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Classification-Based Ridge Estimation Techniques of Alkhamisi Methods

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ABSTRACT Following Lukman and Ayinde [9]: review and classification of methods of estimating ridge parameters into different forms and various types, this study proposed some new ridge parameter estimation using the idea of Alkhamisi *et al*. [1]. The performance of the techniques was evaluated by conducting Monte-Carlo experiments under certain conditions and compared using relative efficiency. Results show that increase in the strength of multicollinearity resulted in increase in mean square error (MSE), which decreases as the sample size increases. Furthermore, the most preferred technique is generally in the different forms in the original and square root types. Moreover, Fixed Maximum Original (FMO) for Alkhamisi *et al*. [1], the proposed Varying Maximum Original (VMO) for AL4, VMO for AL6 and Harmonic Mean Original (HMO) for AL5 competes favorably.

Keywords Mean square error; Monte-Carlo experiment; Ridge parameter; Relative efficiency.

1. Introduction

Consider the standard linear regression model:

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{U} \tag{1}$$

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where $\boldsymbol{X}$ is an $n \times p$ matrix with full rank, $\boldsymbol{Y}$ is an $n \times 1$ vector of dependent variable, $\boldsymbol{\beta}$ is a $p \times 1$ vector of unknown parameters, and $\boldsymbol{U}$ is the error term such that

$$E(\boldsymbol{U}) = \mathbf{0}_{n\times 1} \quad \text{and} \quad E(\boldsymbol{U}\boldsymbol{U}') = \sigma^2 I_n .$$

The Ordinary Least Square (OLS) estimator of $\boldsymbol{\beta}$ of (1) is defined as:

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} \tag{2}$$

The estimator defined in (2) is most efficient under certain assumptions. A basic assumption is that the explanatory variables are not linearly related. A violation of this assumption is referred to as multicollinearity. Computational difficulties arise with this problem. Also, the performance of ordinary least square (OLS) estimator is unsatisfactory in the presence of multicollinearity in that although the regression estimates is unbiased they possess large standard errors which cause the regression estimates to be statistically insignificant and at times have wrong signs (Gujarati [4]). Several methods have been suggested in literature to solve this problem. Hoerl and Kennard [5] introduced the Ordinary Ridge Regression (ORR) estimator as alternative to the OLS estimator to handle the problem of multicollinearity. They suggested the addition of ridge parameter *k* to the diagonal of $\boldsymbol{X}'\boldsymbol{X}$ matrix in (2). Therefore, the ORR estimator is defined as:

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X} + kI_p)^{-1}\boldsymbol{X}'\boldsymbol{Y} \tag{3}$$

where kI_p is a $p \times p$ diagonal matrix of non-negative constants (that is, $k \geq 0$). It is observed that when $k = 0$, (3) returns to OLS estimates. If all the values of the ridge parameter are the same, the resulting estimator is called Ordinary Ridge Regression Estimator (Dorugade, [2]). Ridge regression estimator gives a smaller mean squared error when compared to the OLS estimator for a positive value of *k* (Hoerl and Kennard, [5]). The use of ridge regression estimator depends on the ridge parameter, *k*. Different methods for estimating this ridge parameter has been considered by many authors. Some of these authors are: Hoerl and Kennard [5], McDonald and Galarneau [11], Lawless and Wang [8], Gibbons [3], Kibria [7], Khalaf and Shukur [6], Alkhamisi *et al*. [1], Muniz and Kibria [12], Mansson *et al*. [10], Dorugade [2] and recently, Lukman and Ayinde [9]. In this study, new ridge parameters are proposed and their performances are compared with some of the existing ones based on a simulation study. Data were generated from a normal distribution with two different number of regressors ($p = 3, 7$) under three different error variances. The mean square error (MSE) was used as a performance criterion.

2. Ridge Regression and Proposed Estimators for Ridge Parameter

Ridge Regression was introduced by Hoerl and Kennard [5] as an alternative to OLS when there is problem of multicollinearity. Lukman and Ayinde [9] reviewed the several methods of estimating the ridge parameters earlier mentioned and observed that the existing ridge parameters followed some different forms and various types. This is further explained as follows and illustrated by Table 1 in light of Alkhamisi *et al.* [1].

