J. Appl. Comput. Mech., 6(1) (2020) 26-36
DOI: 10.22055/JACM.2019.28216.1460

ISSN: 2383-4536
jacm.scu.ac.ir

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Shahid Chamran
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Journal of
Applied and Computational Mechanics

Research Paper

Analysis of Transient Rivlin-Ericksen Fluid and Irreversibility of Exothermic Reactive Hydromagnetic Variable Viscosity

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Received January 14 2019; Revised March 14 2019; Accepted for publication March 15 2019.
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Abstract. This study analyzes the unsteady Rivlin-Ericksen fluid and irreversibility of exponentially temperature dependent variable viscosity of hydromagnetic two-step exothermic chemical reactive flow along the channel axis with walls convective cooling. The non-Newtonian Hele-Shaw flow of Rivlin-Erickson fluid is driven by bimolecular chemical kinetic and unvarying pressure gradient. The reactive fluid is induced by periodic changes in magnetic field and time. The Newtons law of cooling is satisfied by the constant heat coolant convection exchange at the wall surfaces with the neighboring regime. The dimensionless non-Newtonian reactive fluid equations are numerically solved using a convergent and consistence semi-implicit finite difference technique which are confirmed stable. The response of the reactive fluid flow to variational increase in the values of some entrenched fluid parameters in the momentum and energy balance equations are obtained. A satisfying equations for the ratio of irreversibility, entropy generation and Bejan number are solved with the results presented graphically and discussed quantitatively. From the study, it was obtained that the thermal criticality conditions with the right combination of thermo-fluid parameters, the thermal runaway can be prevented. Also, the entropy generation can minimize at low dissipation rate and viscosity.

Keywords: Non-Newtonian; Hydromagnetic; Convective cooling; Irreversibility; Viscosity.

1. Introduction

Hydromagnetic flows arise often in nature. The uses of viscous incompressible hydromagnetic liquid flow in industry has connection with heat transfer under the effect of chemical reaction, and are of countless prominence to numerous areas of science and engineering [1,2]. The flow of variable reactive viscosity liquid along a channel with small Reynolds numbers is an essential subject in the area of environmental, biomedical and chemical science and engineering. This occurrence is significant in nature and practically eminent in several different applications, such as renewing heat conversation, oil and gas production from biological make-up, chromatography, and exchange of ion and chemical catalysis of surface reaction. Many of the mentioned applications contain two or more fluids, transient flows and multidimensional. Some applications are concern with exclusive information on the velocity distribution by Kim [3] and Muthtamilselvan et al. [4].

The elastic-viscous fluids have shown to be dawdling and the strictures embodying the elastic properties are considered small. The usefulness of non-Newtonian fluids for example pulps, emulsions, aqueous solutions of polyisobutylene and polyacrylamide etc., as chemical products and raw materials in several industrial processes has prompted researchers into the study of non-Newtonian liquids and their associated transport processes. The stress-strain rate relatives to the Rheological characteristic and the traditional magnetohydrodynamics of third-grade liquids was examined by Ravikumar et al. [5]. The problem was solved using perturbation and it was obtained that a rise in constant scalar and Rivlin-Ericksen material term of the liquid diminished the flow velocity. In the area of conventional model of fluid mechanics, Hele-Shaw flow of fluid problem has been studied by several scholars. Dada and Agunbiade [6] examined chemical reaction influence on transient convective Hele-Shaw flow of Rivlin-Ericksen fluid through a vertical plate. In the study, it was reported that the flow rate reduces with a rise in viscoelasticity parameter. Daleep et al. [7] investigated the flow of complex bounds for the rate of growth in viscoelastic Rivlin-Ericksen thermosolutal fluid past permeable media. Noushima et al.[8] studied Rivlin-Ericksen flow on MHD fluid with capricious porousness. Rana [9] and Sharma and Sunil [10] considered a saturated dust particles Rivlin-Ericksen flow liquid and its thermal instability in porous media. Nidhish [11] considered magnetohydrodynamic unstable flow of energy and species transport influence on Rivlin-Ericksen fluid along porous media with permeable plate. It is seen from the solution that viscoelastic material term decreases fluid momentum but the fluid heat and species were not affected by the rises in the parameter. Seth et al. [12] examined natural radiative convection hydromagnetic flow through an unwarily motioning ramped plate with heat transfer. Emitting/Absorbing radiation and hydromagnetic slanted impacts on micropolar liquid containing gyrostatic microorganisms was examined by Ramya et al. [13].

Meanwhile, effectiveness computation of systems heat conversation has been far delimited to the thermodynamics first law. Computations involving the thermodynamics second law, which is associated with entropy production, are more dependable based on computations than the first law. Thus, the thermodynamics second law may be applicable in the study of the rate at which entropy is generated in the flow system as a result of heat transport and friction in the fluid. Obtaining entropy generation is very essential in improvement of system enactment since entropy generation has do with the degree of the obliteration of the existing effort of the system [14]. As entropy production is observed, the quality of energy reduces. Therefore, it is necessary to examine the diffusion of the generated entropy within the volume of the liquid processes to conserve the worth of the flow fluid energy or decrease the entropy production. Substantial inquiry works were investigated by various scholars' on the importance of the thermodynamics second law to several areas of flow liquid and energy transport problems [15-20]. Irreversibility process encourages entropy generation as experienced in many thermal engineering systems such as peristaltic MHD compressor, heat exchangers, optical switches, turbo machinery and so on. Nevertheless, to the knowledge of the authors, no study has examined the combined consequences of exponentially temperature dependent viscosity and Bimolecular kinetics on a flow systems, entropy generation and thermal stability of transient two-step exothermic chemical reaction which are of huge significance in voluminous engineering fields that include electronic cooling devices, lubrication of automobile, nuclear reactor cooling, storage of energy systems and heat exchangers [21-24]. Exothermic two-step chemical reaction is a process that assists in completing combustion of unburned ethanol. The process aids the reduction of environmental pollution as a result of carbon (II) oxide by Salawu et al. [25]. In an industry and engineering processes where Rivlin-Ericksen fluid is used, the environmental degradation is minimized when two-step exothermic reaction is considered.

