products and the turn over for the same products in order to find the trend of its’ production and predict the possible economics future using Simple Linear programming for an effective decision making in Landmark University poultry farm production.

[6] established optimal principle in solving over-allocation and under-allocation of the classroom space using Linear Programming based on the data obtained from the examination and lecture timetable committee on the classroom facilities, capacities and the number of students per programme to maximize the available classroom space and minimizes the congestion and overcrowding in a
particular lecture room using AMPL software.

Likewise, [7] applied optimization principle in optimizing profits of a production industry using linear programming where they examined and evaluated production costs to determine the optimal profit using secondary data collected from the records of the Landmark University Bakery on five types of bread produced in the firm where it was revealed through the application of AMPL software that Family loaf and the Chocolate bread contributed objectively to the profit. Hence, more of Family loaf and Chocolate bread are needed to be produced and sold in order to maximize the profit.

In this paper, we develop and formulate Linear and Integer Programming models to solve a multi-period capital rationing (MCR) with divisible and indivisible project problems. The model seeks to produce optimum solution quantities (i.e. total NPV) and the shadow cost (i.e. opportunity cost of building constraints).

2. Linear Programming

We consider the following standard form of linear programming:

Maximize \( F = \sum_{j=1}^{n} C_j X_j \)

Subject to \( \sum_{j=1}^{n} a_{ij} X_j = b_i, \) \( i = 1, 2, \ldots, n \)
\( l_j \leq X_j \leq u_j, \) \( j = 1, 2, \ldots, n \)

where \( C_j \) is the \( n \) objective function coefficient, \( a_{ij} \) and \( b \) are parameters in the \( m \) linear inequality constraints and \( l_j \) and \( u_j \) are lower and upper bounds with \( l_j \leq u_j. \) Both \( l_j \) and \( u_j \) may be positive or negative.

The specified Linear Programming model for the attainment of the objective function is as follows:

Minimize \( Z = \sum C_j X_j \)

Subject to \( x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = b_1 \)
\( a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 + a_{17}x_7 + a_{18}x_8 \leq b_2 \)
\( a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + a_{26}x_6 + a_{27}x_7 + a_{28}x_8 \leq b_3 \)
\( a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + a_{36}x_6 + a_{37}x_7 + a_{38}x_8 \leq b_4 \)
\( a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 + a_{46}x_6 + a_{47}x_7 + a_{48}x_8 \leq b_5 \)
\( a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 + a_{56}x_6 + a_{57}x_7 + a_{58}x_8 \leq b_6 \)
\( a_{61}x_1 + a_{62}x_2 + a_{63}x_3 + a_{64}x_4 + a_{65}x_5 + a_{66}x_6 + a_{67}x_7 + a_{68}x_8 \leq b_7 \)
\( a_{71}x_1 + a_{72}x_2 + a_{73}x_3 + a_{74}x_4 + a_{75}x_5 + a_{76}x_6 + a_{77}x_7 + a_{78}x_8 \leq b_8 \)
\( a_{81}x_1 + a_{82}x_2 + a_{83}x_3 + a_{84}x_4 + a_{85}x_5 + a_{86}x_6 + a_{87}x_7 + a_{88}x_8 \leq b_9 \)
\( a_{91}x_1 + a_{92}x_2 + a_{93}x_3 + a_{94}x_4 + a_{95}x_5 + a_{96}x_6 + a_{97}x_7 + a_{98}x_8 \leq b_{10} \)
\( a_{101}x_1 + a_{102}x_2 + a_{103}x_3 + a_{104}x_4 + a_{105}x_5 + a_{106}x_6 + a_{107}x_7 + a_{108}x_8 \leq b_{11} \)
\( x_i, j = 1, 2, 3, \ldots, n \)
Linear and Integer Programming Model for the Project Selection Problem

In this Linear Programming model, we let \( Q_j \) be the capital available in LU for investment at time period \( t \). Then the problem facing LU is to determine which project or portion of the project it should initiate with \( Q_j \). Thus the following algorithms will be strictly follow in determine and solving the challenges.