2.1 Different Forms and Various Types of Ridge Parameter *k*

Different Forms of Ridge Parameter *k*:

1) Fixed Maximum (FM): This is obtained by using the highest value of the estimated regression coefficient or the eigenvalue or both.
2) Varying Maximum (VM): This allows the estimated regression coefficient and the eigenvalue to vary and eventually the maximum of the estimated ridge parameter is chosen. That is the ridge parameter with the highest estimated ridge parameters or eigenvalues or both.
3) Arithmetic Mean (AM): It involves taking the average of the estimated ridge parameter.
4) Harmonic Mean (HM): The ridge parameter is expressed in harmonic mean of the estimated ridge parameters.
5) Geometric Mean (GM): The ridge parameter is expressed as the geometric mean of the estimated ridge parameters.
6) Median (M): This involves taking the median of the estimated ridge parameters.

Various Types of Ridge Parameter *k*:

1) Original form (O)
2) Reciprocal form (R)
3) Square root form (SR)
4) Reciprocal of Square root (RSR)

Alkhamisi *et al.* [1] proposed the ridge parameter

$$k_{\mathrm{ALK}} = \frac{\sigma^2 \lambda_i}{(n-p)\sigma^2 + \lambda_i \alpha_i^2}.$$

Their estimator in different forms and of various types are summarized in Table 1.

Table 1 Summary of Different Forms and Various Types for $\hat{k}_{\mathrm{ALK}} = \dfrac{\widehat{\sigma^2}\lambda_i}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}$

Different Forms	Various Types of *k*	
FM	Type O / Khalaf and Shukur [6] $\hat{k}_{\mathrm{ALK}}^{\mathrm{FMO}} = \dfrac{\mathrm{Max}(\lambda_i)\widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \mathrm{Max}(\lambda_i)\mathrm{Max}(\widehat{\alpha_i}^2)}$	Type R / Lukman and Ayinde [9] $\hat{k}_{\mathrm{ALK}}^{\mathrm{FMR}} = \dfrac{1}{\hat{k}_{\mathrm{ALK}}^{\mathrm{FMO}}}$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\mathrm{ALK}}^{\mathrm{FMSR}} = \sqrt{\hat{k}_{\mathrm{ALK}}^{\mathrm{FMO}}}$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\mathrm{ALK}}^{\mathrm{FMRSR}} = 1\Big/\sqrt{\hat{k}_{\mathrm{ALK}}^{\mathrm{FMO}}}$

Table 1 (Continued)

Different Forms	Various Types of k	
VM	Type O / Muniz *et al*. [13] $\hat{k}_{\text{ALK}}^{\text{VMO}} = \dfrac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}$	Type R / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{VMR}} = \text{Max}\left(\dfrac{1}{\hat{k}_{\text{ALK}}}\right)$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{VMSR}} = \text{Max}\left(\sqrt{\hat{k}_{\text{ALK}}}\right)$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{VMRSR}} = \text{Max}\left(1\Big/\sqrt{\hat{k}_{\text{ALK}}}\right)$
AM	Type O / Muniz *et al*. [13] $\hat{k}_{\text{ALK}}^{\text{AMO}} = \dfrac{1}{p}\sum_{i=1}^{p} \dfrac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}$	Type R / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{AMR}} = \dfrac{1}{\hat{k}_{\text{ALK}}^{\text{AMO}}}$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{AMSR}} = \sqrt{\hat{k}_{\text{ALK}}^{\text{AMO}}}$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{AMRSR}} = 1\Big/\sqrt{\hat{k}_{\text{ALK}}^{\text{AMO}}}$
HM	Type O / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{HMO}} = p\sum_{i=1}^{p} \dfrac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}$	Type R / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{HMR}} = \dfrac{1}{\hat{k}_{\text{ALK}}^{\text{HMO}}}$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{HMSR}} = \sqrt{\hat{k}_{\text{ALK}}^{\text{HMO}}}$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{HMRSR}} = 1\Big/\sqrt{\hat{k}_{\text{ALK}}^{\text{HMO}}}$
GM	Type O / Muniz and Kibria [12] $\hat{k}_{\text{ALK}}^{\text{GMO}} = \left(\prod_{i=1}^{p} \dfrac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}\right)^{1/p}$	Type R / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{GMR}} = \dfrac{1}{\hat{k}_{\text{ALK}}^{\text{GMO}}}$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{GMSR}} = \sqrt{\hat{k}_{\text{ALK}}^{\text{GMO}}}$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{GMRSR}} = 1\Big/\sqrt{\hat{k}_{\text{ALK}}^{\text{GMO}}}$
M	Type O / Alkhamisi *et al*. [1] $\hat{k}_{\text{ALK}}^{\text{MO}} = \text{Median}\left(\dfrac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i}^2}\right)$	Type R / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{MR}} = \text{Median}\left(\dfrac{1}{\hat{k}_{\text{ALK}}}\right)$
	Type SR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{MSR}} = \text{Median}\left(\sqrt{\hat{k}_{\text{ALK}}}\right)$	Type RSR / Lukman and Ayinde [9] $\hat{k}_{\text{ALK}}^{\text{MRSR}} = \text{Median}\left(1\Big/\sqrt{\hat{k}_{\text{ALK}}}\right)$