In this study, unsteady two step exothermic chemical reaction with temperature dependent viscosity of a Rivlin-Ericksen reactive exothermic chemical reaction is examined. The essential properties of the energy distribution such as inherent second law of thermodynamics, thermal criticality and bifurcations conditions of the flow system are also considered. The flow model is presented in section 2. In section 3, the semi-implicit finite difference method is employed in space and time for the computation. In section 4, the computation with the graphical results are established and quantitatively explained with respect to existing fluid parameters entrenched in the flow.

2. Rheological Equations of State

The constitutive equation for a Hele-Shaw flow of a Rivlin-Ericksen according to Sharma and Sunil [10] is defined as

$$T = -PI + \psi_1 a + \psi_2 c + \psi_3 a^2 \tag{1}$$

where $T = \|T_{ij}\|$, T_{ij} is the tensor stress. $I = \|\delta_{ij}\|$, δ_{ij} is the Kroneckar delta, $a = \|a_{ij}\|$, $a_{ij} = (f_{i,j} + f_{j,i})/2$ is the tensor deformation rate and $c = \|c_{ij}\|$, $c_{ij} = d_{i,j} + d_{j,i} + 2F_{n,i}F_{n,j}$ is the gradient tensor acceleration with $a_i = \partial f_i / \partial \tau + f_{ij} f_j$ is the vector acceleration. The coefficients ψ_1, ψ_2, ψ_3 stand for, respectively, the viscosity, viscoelasticity, cross-viscosity are the general temperature functions, properties of material and invariants of a, c, a^2. For polybutylene and polyacrylamid acqueous solutions ψ_1, ψ_2, ψ_3 can be assumed as unchanged parameters.

3. Mathematical Formulation

Consider a chemical kinetics of Bimolecular rate law, isothermal, and absolute exponentially variable viscosity temperature dependent of two-step exothermic combustible reactive Hele-Shaw fluid flow between fixed horizontal walls of width *b*. Since the problem is modeled by Bimolecular kinetic law, the viscosity function reduces exponentially with temperature and the chemical reactive fluid is encouraged by the fully developed thermal hydromagnetic field along with axial uniform pressure gradient. The viscoelastic effect is stimulated by the implementation of Rivlin-Ericksen non-Newtonian third grade fluid model. Assuming the applied voltage is absent as a result of lack of electrical field. The Hall effects are insignificant due to smaller portion of the magnetic Reynolds numbers in the reactive fluid from the Maxwell's electromagnetic equation. The walls are subjected to the same coolant heat convection with the ambient temperature, thus providing an asymmetric cooling effect. The x-axis is assumed to be along the center of the channel and the y-axis is perpendicular to it.

Considering the above assumption, the flow physical geometry is presented in figure 1, the two-step chemical reaction mechanisms and the transport equations of the problem are given as follows [22, 23]:

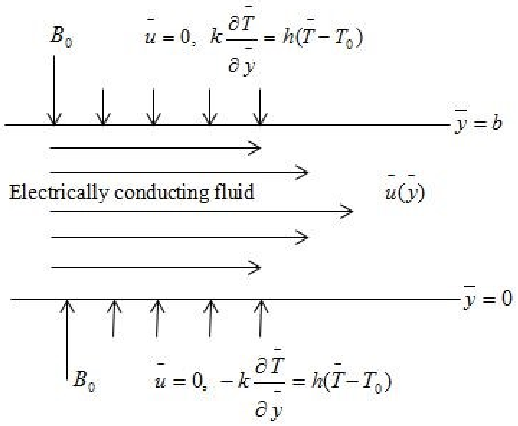

Fig. 1. The formulation geometry

Mechanisms of two-step chemical reaction of ethanol oxidation is given as

$$\begin{gathered} C_2H_5OH+2\left(O_2+3.76N_2\right)\rightarrow 2CO+3H_2O+7.52H_2 \\ CO+0.5\left(O_2+3.76N_2\right)\rightarrow 1.88N_2+CO_2 \end{gathered} \tag{2}$$

The equations governing the flow

$$\rho\frac{\partial \bar{u}}{\partial \bar{t}}=-\frac{\partial \bar{p}}{\partial \bar{x}}+\frac{\partial}{\partial \bar{y}}\left[\bar{\mu}\left(\bar{T}\right)\frac{\partial \bar{u}}{\partial \bar{y}}\right]-\sigma B_0^2\bar{u}+\beta\frac{\partial^3\bar{u}}{\partial \bar{t}\partial \bar{y}^2} \tag{3a}$$

$$\rho C_p\frac{\partial \bar{T}}{\partial \bar{t}}=k\frac{\partial^2\bar{T}}{\partial \bar{y}^2}+\sigma B_0^2\bar{u}^2+\bar{\mu}\left(\bar{T}\right)\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2+Q_1C_1A_1\left(\frac{K\bar{T}}{\nu l}\right)^n e^{\frac{E_1}{R\bar{T}}}+Q_2C_2A_2\left(\frac{K\bar{T}}{\nu l}\right)^n e^{\frac{E2}{R\bar{T}}} \tag{3b}$$