**1) Algorithms for Linear Programming**

**Step a).** Determine the project’s NPV using
\[
\beta_j (NPV) = \sum_{i=1}^{n} \left[ \frac{C_i}{(1+r)^t} \right]
\]
where \( t = 0,1; j = 1,2,3,4,5 \)

\( C \) is the cash flows

We proposed that the NPV of five (5) projects to be initiated as Agriculture (A) = \( \beta_1 \), Electrification (B) = \( \beta_2 \), Lecture Hall (C) = \( \beta_3 \), Lab. Equipment (D) = \( \beta_4 \), Staff/Student Quarters (E) = \( \beta_5 \).

**Step b).** Formulate the Linear Programming problems by defining the objective functions, decision variables and the constraints.

Thus:
\[
\text{Maximize } Z = \beta_1 X_A + \beta_2 X_B + \beta_3 X_C + \beta_4 X_D + \beta_5 X_E
\]

While the decision variables ( \( X_j \) ) are characterized as follows
\( X_A \) is the proportion of project A to be initiated when \( j = 1 \)
\( X_B \) is the proportion of project B to be initiated when \( j = 2 \)
\( X_C \) is the proportion of project C to be initiated when \( j = 3 \)
\( X_D \) is the proportion of project D to be initiated when \( j = 4 \)
\( X_E \) is the proportion of project E to be initiated when \( j = 5 \)

**2) Algorithms for Integer Programming for the project selection problem**

For \( Q_j \) be the capital available in LU for investment at time period \( t \) and the problem facing LU is to determine which project or portion of the project it should initiate with \( Q_j \). Thus LU must take into consideration that:

a) It cannot invest in all \( N \) projects suitable for investment which run for \( n \) year.

b) The project characteristics show that \( \sum_i d_{(i,j)} \) is greater than \( R_j \) where \( d_{(i,j)} \) is the least requirement for \( j \) projects and \( R_j \) is the capital for investment.

c) All the projects and the constraints are independent on one another.

d) Equal investment opportunities are assumed for the project for each period.

e) The cash flows, resources and constraints are well known.

Our main decision problem is to determine which project the LU should select in order to maximize the total returns. To formulate this Integer Programming, we follow these algorithms:

**Step i).** Define the decision variable as follows
Let
\[
X_j = \begin{cases} 1, & \text{if } LU \text{ invest in project } j \\ 0, & \text{if } LU \text{ does not invest in project } j \end{cases}
\]
\[j = 1, 2, \ldots, n\]  
\[\tag{5}\]

where \(X_j\) are integer variable which takes one of two possible values \((0, 1)\) and represents a binary decision.

**Step ii).** Define the constraints as follows

We let \(d_{i,j}\) be the capital requirement for \(j\) project, \(R_j\) be available capital for \(j\) project for each year.

\[
\sum_{j=1}^{N} d(i,j)X_j \leq R_i \quad \text{for } j = 1, 2, \ldots, N; \; i = 1, 2, \ldots, m
\]
\[\tag{6}\]

Then the constraints relating to availability of capital funds each year are:

**Step. iii) Objective function.**

We let the total profit be

\[
\sum_{j=1}^{N} P_jX_j
\]
\[\tag{7}\]

Maximize

\[
Z = \sum_{j=1}^{N} P_jX_j
\]
\[\tag{8}\]

Subject to

\[
\sum_{j=1}^{N} d(i,j)X_j \leq R_j
\]
\[\tag{9}\]

3. **Mathematical Model of Project Selection under Multi-Period Capital Rationing**

Since the problem facing LU is to determine which project or portion of the project, it should initiate with \(Q_j\) and subject to these constraints, they were faced with budgetary limitation. Thus

1) For the capital project at the initial time \((\theta = 0)\),

\[
a_{(1,1)}X_A + a_{(1,2)}X_B + a_{(1,3)}X_C + a_{(1,4)}X_D + a_{(1,5)}X_E \leq Q_1
\]
\[\tag{10}\]

2) For the capital project at the take up time \((\theta = 1)\),

\[
a_{(2,1)}X_A + a_{(2,2)}X_B + a_{(2,3)}X_C + a_{(2,4)}X_D + a_{(2,5)}X_E \leq Q_2
\]
\[\tag{11}\]

3) Then we specified the following proportion constraints to ensure that a project is not accepted more than once or negative projects are not accepted:

\[
X_A, X_B, X_C, X_D, X_E \leq 1
\]

\[
X_A, X_B, X_C, X_D, X_E \leq 0
\]

where \(a_{(i,j)}\) are cash flows for each period and for each project.

