ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, *Vol 8(31), 59158, November 2015* ISSN (Online) : 0974-5645

Improved Constrained Portfolio Selection Model using Particle Swarm Optimization

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Abstract

Objective: The main objective of this study is to improve the extended Markowitz mean-variance portfolio selection model by introducing a new constraint known as expert opinion practicable for portfolio selection in real-life situation. **Methods:** This new extended model consists of four constraints namely: bounds on holdings, cardinality, minimum transaction lots, and expert opinion. The first three constraints have been presented in other researches in literature. The fourth constraint introduced in this study is an essential parameter in making and guiding a realistic portfolio selection. To solve this new extended model an efficient heuristic method of Particle Swarm Optimization (PSO) was engaged with existing benchmark data in the literature. **Results:** The outcome of the computational results obtained in this study with the new extended Markowitz mean-variance portfolio selection model proposed in this study and solved with PSO showed an improved performance over existing algorithm in particular GA in different instances of the data set used. **Conclusion:** The study evolves a new extended portfolio selection model and the findings demonstrate the superiority of PSO performance in solving portfolio selection problem in comparison with GA algorithm.

Keywords: Constraints, Expert Opinion, Genetic Algorithm, Particle Swarm Optimization, Portfolio Selection

1. Introduction

Portfolio Selection Problem (PSP) has remained one of the pertinent research areas in the domain of finance and economics over the years and currently still drawing interest of several researchers in the subject domain. Portfolio can be defined as various ways of diversifying money over different assets. PSP is all about investing particular money over a given set of assets in order to maximize return and minimize risk which is purely optimization problem. In other words, PSP is the process of selecting a given set of assets, and the share invested in each asset, which offers the investor a minimum expected return and minimizes the risk[1]. Knowing appropriate portfolio of assets to select has remained a mirage to fund management organizations and as well as individual investors[2-4]. The first pioneering work on PSP by[5]was on the well known mean-variance model that requires one to minimize risk of the selected portfolio while maximizing the predetermined expected rate of return and efficient use of the available capital[6]. The work of[5]has gained a wide acceptance as a useable tool in portfolio selection optimization. Though there are many extension of the model, it remains the general model of reference.

As the PSP assumed increasing dimension and computational complexity due to increasing number assets in portfolio selection, new Metaheuristics algorithms becomes a promising alternative method to portfolio selection in overcoming the challenges of existing methods such as Tabu Search (TS), Simulated Annealing (SA), Goal Programming (GP), Multiple Objective Programming (MOP), Quadratic Programming (QP)[6,7] to mention a few. Several methods, however have surfaced to tackle the portfolio selection problem with one short coming or the other. The following are the prevalent methods that have been applied to PSP in

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literature. Fuzzy set theory had been immensely engaged in portfolio selection; among the few works reported in literature are the works of[8-13]. Genetic Algorithm (GA) has also been extensively used to solve PSP as reported in work of[7,14-18]. In the work of[2] engaged a heuristic technique of Particle Swarm Optimization (PSO) to extend Markowitz mean variance portfolio selection problem. Their findings compared with GA revealed a superior performance over GA model. Also, in a similar work by[3], developed PSO model for PSP and compared their results with GA model. Their finding showed that PSO model demonstrated high computational efficiency in building optimal risky portfolios. Others related works that engaged PSO for PSP are[19-22].

Due to the shortcomings of existing methods in tackling PSP as result of increasing computation complexity in terms of size of portfolio selection. For instance, TS, SA, QP has failed to handle portfolio selection effectively because of the complexity of the problem when the number of assets in portfolio selection increases[6,7]. Also, inability of fuzzy set theory to learn and inability of GA to converge on time in the face of harder and bigger problems in order to obtain suitable solutions remains a drawback[3]. The main contribution of this work stems from the successfully introduction of a new constraint known as expert opinion for selection of profitable portfolios which is practicable in real-life scenarios make this work unique to other research works that has been presented in literature. The computational results obtained in this work show improve performance over existing methods particularly GA.

The rest of the paper is organized as follows. Section 2 presents the portfolio selection problem and the new model evolved. The methodology used to address the research problem is explained in section 3. Section 4 discussed and analyzed the computational results obtained in this work and the paper concluded in section 5.