Subject to relevant boundary conditions

$$\begin{gathered} \bar{u}\left(\bar{y},0\right)=0,\bar{u}\left(0,\bar{t}\right)=0,\bar{u}\left(b,\bar{t}\right)=0,\bar{T}\left(\bar{y},0\right)=T_0, \\ -k\frac{\partial \bar{T}}{\partial \bar{y}}\left(0,\bar{t}\right)=h\left(\bar{T}\left(0,\bar{t}\right)-T_w\right),k\frac{\partial \bar{T}}{\partial \bar{y}}\left(b,\bar{t}\right)=h\left(\bar{T}\left(b,\bar{t}\right)-T_w\right), \end{gathered} \tag{4}$$

where B_0, σ, k, ρ, $\bar{u}$, $\bar{\mu}$, $\bar{T}$, T_w, β, K, C_p, l, b, ν, h and R are respectively the magnetic strength, electrical conductivity, thermal conductivity, fluid density, velocity of the fluid, temperature dependent fluid viscosity, fluid temperature, temperature of ambient, material coefficient, Boltzmann's constant, specific heat, planck's number, half channel width, vibration frequency, coefficient of heat transfer and constant universal gas. While the parameters A_1, A_2, C_1, C_2, E_1, E_2 and Q_1, Q_2 are respectively first and second rate constant, reacting species, activation energy and reaction heat. The parameter n is the computational exponent such that $n\in\{-2,0,0.5\}$ which represents sensitized, arrhenius and bimolecular chemical kinetics respectively that determine the energy of the reactants and their orientation. The dependent temperature variable viscosity $\bar{\mu}(\bar{T})$ is defined as

$$\bar{\mu}\left(\bar{T}\right)=\mu_1 e^{-c\left(\bar{T}-T_w\right)} \tag{5}$$

The following non-dimensional quantities are used

$$P=\frac{b\bar{p}}{U\mu_1},\eta=\frac{\bar{y}}{b},G=-\frac{\partial p}{\partial x},x=\frac{\bar{x}}{b},T=\frac{E_1\left(\bar{T}-T_w\right)}{RT_w^2},H^2=\frac{b^2\sigma B_0^2}{\mu_1},\varepsilon=\frac{RT_w}{E_1},m=\frac{E_2}{E_1}, \tag{6}$$

$$\Pr = \frac{\mu_1 C_p}{k}, \lambda = \frac{A_1 Q_1 C_1 b^2 E_1}{RkT_w^2}\left(\frac{kT_w}{\nu l}\right)^n e^{\frac{1}{\varepsilon}}, Br = \frac{\mu_1 E_1 U^2}{RkT_w^2}, \delta = \frac{C_2 A_2 Q_2}{C_1 A_1 Q_1}, u = \frac{\bar{u}}{U}, \mu = \frac{\bar{\mu}}{\mu_1},$$

$$t = \frac{\mu_1 \bar{t}}{b^2\rho}, Bi = \frac{bh}{k}, a = \frac{cRT_w^2}{E_1}, \omega = \frac{\beta}{b^2\rho}, T_a = \frac{E_1\left(T_0 - T_w\right)}{RT_w^2}.$$

Introducing Eqs. (5) and (6) into Eqs. (2)-(4) obtains

$$\frac{\partial u}{\partial t} = G - H^2 u - ae^{-aT}\frac{\partial T}{\partial \eta}\frac{\partial u}{\partial \eta} + e^{-aT}\frac{\partial^2 T}{\partial \eta^2} + \omega\frac{\partial^3 u}{\partial \eta^2 \partial t}, \tag{7}$$

$$\Pr\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \eta^2} + \lambda\left(1+\varepsilon T\right)^n\left(e^{\frac{T}{1+\varepsilon T}} + \delta e^{\frac{mT}{1+\varepsilon T}}\right) + Br\left[H^2 u^2 + e^{-aT}\left(\frac{\partial u}{\partial \eta}\right)^2\right], \tag{8}$$

with the corresponding initial and boundary conditions

$$\begin{gathered} u(\eta,0) = 0, u(1,t) = 0, u(0,t) = 0, \\ T(\eta,0) = T_a, \frac{\partial T}{\partial \eta}(0,t) = -BiT(\eta,0), \frac{\partial T}{\partial \eta}(1,t) = BiT(1,t) \end{gathered} \tag{9}$$

where T, u, T_a, G, ω, Pr, H, Br, ε, δ, λ, Bi, m and a are respectively the dimensionless temperature, dimensionless velocity, initial temperature parameter, pressure gradient term, material parameter, Prandtl number, Hartmann number, Brinkman number, activation energy parameter, two-step chemical reaction term, Frank-Kamenetskii parameter, Biot number, activation energy ratio and variable viscosity parameter.

4. Method of Solution

The computational system engaged with the momentum and energy equations is finite difference of semi-implicit scheme as in Chinyoka [24], the approach takes implicit terms between the time level $(\xi + N)$ for $0 \le \xi \le 1$. So as to make huge time steps, ξ is thought to be a unit. Indeed, being totally implicit, the implemented computational method presented in this work is conjectured to be appropriate for any estimation of time steps. The dimensionless equations are discretized on a Cartesian uniform grid with linear mesh on which the finite differences are taken. Approximating the first and second spatial derivatives with second order central differences, the resulting equations of the first and last grid points are modified to incorporate the boundary conditions. The velocity module in semi-implicit technique can be written as:

$$\frac{\partial}{\partial t}\left(u - \omega\frac{\partial^2 u}{\partial \eta^2}\right) = G - a\left(e^{-aT}\frac{\partial T}{\partial \eta}\frac{\partial u}{\partial \eta}\right)^{(N)} - H^2 u^{(N+\xi)} + \left(e^{-aT}\frac{\partial^2 u}{\partial \eta^2}\right)^{(N)}, \tag{10}$$

the equation for u^{N+1} becomes:

$$\begin{gathered} -f_1 u_{j-1}^{N+1} + \left[1 + H^2\Delta t + 2f_1\right]u_j^{N+1} - f_1 u_{j+1}^{N+1} = \\ \left(u + \omega u_{\eta\eta}\right)^{(N)} - a\Delta t\left(e^{-aT}T_\eta u_\eta\right)^{(N)} + \Delta tG - \Delta tH^2\left(1-\xi\right)u^{(N)} + \Delta t\left(e^{-aT}u_{\eta\eta}\right)^{(N)}. \end{gathered} \tag{11}$$