2.2 Proposed Estimators for Ridge Parameter

Following Alkhamisi *et al*. [1] and the classification of Lukman and Ayinde [9], the following estimators are proposed and considered in their different forms with various types as done.

$$\hat{k}_{\text{AL1}} = \frac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i[\text{Max}(\alpha_i)]^2}. \tag{4}$$

$$\hat{k}_{\text{AL2}} = \frac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + [\text{Max}(\lambda_i)]\widehat{\alpha_i^2}}. \tag{5}$$

$$\hat{k}_{\text{AL3}} = \frac{[\text{Max}(\lambda_i)]\widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \widehat{\alpha_i^2}}. \tag{6}$$

$$\hat{k}_{\text{AL4}} = \frac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \text{Max}(\lambda_i)\text{Max}\left(\widehat{\alpha_i^2}\right)}. \tag{7}$$

$$\hat{k}_{\text{AL5}} = \frac{\text{Max}(\lambda_i)\widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \text{Max}\left(\widehat{\alpha_i^2}\right)}. \tag{8}$$

$$\hat{k}_{\text{AL6}} = \frac{\lambda_i \widehat{\sigma^2}}{(n-p)\widehat{\sigma^2} + \lambda_i \text{Max}\left(\widehat{\alpha_i^2}\right)}. \tag{9}$$

where $\widehat{\sigma^2} = (\boldsymbol{U'U})/(n-p)$, λ_i are the eigenvalues of $\boldsymbol{X'X}$ matrix, and α_i is the i^{th} element of the vector $\hat{\alpha} = \boldsymbol{Q'\hat{\beta}}$ where $\boldsymbol{Q}$ is an orthogonal matrix.

3. Monte Carlo Design

Simulation procedure used by McDonald and Galarneau [11], Wichern and Churchill [15], Gibbons [3], Kibria [7], Muniz and Kibria [12], Lukman and Ayinde [9] was also used to generate the explanatory variables in this study. This procedure is based on

$$X_{ij} = (1-\rho^2)^{1/2} Z_{ij} + \rho Z_{ip}, \tag{10}$$

$i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, p$ where Z_{ij} is independent standard normal distribution with mean zero and unit variance, ρ is the correlation between any two explanatory variables and *p* is the number of explanatory variables. The value of ρ is taken as 0.8, 0.9, 0.95, 0.99, 0.999 respectively. Thus, the correlations between the variables is the same. In this study, the number of explanatory variables (*p*) is taken to be three and seven.

The considered regression model is of the form

$$Y_t = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + U_t \tag{11}$$

where $t = 1, 2, \cdots, n$, $p = 3, 7$. The error term U_t was generated to be normally distributed with mean zero and variance σ^2, i.e., $U_t \sim N(0, \sigma^2)$. β_0 was taken to be identically zero.

When $p=3$, the values of $\boldsymbol{\beta}$ were chosen to be $\beta_1=0.8$, $\beta_2=0.1$, and $\beta_3=0.6$. When $p=7$, the values of $\boldsymbol{\beta}$ were chosen to be: $\beta_1=0.4$, $\beta_2=0.1$, $\beta_3=0.6$, $\beta_4=0.2$, $\beta_5=0.25$, $\beta_6=0.3$, $\beta_7=0.53$. The parameter values were chosen such that $\boldsymbol{\beta}'\boldsymbol{\beta}=1$ which is a common restriction in simulation studies of this type (Muniz and Kibria [12]). We varied the sample sizes between 10, 20, 30, 40 and 50. Three different values of σ: 0.5, 1 and 5 were also used. At a specified value of *n*, *p* and σ, the fixed $\boldsymbol{X}$s are first generated; followed by the $\boldsymbol{U}$, and the values of $\boldsymbol{Y}$ are then obtained using the regression model. The experiment is repeated 1000 times.