We then transform the formula into the compact form as:

Maximize \(Z = \beta_1X_A + \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5\)
Subject to:

\[
\begin{align*}
    a_{(1,1)}X_1 + a_{(1,2)}X_2 + a_{(1,3)}X_3 + a_{(1,4)}X_4 + a_{(1,5)}X_5 & \leq Q_1 \\
    a_{(2,1)}X_1 + a_{(2,2)}X_2 + a_{(2,3)}X_3 + a_{(2,4)}X_4 + a_{(2,5)}X_5 & \leq Q_2 \\
    X_1 & \leq 1 \\
    X_2 & \leq 1 \\
    X_3 & \leq 1 \\
    X_4 & \leq 1 \\
    X_5 & \leq 1 \\
\end{align*}
\] (12)

Generally, we then have the following form of equation:

\[
\text{Maximize } Z = \sum_{j=1}^{N} B_j X_j
\] (13)

Subject to \( \sum_{j=1}^{N} a(i, j) X_j = Q_i \) \( 0 \leq X_j \leq 1, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \) (14)

where \( (a(i, j), Q_j, \beta_j) \) are given and \( n \) is the number of the projects to be invested and \( Z \) constitutes the objective function.

3.1. Implementation of LP and IP Models in LU Project Selection

3.1.1. Sources and Data Collection

Information about cash inflows of the LU small and capital projects with distribution of capital requirements for the Small Project from 2012-2017 as shown in Table 1 and Table 2 below were extracted from the LU financial statement, monthly and annual report from 2012 to 2017. The LU project were classified into small and capital project and the discount factors were estimated at cost of capital of 10% for each cash flow for each period and the corresponding NPV at 10%. While Table 3 below shows the results for the above LP model for large scale using MATLAB Package.

3.1.2. LP Model Implementation

From Equations (7) and (8) we applied LP model to LU capital rationing data in Table 1 and formulate LU project selection problem as shown below.

Maximize \( Z = \sum_{j=1}^{N} B_j X_j \)

Subject to \( \sum_{j=1}^{N} a(i, j) X_j = Q_i \) \( 0 \leq X_j \leq 1, i = 1, 2, \ldots, M; j = 1, 2, \ldots, N \)
Table 1. The net cash flow of LU Large scale project for 2012-2017.

<table>
<thead>
<tr>
<th>Year</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project period</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Agriculture (x1)</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>400</td>
<td>600</td>
<td>500</td>
</tr>
<tr>
<td>Electrification (x2)</td>
<td>400</td>
<td>500</td>
<td>1000</td>
<td>1200</td>
<td>1400</td>
<td>1200</td>
</tr>
<tr>
<td>Lecture hall (x3)</td>
<td>250</td>
<td>200</td>
<td>360</td>
<td>500</td>
<td>400</td>
<td>0</td>
</tr>
<tr>
<td>Lab. equipment (x4)</td>
<td>30</td>
<td>50</td>
<td>60</td>
<td>60</td>
<td>150</td>
<td>90</td>
</tr>
<tr>
<td>Staff/std quarters (x5)</td>
<td>10</td>
<td>20</td>
<td>10</td>
<td>0</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>Discount factors</td>
<td>1.00</td>
<td>0.909</td>
<td>0.826</td>
<td>0.751</td>
<td>0.683</td>
<td>0.621</td>
</tr>
<tr>
<td>Capital Limitation Q1</td>
<td>550</td>
<td>500</td>
<td>450</td>
<td>400</td>
<td>650</td>
<td>700</td>
</tr>
</tbody>
</table>

Table 2. The distribution of capital requirement for Small Project for 2012-2017.