where $f_1 = [\xi\Delta t(\mu^{(N)} + \omega)]/(\Delta\eta^2)$ with $\mu^{(N)} = e^{(-aT)^{(N)}}$ and the forward difference procedures are adopted for all derivatives of time. The solution technique for $u^{(N+1)}$ reduces to tri-diagonal inversion matrices.
The semi-implicit method for the temperature component takes after that of momentum equation. The unchanged second derivatives of the energy equation can be written as:

$$\Pr\frac{T^{(N+1)} - T^{(N)}}{\Delta t} = \frac{\partial^2}{\partial \eta^2}T^{(N+\xi)} + \lambda\left[\left(1+\varepsilon T\right)^n\left(e^{\frac{T}{1+\varepsilon T}} + \delta e^{\frac{mT}{1+\varepsilon T}}\right)\right]^{(N)} + Br\left[e^{-aT}u_\eta^2 + H^2u^2\right]^{(N)}, \tag{12}$$

the equation for $T^{(N+1)}$ becomes:

$$\begin{gathered} -f_2 T_{j-1}^{(N+1)} + \left(\Pr + 2f_2\right)T_j^{(N+1)} - f_2 T_{j+1}^{(N+1)} = T^{(N)} + \Delta t\left(1-\xi\right)T_{\eta\eta}^{(N)} + Br\Delta t\left[e^{-aT}u_\eta^2 + H^2u^2\right]^{(N)} \\ + \lambda\Delta t\left[\left(1+\varepsilon T\right)^n\left(e^{\frac{T}{1+\varepsilon T}} + \delta e^{\frac{mT}{1+\varepsilon T}}\right)\right]^{(N)}. \end{gathered} \tag{13}$$

where $f_2 = \xi \Delta t / (\Delta \eta^2)$. The solution technique for $T^{(N+1)}$ also decreases to the tri-diagonal inversion matrices. The schemes (11) and (13) were confirmed for regularity. When $\xi = 1$ enables us to take large time steps that is of order two in space but order one in time. As formerly taken, the method gratifies any time step values. The Maple 2016 code was used to carry out the analysis.

5. Entropy Generation Analysis

The fluid physical properties change significantly with temperature when the effects of variable viscosity are taken into consideration. The overall entropy generation equation for the flow per unit volume is defined by Adesanya and Makinde [21].

$$I_G = \frac{k}{T_w^2}\left(\frac{d\bar{T}}{d\bar{y}}\right)^2 + \frac{\bar{\mu}\left(\bar{T}\right)}{T_w}\left(\frac{d\bar{u}}{d\bar{y}}\right)^2 + \frac{\sigma B_0^2 \bar{u}^2}{T_w}. \tag{14}$$

The heat transfer irreversibility is the first term of equation (14) while the second term is the viscous dissipation entropy generation. By equation (14), the dimensionless entropy generation number is obtained as:

$$Ns = \frac{E_1^2 b^2 I_G}{R^2 T_w^2 k} = \left(\frac{dT}{d\eta}\right)^2 + \frac{Br}{\varepsilon}\left[H^2 u^2 + \mu\left(\frac{du}{d\eta}\right)^2\right]. \tag{15}$$

Following Eq. (13), the first term is assigned N_1 and the second term is assigned N_2, i.e,

$$N_1 = \left(\frac{dT}{d\eta}\right)^2, \quad N_2 = \frac{Br}{\varepsilon}\left[H^2 u^2 + \mu\left(\frac{du}{d\eta}\right)^2\right]. \tag{16}$$

The Bejan number (*Be*) is defined mathematically as follows:

$$Be = \frac{N_1}{Ns} = \frac{N_1}{N_1 + N_2} = \frac{1}{1+\varphi}, \quad \varphi = \frac{N_2}{N_1}. \tag{17}$$

The *Ns* and *Be* are graphically reported in the plots 10-17.

Table 1. The different parameters effect on the system thermal runaway.

a	Br	δ	H	m	Bi	G	ε	$\lambda_c(n=-2)$	$\lambda_c(n=0)$	$\lambda_c(n=0.5)$
0.3	0.2	1.0	2.0	1.0	0.1	2.0	0.5	0.0467275091	0.0343955359	0.0322783879
0.5	0.2	1.0	2.0	1.0	0.1	2.0	0.5	0.0439207718	0.0325235148	0.0304986176
0.1	0.2	2.0	2.0	1.0	0.1	2.0	0.5	0.0326423278	0.0239611814	0.0224912114
0.1	0.2	3.0	2.0	1.0	0.1	2.0	0.5	0.0244451756	0.0179761103	0.0159573587
0.1	0.2	1.0	2.0	1.0	0.2	2.0	0.5	0.0990538855	0.0739809738	0.0695735876
0.1	0.2	1.0	2.0	1.0	0.3	2.0	0.5	0.2372892649	0.1781209519	0.1676773956
0.1	0.2	1.0	2.0	1.0	0.1	3.0	0.5	0.0443533322	0.0314525088	0.0292463019
0.1	0.2	1.0	2.0	1.0	0.1	5.0	0.5	0.0320465023	0.0201352795	0.0181996292
0.1	0.5	1.0	2.0	1.0	0.1	2.0	0.5	0.0434489008	0.0306066628	0.0284095347
0.1	1.0	1.0	2.0	1.0	0.1	2.0	0.5	0.0355226324	0.0232098659	0.0211846497

6. Results and Discussions

Following from Makinde et al. [22], the initial temperature of the reactive fluid is taken to be equal to wall temperature, therefore parameter $T_a = 0$. The parameters values adopted for this study based on existing theoretical results are $G = 2.0$, $\Pr = 2.0$, $Br = 0.2$, $\lambda = 1.0$, $\delta = 1.0$, $m = 1.0$, $n = 0.5$, $\omega = 0.3$, $\varepsilon = 0.1$, $H = 2.0$, $a = 0.1$, $Bi = 0.1$.

Table 1 depicts the impact of heat dependent viscosity, viscous heating, second step exothermic reaction, Biot number and constant pressure gradient terms on the development of thermal criticality for various chemical kinetics. A rise in the parameters reduces the Frank-Kamenettski thermal critical values (λ_c) except for Biot number (Bi) that increases the thermal runaway for the chemical kinetics.