3.1 Criterion for Investigation

(Dorugade, [2]). Ridge regression estimator gives a smaller mean squared error when compared to the OLS estimator for a positive value of *k* (Hoerl and Kennard, [5]). The use of ridge regression estimator depends on the ridge parameter, *k*. Different methods for estimating this ridge parameter has been considered by many authors. Some of these authors are: Hoerl and Kennard [5], McDonald and Galarneau [11], Lawless and Wang [8], Gibbons [3], Kibria [7], Khalaf and Shukur [6], Alkhamisi *et al*. [1], Muniz and Kibria [12], Mansson *et al*. [10], Dorugade [2] and recently, Lukman and Ayinde [9].

Several authors in literatures had applied the mean square error (MSE) to evaluate and compare the performance of ridge regression estimator with that of the OLS estimator when there is multicollinearity. Some of these were Hoerl and Kennard [5], Lawless and Wang [8], Saleh and Kibria [14], Kibria [7], Khalaf and Shukur [6], Alkhamisi *et al*. [1], Mansson *et al*. [10]. To investigate whether the ridge estimator is better than the OLS estimator, the AMSE is calculated using the equation defined as:

$$\text{AMSE}(\hat{\boldsymbol{\beta}})=\frac{1}{1000}\sum_{j=1}^{1000}\sum_{i=1}^{p}\left(\hat{\beta}_{ij}-\beta_i\right)^2 \tag{12}$$

where $\hat{\beta}_{ij}$ is the i^{th} element of the estimator $\hat{\boldsymbol{\beta}}$ in the j^{th} replication which gives the estimate of β_i. Each β_i is the true value of the parameter previously mentioned. Estimators with the minimum AMSE are considered best.

This is further examined by computing the relative efficiency of the ridge regression estimator relative to OLS estimator. A sample of the results of $\hat{k}_{\text{AL2}}$ is shown in Appendix 1.

$$\text{Relative Efficiency (RE)}=\frac{\text{AMSE}(\hat{\boldsymbol{\beta}}_{\text{ridge}})}{\text{AMSE}(\hat{\boldsymbol{\beta}}_{\text{OLS}})}. \tag{13}$$

Thus, the smaller the efficiency value the better the ridge parameter. Consequently, ridge parameter estimates whose efficiency was not more than 0.75 are preferred and selected. That is, the ridge estimators whose MSE were better than that of OLS by at least 25% of OLSMSE. Furthermore, the number of times they were preferred (RE ≤ 0.75) over the five levels of multicollinearity and three error variance was counted so as to know the frequency of their efficiency at each level of sample size. Thus, a maximum of fifteen counts was expected. Having further counted over all the sample sizes, most preferred techniques were identified as having high number of counts and the best one has the highest counts.

4. Results and Discussion

Based on the classification of the existing ridge parameters into the different forms and various types of Lukman and Ayinde [9], the frequency of the relative efficiency (RE) of the pro- posed estimators over the levels of multicollinearity and error variance is summarized in Tables 3-9. A sample of the relative efficiency of the ridge parameter and MSE of OLS based on $\hat{k}_{AL2}$ in different forms with various types is given in Appendix 1. Results of five of the most preferred techniques for each of the proposed ridge parameters are summarized in Table 2.

Table 2 Summary of five most preferred techniques over different estimators

Estimators	Preferred Techniques	Estimators	Preferred Techniques
$\hat{k}_{ALK}$	FMO,FMSR, VMO, MSR and VMSR	$\hat{k}_{AL4}$	(FMO, VMO), FMSR, VMSR and AMSR
$\hat{k}_{AL1}$	FMO, VMO, FMSR, VMSR and AMSR	$\hat{k}_{AL5}$	HMO, FMO, FMSR, HMSR and GMSR
$\hat{k}_{AL2}$	FMO, FMSR, GMSR, MSR and VMO	$\hat{k}_{AL6}$	FMO, VMO, FMSR, VMSR and AMSR
$\hat{k}_{AL3}$	FMO, FMSR, GMSR, HMSR and AMSR		

The techniques in parenthesis are equally ranked. The best ridge parameters are in the original and square root types.

Table 3 Frequency of the RE of ridge parameters based on $\hat{k}_{ALK}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	1	15	12	15	13	13	68	1
	R	FMR	7	5	8	8	5	33	21	0	4	6	8	6	24	22
	SR	FMSR	14	14	15	12	10	65	2	7	10	13	13	12	55	2
	RSR	FMRSR	9	8	11	8	5	41	17	0	5	9	8	7	29	18
Varying Maximum	O	VMO	11	11	10	11	10	53	4	2	8	9	11	10	40	6
	R	VMR	6	3	7	7	5	28	22	0	1	2	8	4	15	23
	SR	VMSR	11	12	11	10	10	54	3	2	8	9	11	9	39	7
	RSR	VMRSR	9	5	10	11	9	44	15	0	5	6	11	9	31	15
Arithmetic Mean	O	AMO	13	10	11	7	5	46	13	4	10	10	7	7	38	8
	R	AMR	9	8	11	10	8	46	13	0	5	7	11	10	33	14
	SR	AMSR	11	12	12	8	7	50	6	2	8	12	8	7	37	10
	RSR	AMRSR	11	9	11	8	10	49	8	1	6	9	11	10	37	10