2. Portfolio Selection Problem

The portfolio selection problem can be expressed in the standard Markowitz model as follows[23]:

$$\min \ \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

subject to

$$r_p = \sum_{i=1}^{N} w_i r_i \geq R \tag{2}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{3}$$

$$w_i \geq 0, \forall_i \in \{1,2,\dots,N\} \tag{4}$$

where

N is the number of available assets;

r_p is the expected rate of return;

r_i is the expected rate of return of asset i;

σ_{ij} is the covariance of returns of asset i and j;

R is the investor's expected rate of return and

σ_p^2 is the return variance of the portfolio.

w_i is the decision variable that represents the weight of budget to be invested in asset i.

The constraint in equation (3) ensures that the total budget is invested. Equation (4) ensures that no short sell is allowed. The goal is to minimize the portfolio risk σ_p^2 for a given value of portfolio expected return r_p.

The following constraints such as bounds on holdings, cardinality, and minimum transaction lots are particularly important in making significant investment decision in real-life financial market. The bounds on holding constraint,ensures that the amount invested in each asset lie between predetermined upper and lower bounds. The carnality constraintensures that the total number of assets selected in the portfolio is equal to the predefined number while the minimum transaction lots constraint requires that each asset can only be purchased in batch with a given number of units. The three aforementioned constraints have been well researched in portfolio selection problem[2,15,24,25]. In order to make the model realistic and attaining the goal set in reducing investment risk, an important constraint known as expert opinion is added. The importance of expert opinion in portfolio selection cannot be over-emphasized due to fact that expert is well informed and can do a thorough analysis of each security before selection of an asset to be part of the portfolio. This research differs significantly from the previous studies on portfolio selection problem by the introduction of new feasible constraint of expert opinion to portfolio selection problem.

2.1 The Proposed Model

This section describes the proposed model. The proposed model is an extension of Markowitz's mean variance portfolio selection model in the work of[2]. The Markowitz's model lack real market situation scenario. To explain the proposed model the definition of following variables is of importance. Therefore:

M is the number of assets to be selected from N available assets.

B is the total available budget.

R is the investor's expected rate of return.

B_{lower_i} is the minimum amount of budget that can be invested in asset i.

B_{upper_i} is the maximum amount of budget that can be invested in asset i.

c_i is the minimum transaction lots for asset i.

x_i is the number of c_i's that is purchased.

w_i is the decision variable that represents the weight of budget to be invested in asset i.

z_i is a binary variable {0,1}if 1 asset i is in the portfolio and otherwise 0.

e_i isthe expert opinion, a binary variable of 1 if asset i is selected and otherwise 0.

i is the index of securities.

Investors always desire to minimize risk of investment and maximize possible return. The extended Markowitz model for the portfolio selection problem proposed in this paper is, thus, formulated as follows:

$$\min \quad \sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{5}$$

where

$$w_i = \frac{x_i c_i z_i}{\sum_{j=1}^{N} x_j c_j z_j}, i = 1,\ldots,N \tag{6}$$

and

$$\sum_{i=1}^{N} z_i = M \leq N; M, N \in \mathrm{N} \tag{7}$$

subject to

$$\sum_{i=1}^{N} x_i c_i z_i e_i r_i \geq BR \tag{8}$$

$$\sum x_i c_i z_i e_i \leq B \tag{9}$$

$$0 \leq B_{lower_i} \leq x_i c_i \leq B_{upper_i} \leq B, i = 1,\ldots,N \tag{10}$$

$$\sum_{i=1}^{N} w_i = 1 \tag{11}$$

$$w_i \geq 0, \forall_i \in \{1, 2, \ldots, N\} \tag{12}$$

$$e_i \in \{0,1\} \tag{13}$$

where

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{14}$$

$x_i c_1$ represents the number of units of asset i in the selected portfolio. Z_i is the decision variable in which it is equal to 1 if asset i is upheld in the portfolio and otherwise 0. The inequality in equation (7) denotes cardinality constraint while the inequality in equation (8) is the same as equation (2). Equation (9) represents the budget constraint. Equation (10) indicates the bounds on holdings constraint. The equations (13) and (14) represent the expert opinion constraint. The expert opinion constraint is a practicable and useful constraint in a real life scenario of portfolio selection because the expert has detail information about sector capitalization where each asset i to be selected in the portfolio belongin order to minimize investment risk. Beyond sector capitalization the expert or financial analyst can access other information regarding each asset i to be selected in the portfolio such as price/annual earning, management calibre, dividend rate, book value and so on. An in-depth analysis of thisinformation can guide the expert upon which an opinion is formed whether asset i should be included in the portfolio or not. This paper is the first ever to introduce this important constraint of expert opinion in the portfolio selection problem.