Table 2 represents the analysis of entropy generation numerical values for various chemical kinetics are obtained. The main flow region at the channel surfaces is dominated by fluid friction at $\phi > 1$. Likewise for the chemical kinetics, the Bejan numbers (Be) ranges within 0 and 1. The effects of exponentially temperature dependent viscosity term and magnetic heat source term are very significant when compared to cases where they were not considered.

Table 2. Computational values for entropy analysis under various chemical kinetics

y	N_2	N_1			N_S			Be		
		m = -2	m = 0	m = 0.5	m = -2	m = 0	m = 0.5	m = -2	m = 0	m = 0.5
0.0	1.0449	0.0331	0.0270	0.0257	1.0580	1.0766	1.0794	0.0314	0.0251	0.0239
0.1	0.6375	0.0207	0.0164	0.0157	0.6562	0.6553	0.6551	0.0317	0.0252	0.0239
0.2	0.4023	0.0112	0.0089	0.0085	0.4089	0.4116	0.4113	0.2762	0.0218	0.0207
0.3	0.2751	0.0049	0.0039	0.0037	0.2744	0.2785	0.2785	0.0181	0.0141	0.0133
0.4	0.2138	0.0012	0.0010	0.0009	0.2104	0.2148	0.2155	0.0058	0.0046	0.0042
0.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.6	0.2138	0.0012	0.0010	0.0009	0.2104	0.2148	0.2155	0.0058	0.0046	0.0042
0.7	0.2751	0.0049	0.0039	0.0037	0.2744	0.2785	0.2785	0.0181	0.0141	0.0133
0.8	0.4023	0.0112	0.0089	0.0085	0.4089	0.4116	0.4113	0.2762	0.0218	0.0207
0.9	0.6375	0.0207	0.0164	0.0157	0.6562	0.6553	0.6551	0.0317	0.0252	0.0239
1.0	1.0449	0.0331	0.0270	0.0257	1.0580	1.0766	1.0794	0.0314	0.0251	0.0239

The transient solutions on an even finer mesh ($\Delta t = 0.0001$ with $\Delta y = 0.01$) are presented in Figs. 2 and 3. The plots show a transient increase in both the heat and momentum until a steady state is reached. The time of achieving steady state momentum and energy profiles for different parameter values varies. That is, the time of obtaining no variation in the fluid momentum and energy distributions depend highly on the parameter values.

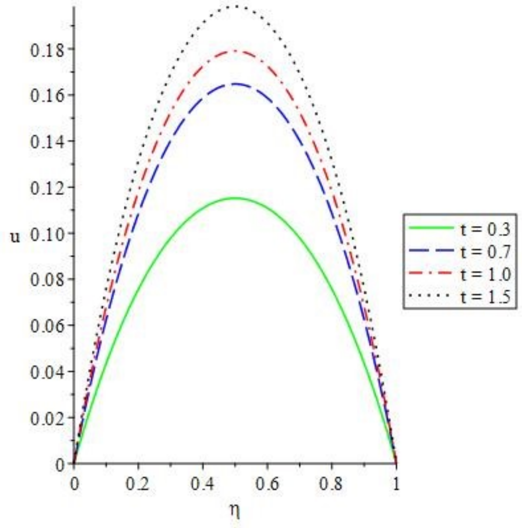

Fig. 2. Transient velocity profile

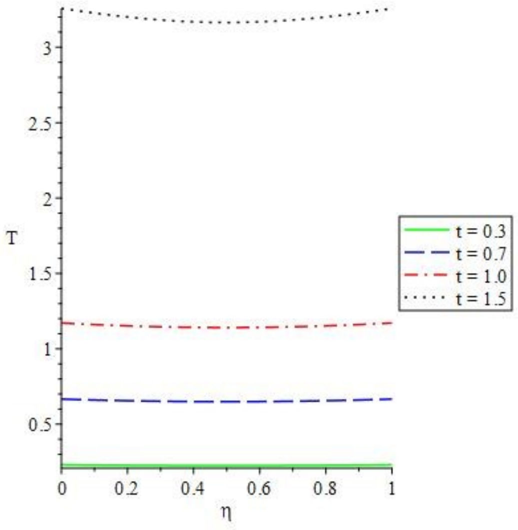

Fig. 3. Transient temperature profile

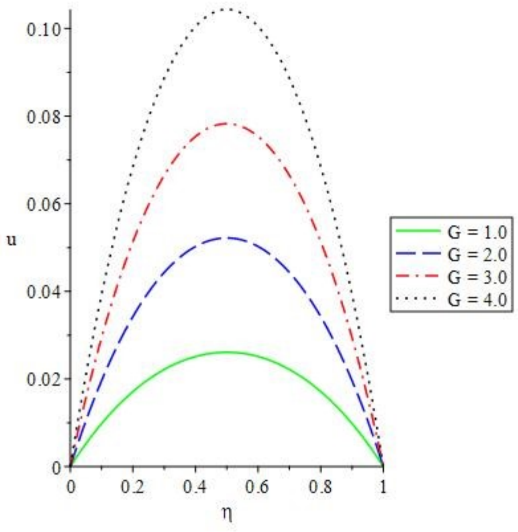

Fig. 4. Velocity profile for values of G

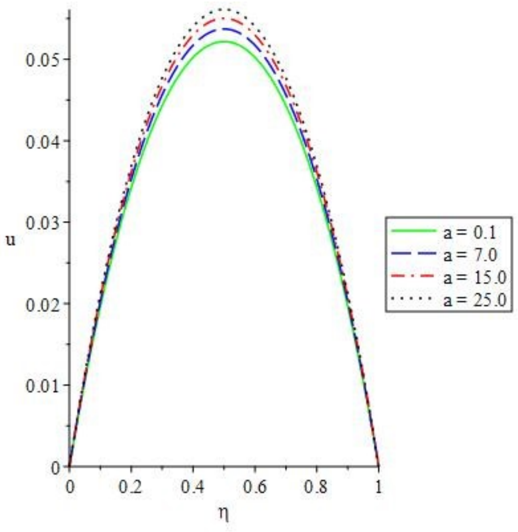

Fig. 5. Velocity profile for values of a

Figure 4 shows the transient velocity profile for variational increase in the values of uniform pressure gradient. An increase in the values of the term (G) leads to an enhancement in free movement at high collision rate of the fluid particles as a result of an increase in the heat within the system that reduces the viscoelastic reactive fluid bonding force.