Table 3 (Continued)

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Harmonic Mean	O	HMO	15	0	0	0	0	15	24	9	4	0	0	0	13	24
	R	HMR	7	4	8	9	8	36	20	1	4	3	9	7	24	22
	SR	HMSR	14	11	12	6	6	49	8	5	10	12	6	9	42	5
	RSR	HMRSR	9	8	10	11	8	46	13	1	6	6	11	9	33	14
Geometric Mean	O	GMO	15	9	10	6	3	43	16	6	10	9	1	3	29	18
	R	GMR	8	5	9	10	7	39	19	0	4	6	11	9	30	16
	SR	GMSR	13	11	10	9	6	49	8	3	10	13	9	9	44	4
	RSR	GMRSR	9	8	11	10	8	46	13	0	6	9	11	9	35	12
Median	O	MO	15	0	8	0	0	23	23	5	10	10	0	0	25	20
	R	MR	7	5	10	9	9	40	18	0	4	6	9	9	28	19
	SR	MSR	13	11	12	9	6	51	5	3	11	13	9	9	45	3
	RSR	MRSR	9	8	11	11	8	47	10	0	6	9	11	9	35	12

Table 4 Frequency of the RE of ridge parameters based on $\hat{k}_{AL1}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	13	15	13	12	68	1.5	10	10	15	13	13	61	1.5
	R	FMR	9	7	10	8	5	39	18	0	5	8	7	7	27	17.5
	SR	FMSR	13	11	13	10	10	57	4	6	9	12	13	12	52	3
	RSR	FMRSR	11	9	11	9	7	47	11	0	6	9	9	7	31	13.5
Varying Maximum	O	VMO	15	13	15	13	12	68	1.5	10	10	15	13	13	61	1.5
	R	VMR	6	3	6	7	5	27	22	0	1	2	8	4	15	23
	SR	VMSR	13	11	13	10	10	57	4	5	9	12	13	12	51	4
	RSR	VMRSR	9	5	9	11	9	43	16	0	5	6	11	9	31	13.5
Arithmetic Mean	O	AMO	15	10	13	9	7	54	6.5	10	10	12	7	7	46	7
	R	AMR	8	8	9	11	8	44	14.5	0	4	6	11	9	30	16
	SR	AMSR	13	12	13	10	9	57	4	5	10	13	10	10	48	5.5
	RSR	AMRSR	9	8	11	8	8	44	14.5	0	5	9	11	10	35	10
Harmonic Mean	O	HMO	13	0	0	0	0	13	24	10	2	0	0	0	12	24
	R	HMR	7	5	7	9	8	36	21	0	3	3	9	7	22	22
	SR	HMSR	14	11	12	6	6	49	9	6	10	12	6	8	42	9
	RSR	HMRSR	9	8	9	11	8	45	13	0	5	6	11	9	31	13.5
Geometric Mean	O	GMO	15	9	9	4	1	38	20	10	8	6	0	0	24	20.5
	R	GMR	7	5	9	9	9	39	18	0	3	5	9	7	24	20.5
	SR	GMSR	14	11	13	9	6	53	8	5	10	12	9	9	45	8
	RSR	GMRSR	9	8	11	11	8	47	11	0	5	6	11	9	31	13.5
Median	O	MO	15	0	6	0	0	21	23	10	8	9	0	0	27	17.5
	R	MR	7	5	9	9	9	39	18	0	4	6	9	7	26	19
	SR	MSR	15	11	13	9	6	54	6.5	5	10	15	9	9	48	5.5
	RSR	MRSR	9	8	11	11	8	47	11	0	5	8	11	9	33	11

Table 5 Frequency of the RE of ridge parameters based on $\hat{k}_{AL2}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	1	15	12	15	13	13	68	1
	R	FMR	7	5	8	8	5	33	15.5	0	4	6	8	6	24	14
	SR	FMSR	14	14	15	12	10	65	2	7	10	13	13	12	55	2
	RSR	FMRSR	9	8	11	8	5	41	11.5	0	5	9	8	7	29	13

Table 5 (Continued)