This extended model requires efficient heuristics to find the solution because it is classified as a quadratic mixed integer programming model. In the next section which contained the methodology used in this work, Particle Swarm Optimization (PSO) is reviewed and use to solve the proposed extended Markowitz model as formulated above.

3. Methodology

This section describes briefly the concept of Metaheuristics used in this work in particularly the PSO and GA. The data used and experimental details.

3.1 Particle Swarm Optimization

PSO is a population based search technique modelled according to social behaviour of organism such as bird flocking and fish schooling. The population of individuals is called particles. Particles in search space move by following the current optimum particles and adjust positions in order find the optima. There are two operators in PSO namely velocity and position update. In each generation each particle position is updated by following the current best position and the global best position of the population. In each iteration, a new velocity value for each particle is computed based on its current velocity, the distance from its previous best position, and the distance from the global best position. The new position of particle in the search space is calculated by the velocity updated at each time. This process is iterated a number of time until minimal error is obtained. The procedural steps of PSO are enumerated as follows[26,27].

Step one: Initialize Population
Step two: ***repeat***
Step three: Compute to evaluate fitness values of particles
Step four: Modify the best particles in the swarm
Step five: Identify and choose the best particle
Step six: Compute the velocities of particles
Step seven: Update the particle position
Step eight: ***Until*** requirements are met

3.2 Genetic Algorithm

GA starts with initial population of a set of binary strings generated randomly by random operator. The strings are candidate solution to the optimization problem being considered. Each string has its own fitness value computed by evaluation unit. The goodness of the solution is a function of the fitness value obtained. The objective of the genetic operator is to make a set of strings into its highest fitness values. In the reproduction phase, the operator copied individual strings from one set to the next as determined by the fitness values. The higher the fitness value, the greater is the chances of being selected in the next generation. The crossover operator selects pairs of strings at random and generates new pairs. The crossover operator selects pairs of strings at random and generates new pair. The crossover rate determined the number of crossover operations. The mutation operator randomly mutates the values of bits in a string. The mutation rate determined the number of mutation operations. Each phase of the algorithm consists of applying fitness evaluation, reproduction, crossover and mutation operations. A new generation of solutions emerge in each phase of the algorithm. In general the basic GA algorithm can be summarised as follows[26]:

Step one: Initialize Population
Step two: ***repeat***
Step three: Evaluation
Step four: Reproduction
Step five: Crossover
Step six; Mutation
Step seven: ***until*** requirement are met

3.3 Data used and Experimental Settings

The proposed extended Markowitz model developed in this work was implemented with efficient heuristics methods of PSO and GA with each set of data of 31 and 85 stocks from the stock markets of Hong Kong HangSeng and the German DAX 100 respectively. The data was obtained from test data from OR-Library[28]. Each data set contains the number of assets (N). The mean return and standard deviation of return for each asset i and correlation between asset i and j for all possible pairs of assets. In order to evaluate the performance of the two algorithms on theproposed portfolio model. It was run on a PC with Intel Pentium 4.3 GHz with 2GB RAM. The parameters settings for each of the data set is as follows: expert opinion was set to greater than 0.5 if the asset is selected in the portfolio, the value of budget was set to 2800, expected rate of returns was set to 0.004, 0.005 and 0.006 respectively. A predetermined upper and lower bound was set for each of the assets selected. The size of portfolio was set to 15, 20 and 25 for each of the data set.

Five criteria were used to compare the performance of the results obtained of the two algorithms used for the proposed portfolio model. The criteria are as follows:

- Best variance; depict lowest risk from algorithm runs, showing the best solution found.

- Mean variance; the average of the objective function found by the algorithm.
- Worst variance, depicts the highest risk from algorithms runs, showing the worst solution.
- Standard deviation of variance, depicts how close the solution found by the algorithms are close to each other and,
- Mean execution time, depicts amount of time needed to arrive to a solution.