Hence, maximum velocity distribution is obtained. The impact of heat dependent exponentially variable viscosity term (a) on the fluid momentum field is illustrated in Fig. 5. Rising in the parameter values (a) diminishes the viscosity of the liquid and therefore consistently weakens fluid flow opposition force. This automatically leads to an encouragement in the velocity of the fluid as shown in the plot. The viscous heating term in the energy equation increases which in turn reduces the viscoelastic reactive bond and correspondingly enhances the velocity distribution.

Figure 6 presents the effect of third grade non-Newtonian material parameter (ω) on the fluid flow rate. It is observed that the flow momentum decreases as the parameter values (ω) increase. This is because the heat presented in the reactive liquid reduces which make the fluid to be more elastic as a result of increase in the reactive bonding force thereby discourage free fluid particles movement. Hence, the flow rate retarded and reactive Rivlin-Ericksen liquid velocity profile reduces. The reaction of the temperature distribution to varying in the Brinkman number (Br) is presented in Fig. 7 for a symmetric convective cooling at fixed values of other fluid parameters. Due to cooling convection, the heat distribution of a non-Newtonian reactive two-step exothermic chemical reaction at the centreline of the channel shows a symmetry curve. A rise in the viscous heating dissipation is noticed as a result of a rise in the Brinkman number, this suitably manifested in greater temperature of the fluid. Therefore, the temperature profile is strengthen.

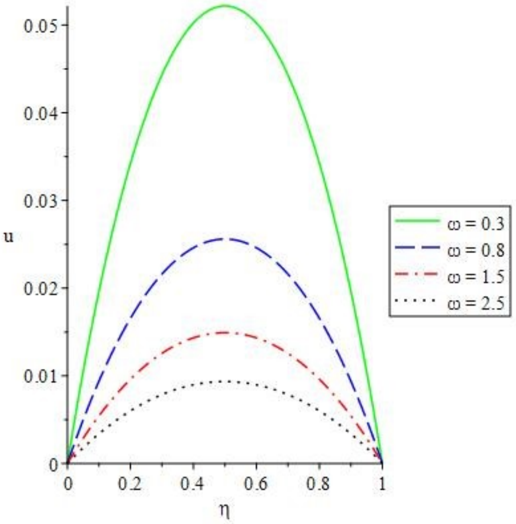

Fig. 6. Velocity profile for values of ω

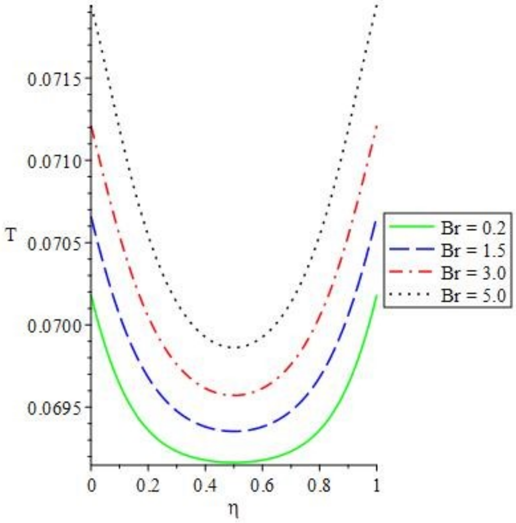

Fig. 7. Heat profile for values of Br

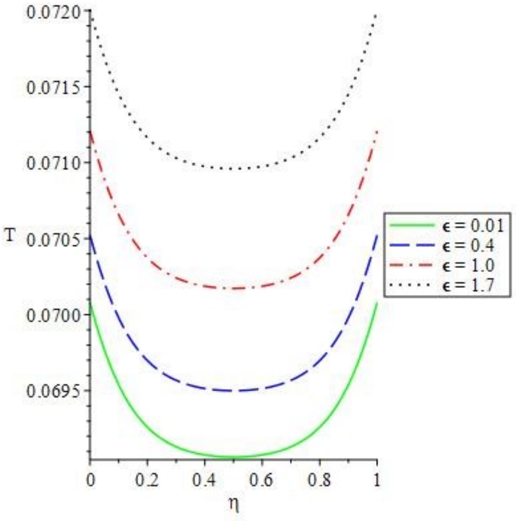

Fig. 8. Heat profile for values of ε

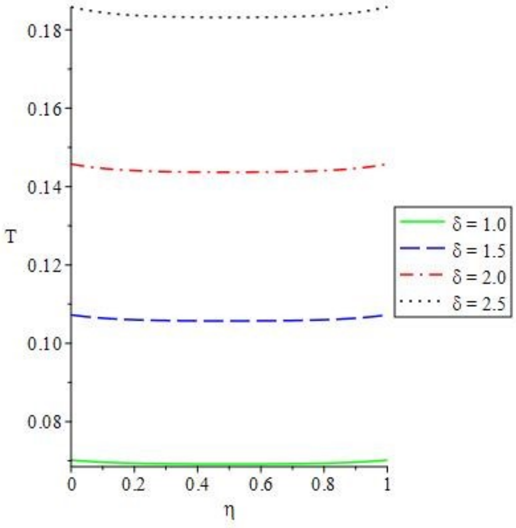

Fig. 9. Heat profile for values of δ

Figure 8 corresponds to the reaction of temperature field to changes in the activation energy term (ε). The temperature of the fluid rises as the parameter values (ε) increase. The pronounced effect of activation energy on the fluid heat distribution is due to the influence of the Bimolecular kinetic that is at $n = 0.5$ for a temperature dependent viscosity in a viscoelastic fluid. Activation energy is an energy requires for a chemical reaction process to occur, hence an increase in the values enhances the temperature field. In Fig. 9, the consequent of variational rise in the two-step reaction parameter (δ) on the heat field is presented. The plot shows a rise in the temperature of the fluid within the system as the second step chemical reaction term increases. The rise in the temperature of the fluid is an evidence that there is an increase in the combustion rate of Rivlin-Ericksen liquid which result in the reduction of toxic substance that polluted the environment from the combustive engine. Hence, two-step exothermic chemical reaction support complete combustion of unburnt ethanol in an automobile engine.