<table>
<thead>
<tr>
<th>Year</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
<th>2017</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project period</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Machineries (x1)</td>
<td>50</td>
<td>30</td>
<td>20</td>
<td>60</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Refuse facility (x2)</td>
<td>10</td>
<td>80</td>
<td>20</td>
<td>20</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>Borehole water (x3)</td>
<td>15</td>
<td>15</td>
<td>30</td>
<td>40</td>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>Shield (x4)</td>
<td>10</td>
<td>40</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Bus stop (x5)</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>Available capital</td>
<td>80</td>
<td>145</td>
<td>90</td>
<td>100</td>
<td>165</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 3. The results for the above LP model for large scale using MATLAB Package. Optimal solution (Max objective function) = $1027.56 \times 10^2$.

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Solution variables</th>
<th>Unit cost or profit</th>
<th>Total contribution</th>
<th>Shadow price</th>
<th>Reduction cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture (x1)</td>
<td>1.00</td>
<td>264</td>
<td>264</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Electrification (x2)</td>
<td>0.662</td>
<td>719</td>
<td>474.56</td>
<td>1.438</td>
<td>0</td>
</tr>
<tr>
<td>Lecture hall (x3)</td>
<td>0</td>
<td>237</td>
<td>0</td>
<td>0</td>
<td>50.6</td>
</tr>
<tr>
<td>Lab. equipment (x4)</td>
<td>1.00</td>
<td>217</td>
<td>217</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Staff/Std quarters (x5)</td>
<td>1.00</td>
<td>72</td>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pmax.</td>
<td></td>
<td></td>
<td></td>
<td>1,509</td>
<td>1,027.56</td>
</tr>
</tbody>
</table>

Thus we have:

Maximize \[ Z = 264X_1 + 719X_2 + 237X_3 + 217X_4 + 72X_5 \]

Subject to \[ 100X_1 + 400X_2 + 250X_3 + 30X_4 + 10X_5 \leq 550 \]
\[ 100X_1 + 500X_2 + 200X_3 + 50X_4 + 20X_5 \leq 500 \]
\[ 200X_1 + 1000X_2 + 360X_3 + 60X_4 + 10X_5 \leq 450 \]
\[ 400X_1 + 1200X_2 + 500X_3 + 60X_4 \leq 400 \]
\[ 600X_1 + 1400X_2 + 400X_3 + 150X_4 + 30X_5 \leq 650 \]
3.1.3. Sensitivity Analysis for Linear Programming
Here the stability or robustness of the model is tested by a slight change in the technological coefficients in order to determine the redundancy or otherwise of one of the constraints, this helps make better recommendations and reduce errors in making decisions. The redundancy of a constraint is also put into test and the solution compared to the original LP problem as shown in Table 4 below by the reduced cost of 50.6 in row \((x_3)\) shows the amount by which the objective function coefficient for the variable \((x_3)\) should be change to make it a non-zero. Hence the coefficient of \((x_i)\) in the objective function is altered by \(-50.6\) and the LP problem will be resolved to yield.

3.1.4. Interpretation of Results for Sensitivity
Addition of the reduced cost of 50.6 on the row of variable \((x_2)\) to its corresponding coefficient in the objective function effect no changes in the shadow prices with solution values for variables \((x_1), (x_4), (x_5)\) and the optimal objective function. However, there were sharp variations in some optimal solution values. The coefficient of variables \((x_2)\) decreased from 0.662 to 0.38042 while \((x_3)\) increases from 0 to 0.69895, increasing the NPV per unit on variable \((x_3)\), impact a sharp change on the optimal solution. Given the sensitivity analysis of one or more of the key factors of project like this, the LU management’s task is to decide whether the project is commendable and worthwhile.

4. Integer Programming Model Implementation
From Table 2, we put LU small scale project selection problem data into the IP model as:

Maximize \(Z = \sum_{j=1}^{N} P_j X_j\)

Subject to \(\sum_{i=1}^{m} d_{(i,j)} X_j \leq R_j, X_j = 0; j = 1, 2, \cdots, N\)

\(0 \leq X_j \leq 1, i = 1, 2, \cdots, m; j = 1, 2, \cdots, n\)

Thus:

Maximize \(Z = 50X_1 + 30X_2 + 50X_3 + 10X_4 + 20X_5\)

Subject to \(50X_1 + 10X_2 + 15X_3 + 10X_4 + 10X_5 \leq 80\)