4. Computational Results and Discussion

The results of GA and PSO algorithm for data set of 31 stocks are tabulated in Table 1. Similarly, the results obtained for data set of 85 stocks with GA and PSO are contained in Table 2 accordingly. The comparison made between PSO and GA on the improved portfolio model developed in this work was tabulated in Table 3. The positive number in table 3 shows the percentage of improvement obtained using PSO when compared the results of the proposed model with GA while the negative numbers shows otherwise.

From the results obtained in Table 3. When the size of portfolio is 15, the best variance found by PSO is 31.58% better than GA in the average. However, the mean execution time taken to find the best variance is 12.98% higher than the time it takes GA on the average. In the case of the portfolio size of 20, PSO shows the best variance of 40.96% improvement over GA on the average and took more meanexecution time of 11.46% to find the best solution to GA on the average. When the size of portfolio is 25, the average percentage of improvement of best variance is 41.43% in PSO compared to GA and similarly, the mean execution time taken to find best solution is 22.06% higher than GA on the average. The results agreed with the results obtained in the work of[2]. It is obvious from Table 3 that for the data set of 31 stocks, the best variance of found by PSO on the proposed improved extended portfolio model are better than those found by GA. With respect to mean variance, worst variance and standard deviation variance, PSO still show better performance than GA.

To further evaluate the performance of improved extended portfolio model in a complex scenario of larger data set of 85 stocks. Table 2 shows the results obtained with 85 stock data set. Similar comparison was also performed and the results obtained are contained in Table 4. The performance of efficient heuristics of PSO shows superior performance with less time taken to find the best variance solutions.

Table 1. Results of GA and PSO algorithm to 31 stocks data set across 50 independent executions

Size of portfolio		Expected rate of return	0.004		0.005		0.006	
15	Variance	Algorithms	GA	PSO	GA	PSO	GA	PSO
		Best	0.35203899	0.15107619	0.34717306	0.214028489	0.35378268	0.25157078
		Mean	0.67218507	0.61424974	0.64333504	0.566691078	0.68509601	0.60348172
		Worst	0.98431799	0.77090457	0.95442150	0.735944859	1.07633423	0.69708422
		Std. Dev.	0.13738563	0.13578541	0.15035748	0.129776277	0.17494801	0.14831623
		Mean exe. time (s)	34.74636	38.32675	29.6409	31.0872	28.78512	35.6251
20		Best	0.52981593	0.32947806	0.51186707	0.373316492	0.315823093	0.287602147
		Mean	0.93358625	0.55694377	0.91360268	0.800158982	0.908991852	0.705545178
		Worst	1.56117885	0.95614994	1.38676291	1.234916183	1.448040121	0.943375093
		Std. Dev.	0.21277851	0.17553432	0.19925772	0.114052924	0.232814076	0.188732521
		Mean exe. time (s)	43.38638	45.93512	30.10292	32.5496	33.25678	39.98751
25		Best	0.65977877	0.45408498	0.63744981	0.510079919	0.635221067	0.434594241
		Mean	1.08509956	0.68236685	1.10107861	0.946176642	1.058870543	0.721922588
		Worst	2.09389079	0.86684947	1.71801513	1.368720214	1.723429881	1.239178664
		Std. Dev.	0.28499070	0.16394986	0.26451288	0.214568917	0.223461266	0.172342988
		Mean exe. time (s)	36.53728	40.18954	25.48038	29.50243	20.6254	28.9627

Table 2. Results of GA and PSO algorithm to 31 stocks data set across 50 independent executions