Figure 10 illustrates the effect of Frank-Kamenetskii parameter (λ) on the difference in the reactive non-Newtonian maximum temperature $(\theta_{\max})$ of the liquid. Most especially, for each value of $0 \le \varepsilon \le 1$ there is a corresponding critical value (λ_{cr}) that give rise to (a turning point) that is, for $0 < \lambda < \lambda_{cr}$, two possible solutions are obtained. As a result of chemical kinetic in the energy balance equation, lower and upper branches solution is noticed. When $\lambda > \lambda_{cr}$, no actual solution is gotten in the system and a classical figure is displayed as thermal runaway.

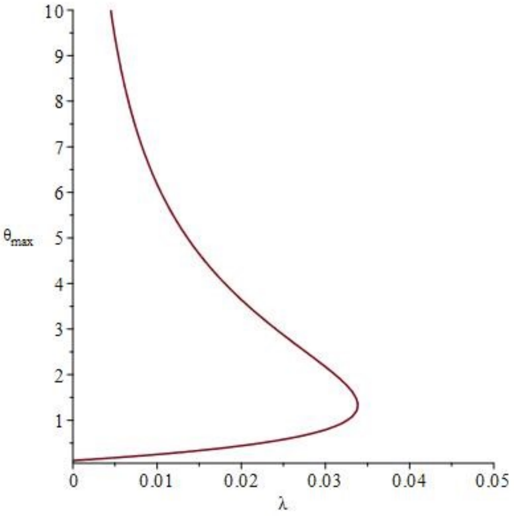

Fig. 10. A plane diagram of a slice approximate bifurcation

The influence of variation in the viscosity term on the rate of entropy volumetric generation is depicted as entropy generation distribution in Fig. 11. At the channel centreline, the gradient velocity and gradient temperature are both zero that is at $(\eta = 0.5)$ but the rate of entropy production at the point varies. An increase in the viscosity parameter (*a*), decreases the viscosity of the fluid but the effect is otherwise when related to its impact on the gradient velocity as it rises as the parameter (*a*) is enhanced (see Fig. 5). Therefore, the dissipation rate is encouraged as (*a*) increases, hence the entropy generation rate increases.

The reaction of heat volumetric generation rate to changes in Brinkman number (*Br*) is illustrated in Fig. 12 as the rate of entropy generation for a convective cooling channel with fixed values of viscosity parameter and gradient pressure. As obtained, diminishing in the entropy generation is experienced as the Brinkman number increases. The rate of entropy production decreases toward the centreline of the channel because low heat is produced in the reactive fluid flow system.

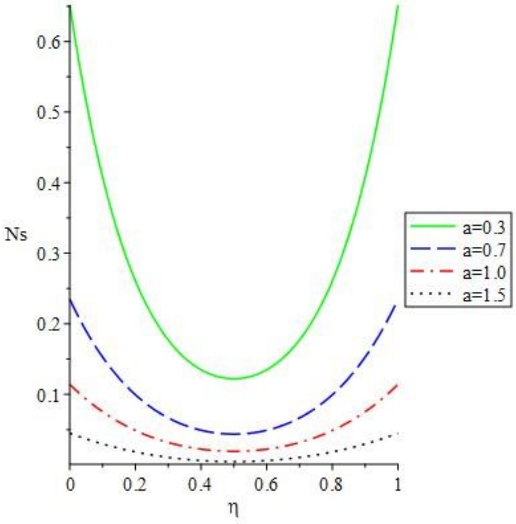

Fig. 11. Entropy generation for values of *a*

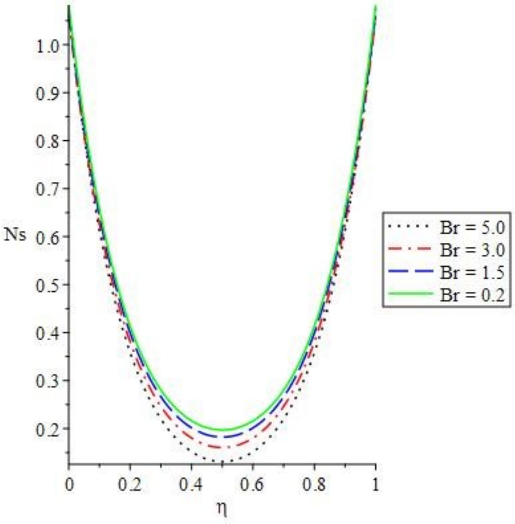

Fig. 12. Entropy generation for values of *Br*

Figure 13 denotes the effect of Hartmann number on the production of entropy in a two-step exothermic chemical reactive fluid flow with temperature dependent exponential viscosity. A rise in the magnetic field parameter (H) opposes the rate of entropy volumetric generation because of the retarding force i.e Lorentz force that resist the entropy production rate in a symmetrically cooling channel. The viselastic fluid get enhanced as it restricts free fluid particles motion in the channel as the parameter values of (H) rises. Thus, entropy generation rate decreases.