Different Forms	Various Types	Methods	*p* = 3							*p* = 7						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Varying Maximum	O	VMO	11	10	11	11	10	53	4.5	1	7	9	11	10	38	5
	R	VMR	2	0	5	4	2	13	21	0	0	0	4	1	5	23
	SR	VMSR	11	11	11	10	10	53	4.5	1	7	9	11	9	37	6.5
	RSR	VMRSR	4	3	7	9	7	30	17	0	1	2	9	6	18	18
Arithmetic Mean	O	AMO	13	8	11	7	5	44	10	5	9	9	7	7	37	6.5
	R	AMR	8	8	11	11	8	46	9	0	5	6	11	10	32	10
	SR	AMSR	11	9	13	8	7	48	8	2	8	12	7	7	36	8.5
	RSR	AMRSR	11	9	11	8	10	49	7	0	6	9	11	10	36	8.5
Harmonic Mean	O	HMO	0	0	0	0	0	0	24	0	0	0	0	0	0	24
	R	HMR	2	0	5	7	3	17	20	0	0	0	7	4	11	20
	SR	HMSR	14	8	9	4	1	36	14	11	6	0	0	0	17	19
	RSR	HMRSR	6	5	7	9	6	33	15.5	0	3	3	9	7	22	15.5
Geometric Mean	O	GMO	5	0	1	0	0	6	22	9	0	0	0	0	9	21
	R	GMR	4	3	7	8	5	27	18	0	2	3	9	6	20	17
	SR	GMSR	15	12	12	9	6	54	3	8	10	12	6	7	43	3.5
	RSR	GMRSR	6	7	9	11	8	41	11.5	0	5	6	11	9	31	11.5
Median	O	MO	2	0	0	0	0	2	23	8	0	0	0	0	8	22
	R	MR	4	2	7	7	3	23	19	0	3	3	9	7	22	15.5
	SR	MSR	15	11	12	6	6	50	6	8	10	12	6	7	43	3.5
	RSR	MRSR	6	5	9	11	8	39	13	0	5	6	11	9	31	11.5

Table 6 Frequency of the RE of ridge parameters based on $\hat{k}_{AL3}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	*p* = 3							*p* = 7						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	1	15	12	15	13	13	68	1
	R	FMR	7	5	8	8	5	33	23	0	4	6	8	6	24	23
	SR	FMSR	14	14	15	12	10	65	2	7	10	13	13	12	55	2
	RSR	FMRSR	9	8	11	8	5	41	19	0	5	9	8	7	29	16
Varying Maximum	O	VMO	7	8	9	11	12	47	14	0	3	3	9	9	24	23
	R	VMR	11	9	10	10	6	46	15	1	7	9	9	10	36	5
	SR	VMSR	10	8	11	11	10	50	11	0	5	8	11	10	34	11
	RSR	VMRSR	11	10	11	10	6	48	12	1	7	9	10	10	37	3
Arithmetic Mean	O	AMO	9	8	11	11	12	51	9	0	3	3	11	9	26	19
	R	AMR	10	8	10	7	3	38	22	6	5	6	6	5	28	17
	SR	AMSR	11	9	11	12	10	53	5	0	6	8	11	10	35	8
	RSR	AMRSR	11	8	10	7	6	42	17	4	6	9	7	8	34	11
Harmonic Mean	O	HMO	11	8	11	11	11	52	7	0	5	5	11	9	30	14
	R	HMR	10	8	10	7	5	40	20	4	6	6	4	4	24	23
	SR	HMSR	11	10	11	11	10	53	5	0	6	9	11	10	36	5
	RSR	HMRSR	11	10	11	9	6	47	14	2	8	9	7	8	34	11
Geometric Mean	O	GMO	9	8	11	11	12	51	9	0	4	5	11	9	29	16
	R	GMR	10	8	10	7	3	38	22	5	5	6	6	4	26	19
	SR	GMSR	11	9	11	13	10	54	3	0	6	9	11	10	36	5
	RSR	GMRSR	11	8	10	7	6	42	17	3	8	9	7	8	35	8
Median	O	MO	9	8	10	11	12	50	11	0	3	3	10	9	25	21
	R	MR	10	5	7	4	3	29	24	6	5	3	6	6	26	19
	SR	MSR	11	9	11	11	10	52	7	0	5	8	11	10	34	11
	RSR	MRSR	11	8	10	7	6	42	17	4	6	9	7	8	34	11