Size of portfolio		Expected rate of return	0.004		0.005		0.006	
15	Variance	Algorithms	GA	PSO	GA	PSO	GA	PSO
		Best	0.14043814	0.13438406	0.09654804	0.079737446	0.198249716	0.075848570
		Mean	0.29943517	0.26523937	0.28107119	0.254470528	0.322653323	0.278438959
		Worst	0.55895829	0.48650518	0.43885621	0.394134793	0.598769274	0.356018608
		Std. Dev.	0.08599229	0.07007661	0.07421649	0.067178084	0.083048395	0.064038095
		Mean exe. time (s)	39.26028	42.63432	45.05456	47.11035	44.13692	46.98563
20		Best	0.20805088	0.17834219	0.24077569	0.158127063	0.217592437	0.137872202
		Mean	0.41541530	0.35715149	0.40527862	0.326921371	0.427922810	0.283161073
		Worst	0.70692483	0.59088845	0.70969466	0.527231052	0.743734672	0.487801885
		Std. Dev.	0.10504707	0.09101289	0.11433733	0.083268498	0.124465355	0.074373463
		Mean exe. time (s)	42.44526	43.56093	38.62946	41.78632	37.14916	39.9572
25		Best	0.27617457	0.19016071	0.30835802	0.232138784	0.325238055	0.304761661
		Mean	0.52020104	0.35529302	0.52260675	0.488508212	0.529653841	0.410432784
		Worst	0.78384814	0.68452987	0.95231780	0.720440502	0.789133804	0.633696336
		Std. Dev.	0.11005574	0.08231027	0.13102051	1.028592817	0.113633008	0.10946686
		Mean exe. time (s)	44.0845	45.94935	41.6266	44.889324	50.19838	53.33126

Table 3. Comparison of PSO and GA for data set of 31 stocks. Positive numbers show the percentage of improvement obtained when using PSO compared to GA.

Size of portfolio		Expected rate of return	0.004	0.005	0.006	Average (%)
15	Variance	Best	33.02	21.08	40.63	31.58
		Mean	8.62	11.91	11.91	10.81
		Worst	21.68	22.89	35.24	26.6
		Std. Dev.	1.16	13.69	15.22	10.02
		Mean exe. time (s)	-10.30	-4.88	-23.76	-12.98
20		Best	60.80	52.27	9.81	40.96
		Mean	40.34	12.42	22.38	25.05
		Worst	38.75	10.95	34.85	28.18
		Std. Dev.	17.50	42.76	18.93	26.4
		Mean exe. time (s)	-5.87	-8.13	-20.24	-11.41
25		Best	45.29	32.83	46.16	41.43
		Mean	37.11	14.07	31.82	27.67
		Worst	58.60	20.33	28.10	35.68
		Std. Dev.	42.47	18.88	22.88	28.08
		Mean exe. time (s)	-9.99	-15.78	-40.42	-22.06

Table 4. Comparison of PSO and GA for data set of 85 stocks. Positive numbers show the percentage of improvement obtained when using PSO compared to GA.

Size of portfolio		Expected rate of return	0.004	0.005	0.006	Average (%)
15	Variance	Best	4.50	21.08	61.37	28.98
		Mean	11.42	9.46	13.70	11.53
		Worst	12.96	10.19	40.54	21.23
		Std. Dev.	18.51	9.48	22.89	16.96
		Mean exe. time (s)	-8.59	-4.56	-6.45	-6.533
20		Best	16.66	52.27	57.82	42.25
		Mean	14.05	19.33	33.83	22.4
		Worst	16.41	25.71	34.41	25.51
		Std. Dev.	13.36	27.17	40.24	26.92
		Mean exe. time (s)	-2.62	-8.17	-7.56	-6.117
25		Best	45.23	32.83	6.72	28.26
		Mean	31.70	6.52	22.51	20.24
		Worst	12.67	24.34	19.70	18.9
		Std. Dev.	25.21	-685.06	3.67	-218.7
		Mean exe. time (s)	-4.23	-7.84	-6.24	-6.103

5. Conclusion

This paper presents an improved extended Markowitz mean-variance portfolio model with the introduction of new constraint known as expert opinion. This work would the first in literature pioneering this new innovation to portfolio selection model since advent of portfolio selection problem. This new extended model consists of four constraints namely: bounds on holdings, cardinality, minimum transaction lots, and expert opinion. An efficient heuristic method of particle swarm optimization algorithm was engaged and genetic algorithm to make a comparison of the results obtained. Five performance evaluation criteria already used in similar works by other researchers in literature was used tocompare performance between particle swarm optimization algorithm and genetic algorithmon both small and large data set for the improved extended portfolio model developed. In all the test cases PSO achieve better solution than GA, however with higher computational mean execution time. Further studies are to engage comparative study of other swarm intelligence techniques to the new extended portfolio model developed in this paper.

6. Acknowledgment

The authors to acknowledge the support of OluwoleAdekambi for the codes provided which enhance the completion of the work in record time.

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