\(X_1 + 80X_2 + 15X_3 + 40X_4 + 0X_5 \leq 145\)

\(20X_1 + 20X_2 + 30X_3 + 10X_4 + 10X_5 \leq 90\)

\(20X_1 + 20X_2 + 40X_3 + 10X_4 + 20X_5 \leq 100\)

\(60X_1 + 30X_2 + 60X_3 + 0X_4 + 50X_5 \leq 145\)

\(40X_1 + 60X_2 + 0X_3 + 0X_4 + 10X_5 \leq 80\)

\(X_j = 0 \text{ or } 1, j = 1, 2, 3, 4, 5\)

The above LP was solved using MATLAB and the results of the binary decision are shown in Table 5 below.
Table 4. The sensitivity analysis of the Large Scale project. Optimal solution (Max. objective function) = $1027.72.

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Solution variables</th>
<th>Unit cost or profit</th>
<th>Total contribution</th>
<th>Shadow price</th>
<th>Reduction cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture (x₁)</td>
<td>1.00000</td>
<td>264</td>
<td>264</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Electrification (x₂)</td>
<td>0.38042</td>
<td>719</td>
<td>273.52</td>
<td>1.438</td>
<td>0</td>
</tr>
<tr>
<td>Lecture hall (x₃)</td>
<td>0.69895</td>
<td>287.6</td>
<td>201.20</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Lab. equipment (x₄)</td>
<td>1.00000</td>
<td>217</td>
<td>217</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Staff/Std. quarters (x₅)</td>
<td>1.00000</td>
<td>72</td>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Pmax.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1,027.72</td>
</tr>
</tbody>
</table>

Table 5. Optimal objective function value = 110 × 10².

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Solution variables</th>
<th>Unit cost or profit</th>
<th>Total contribution</th>
<th>Reduction cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machineries (x₁)</td>
<td>1</td>
<td>50</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Refuse facility (x₂)</td>
<td>0</td>
<td>30</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Borehole water (x₃)</td>
<td>1</td>
<td>50</td>
<td>50</td>
<td>0</td>
</tr>
<tr>
<td>Shield (x₄)</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Bus stop (x₅)</td>
<td>0</td>
<td>20</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Pmax.</td>
<td></td>
<td></td>
<td></td>
<td>160</td>
</tr>
</tbody>
</table>

Interpretation of IP Results

The optimal decision is to choose (x₁), (x₃), (x₄), while LU can provide (x₂), (x₅) with $n$ capital for the next five years unless the LU investment is reviewed. The optimal decision achieves maximum returns of 110 × 10². It is evident that the model has assisted the project manager to select a large number of the variable projects that can maximize profit. This is larger than relying on an ad-hoc judgmental approach to project investment that could have cost 160 × 10² for the same project.

5. Conclusions

In this paper we have successfully examined optimization principles and its applications in selecting potential projects in LU in order to maximize the returns and the profits from the batch of projects by maximizing the Net present Value (NPV) and maintain the net discount cash overflow for each project per period as contained in data collected from LU monthly financial statement and annual report from 2011 to 2016 revealed that LU will incur 1509 × 10² as unit cost or profit for a total contribution of 1027.56 × 10².

The discount factors were estimated at cost of capital of 10% for each cash flow per period with the corresponding NPV at 10% which revealed that the optimal decision achieves maximum returns of $110 × 10² and this will help the project manager to select a large number of the variable projects that can maximize the profits which is far better than relying on an ad-hoc judgmental ap-
An approach to project investment that could have cost $160 \times 10^2$ for the same project.

Sensitivity analysis on the project parameters revealed that a little reduction and/or addition of reduced cost by certain amount or percentages to its corresponding coefficient in the objective function effect changes in the shadow prices with solution values for variables $(x_1), (x_4), (x_5)$ and the optimal objective function. However, there were sharp variations in some optimal solution values where the coefficient of variables $(x_1)$ decreased while $(x_5)$ increased and an increase in NPV per unit on variable $(x_5)$, has a sharp change on the optimal solution.

This will give some guidance to the firm management in their consideration of many options with regards to the limited resources and for the decision-making process.

**Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

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