Table 7 Frequency of the RE of ridge parameters based on $\hat{k}_{\mathrm{AL4}}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	2	15	12	15	13	13	68	2
	R	FMR	7	5	8	8	5	33	14	0	4	6	8	6	24	9
	SR	FMSR	14	14	15	12	10	65	4	7	10	13	13	12	55	4
	RSR	FMRSR	9	8	11	8	5	41	9	0	5	9	8	7	29	8
Varying Maximum	O	VMO	15	15	15	13	11	69	2	15	12	15	13	13	68	2
	R	VMR	2	0	3	3	1	9	21	0	0	0	4	0	4	21
	SR	VMSR	14	14	15	12	10	65	4	7	10	13	13	12	55	4
	RSR	VMRSR	4	3	7	9	9	32	16	0	1	2	9	5	17	16
Arithmetic Mean	O	AMO	15	9	12	7	7	50	6	10	4	2	1	4	21	14
	R	AMR	5	4	7	9	7	32	16	0	1	2	9	7	19	15
	SR	AMSR	15	11	13	10	9	58	5	9	10	12	9	9	49	5
	RSR	AMRSR	9	8	9	10	8	44	8	0	5	6	11	9	31	6
Harmonic Mean	O	HMO	0	0	0	0	0	0	23	0	0	0	0	0	0	23
	R	HMR	2	0	5	4	2	13	20	0	0	0	5	2	7	20
	SR	HMSR	11	5	7	3	0	26	17	10	0	0	0	0	10	18
	RSR	HMRSR	7	4	7	9	9	36	13	0	2	3	9	7	21	14
Geometric Mean	O	GMO	0	0	0	0	0	0	23	0	0	0	0	0	0	23
	R	GMR	3	0	6	7	4	20	18	0	0	0	7	4	11	17
	SR	GMSR	15	9	10	6	6	46	7	15	6	6	0	3	30	7
	RSR	GMRSR	7	5	9	11	8	40	10	0	3	3	9	7	22	12
Median	O	MO	0	0	0	0	0	0	23	0	0	0	0	0	0	23
	R	MR	2	0	5	5	2	14	19	0	0	0	6	3	9	19
	SR	MSR	13	8	10	4	1	36	13	15	5	3	0	0	23	10
	RSR	MRSR	7	5	9	9	9	39	11	0	3	3	9	7	22	12

Table 8 Frequency of the RE of ridge parameters based on $\hat{k}_{\mathrm{AL5}}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	$p = 3$							$p = 7$						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	2	15	12	15	13	13	68	1
	R	FMR	7	5	8	8	5	33	21.5	0	4	6	8	6	24	19.5
	SR	FMSR	14	14	15	12	10	65	3	7	10	13	13	12	55	3
	RSR	FMRSR	9	8	11	8	5	41	18	0	5	9	8	7	29	14.5
Varying Maximum	O	VMO	8	8	9	11	11	47	12	0	3	3	10	9	25	17.5
	R	VMR	7	5	8	8	6	34	20	0	4	6	8	6	24	19.5
	SR	VMSR	11	8	11	11	10	51	9	0	6	8	11	10	35	8
	RSR	VMRSR	9	8	11	8	6	42	17	0	5	9	8	7	29	14.5
Arithmetic Mean	O	AMO	9	8	11	11	11	50	10.5	0	5	5	11	9	30	13
	R	AMR	10	8	10	7	4	39	19	2	5	6	6	4	23	21
	SR	AMSR	11	10	11	13	10	55	6	0	6	9	11	10	36	7
	RSR	AMRSR	11	8	9	7	8	43	15.5	2	8	9	7	7	33	10
Harmonic Mean	O	HMO	15	15	15	15	13	73	1	10	10	15	15	15	65	2
	R	HMR	4	5	8	5	2	24	24	0	2	2	2	1	7	24
	SR	HMSR	13	12	13	12	12	62	4	4	9	12	13	13	51	4
	RSR	HMRSR	11	8	11	7	8	45	13	0	6	6	7	6	25	17.5
Geometric Mean	O	GMO	11	11	12	13	11	58	5	1	6	6	11	10	34	9
	R	GMR	8	6	8	7	4	33	21.5	0	5	6	4	4	19	22.5
	SR	GMSR	11	11	11	11	10	54	7	1	7	9	11	10	38	5
	RSR	GMRSR	11	9	8	7	8	43	15.5	1	7	7	7	6	28	16

Table 8 (Continued)

Different Forms	Various Types	Methods	p = 3							p = 7						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Median	O	MO	9	8	11	11	11	50	10.5	1	5	5	11	9	31	12
	R	MR	10	7	7	4	4	32	23	0	5	4	6	4	19	22.5
	SR	MSR	11	9	11	12	10	53	8	1	6	9	11	10	37	6
	RSR	MRSR	11	8	10	7	8	44	14	1	8	9	7	7	32	11