Following from the data of Figs 11 and 12 of the entropy generation profiles, the datas are respectively provided for Figs. 14 and 15. In Fig. 14, the Bejan number profile portrays that an increase in the exponential variable viscosity term (a) encourages reactive fluid irreversibility. The viscosity effect is well pronounced in the Bejan number distribution as it increases the irreversibility ratio. As earlier observed, magnifying the parameter values (a) depresses the fluid momentum that in turn boost the Bejan number, and the prevailing influence of the irreversibility fluid friction within the main area of the walls. Fig. 15 represents the significance of Brinkman number on the irreversibility rate of the fluid. A rise in Brinkman number advances the Bejan number profile, the increased viscous dissipation transforms into greater irreversibility of the reactive fluid as a result of friction in the fluid channel center area and enhanced at the walls the Bejan number.

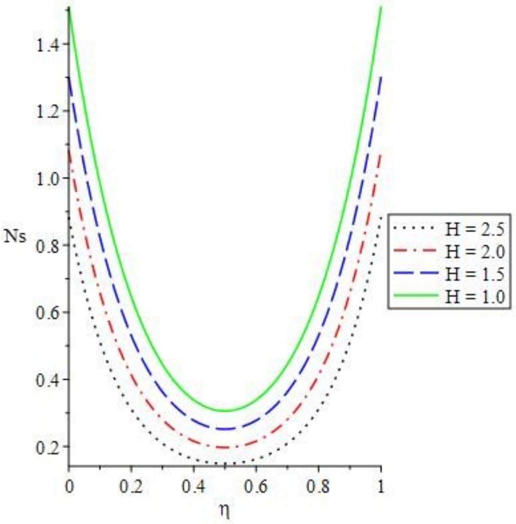

Fig. 13. Entropy generation for values of H

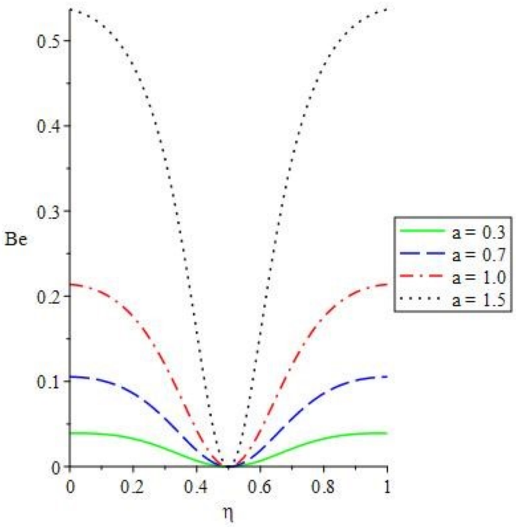

Fig. 14. Bejan number for values of a

Figure 16 demonstrates the solution plots for different values of Frank-Kamenettski term (λ) for Bejan number distribution. Increasing the values of (λ) qualitative changes the rate of the liquid irreversibility in the system by enhancing it due to the influence of second step exothermic reaction parameter. The Frank-Kamenettski parameter is a strong heat source term in a reactive fluid energy equation that encourages irreversibility of Bimolecular and pressure driven chemical reaction. The momentum and heat gradient are zero at the channel centreline at ($\eta = 0.5$), and the Bejan number is also zero at the location.

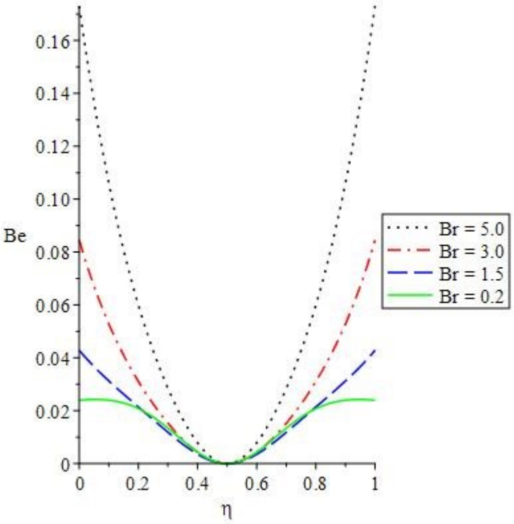

Fig. 15. Bejan number for values of Br

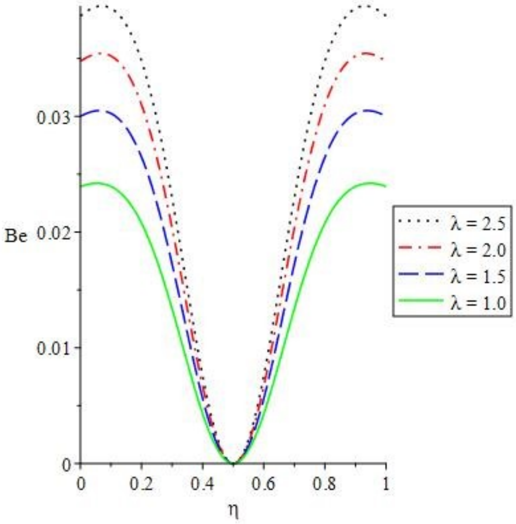

Fig. 16. Bejan number for values of λ

7. Conclusions

The study analyzed the pressure driven transient Rivlin-Ericksen fluid and irreversibility of variable viscosity of hydromagnetic two-step exothermic chemical reactive flow through a convective cooling channel. The momentum and energy distributions were adopted for the evaluation of entropy generation and irreversibility of the fluid flow. From the investigation, it was seen that the rate of entropy production is greater for increasing Brinkman number and a symmetrical convective cooled walls. Hence, it is possible for a system to operate at decreased entropy generation rate with the support of convective cooling of the channel. An increment in the temperature dependent exponential viscosity enhances the velocity gradient and the fluid velocity is at the peak, it also boosted the viscous dissipation which encourages huge entropy generation and irreversibility of fluid friction within the main channel of the system. It was found that among others, the thermal criticality conditions and with the right combination of thermo-fluid parameters controlling the system, the thermal runaway can be prevented. These will be of interest to lubrication companies in improving the efficiency and effectiveness of hydromagnetic materials used in engineering systems.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The authors received no financial support for the research, authorship and publication of this article.

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