Table 9 Frequency of the RE of ridge parameters based on $\hat{k}_{AL6}$ estimator while counting over 5 levels of multicollinearity and 3 levels of error variances

Different Forms	Various Types	Methods	p = 3							p = 7						
			10	20	30	40	50	Total	Rank	10	20	30	40	50	Total	Rank
Fixed Maximum	O	FMO	15	15	15	13	11	69	1.5	15	12	15	13	13	68	1.5
	R	FMR	7	5	8	8	5	33	20	0	4	6	8	6	24	19.5
	SR	FMSR	14	14	15	12	10	65	3.5	7	10	13	13	12	55	3.5
	RSR	FMRSR	9	8	11	8	5	41	15	0	5	9	8	7	29	15.5
Varying Maximum	O	VMO	15	15	15	13	11	69	1.5	15	12	15	13	13	68	1.5
	R	VMR	4	2	5	7	5	23	22	0	1	2	8	4	15	23.5
	SR	VMSR	14	14	15	12	10	65	3.5	7	10	13	13	12	55	3.5
	RSR	VMRSR	9	5	9	11	9	43	14	0	5	5	11	9	30	14
Arithmetic Mean	O	AMO	15	12	13	9	7	56	6	15	10	9	4	6	44	8.5
	R	AMR	7	5	9	11	8	40	16	0	3	5	11	9	28	17
	SR	AMSR	15	14	15	10	9	63	5	7	10	15	10	10	52	5
	RSR	AMRSR	9	8	11	8	8	44	13	0	5	6	11	9	31	11.5
Harmonic Mean	O	HMO	13	0	0	0	0	13	24	15	0	0	0	0	15	23.5
	R	HMR	7	3	7	9	8	34	19	0	3	3	9	7	22	21.5
	SR	HMSR	14	11	12	6	6	49	9	7	10	12	6	9	44	8.5
	RSR	HMRSR	9	8	9	11	8	45	12	0	5	6	11	9	31	11.5
Geometric Mean	O	GMO	15	5	7	1	0	28	21	15	8	3	0	0	26	18
	R	GMR	7	5	7	9	9	37	18	0	3	3	9	7	22	21.5
	SR	GMSR	15	11	13	9	6	54	7.5	7	10	12	9	9	47	7
	RSR	GMRSR	9	8	10	11	8	46	10.5	0	5	6	11	9	31	11.5
Median	O	MO	15	0	6	0	0	21	23	15	8	6	0	0	29	15.5
	R	MR	7	4	9	9	9	38	17	0	3	5	9	7	24	19.5
	SR	MSR	15	11	13	9	6	54	7.5	7	10	15	9	9	50	6
	RSR	MRSR	9	8	10	11	8	46	10.5	0	5	6	11	9	31	11.5

5. Conclusion

Some new classification based ridge regression parameter estimation techniques have been proposed and investigated in a linear regression model. A simulation study was conducted to evaluate the performance of these estimators. The performances of the estimators depend on sample size, level of multicollinearity and error variances. Moreover, Fixed Maximum Original (FMO) of Alkhamisi (ALK); Varying Maximum Original (VMO) for AL4 and AL6 and Harmonic Mean Original (HMO) for AL5 are consistently the best. It was observed that the HMO for AL5 perform better than FMO of ALK when the number of explanatory (p) is three and compete favourably as p increase. Consequently, VMO for AL4, VMO for AL6 and HMO for ALS can be used alternatively to the Fixed Maximum original proposed by Alkhamisi *et al.* [1].

Appendix 1

Method	$\sigma = 0.5$					$\sigma = 1$					$\sigma = 5$				
	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999
	$N = 10 \quad \hat{k}_{\text{AL2}} \quad p = 3$														
MSEOLS	0.920	0.864	4.030	22.661	248.616	0.985	2.043	4.315	24.262	266.182	3.034	6.292	13.291	74.738	819.954
FMO	0.548	0.453	0.407	0.380	0.374	0.540	0.447	0.403	0.377	0.370	0.503	0.444	0.410	0.374	0.354
FMR	0.746	0.826	0.929	1.002	0.998	0.746	0.815	0.908	0.973	0.966	0.700	0.682	0.713	0.707	0.667
FMSR	0.550	0.454	0.430	0.537	0.764	0.545	0.451	0.426	0.529	0.744	0.527	0.451	0.424	0.463	0.557
FMRSR	0.647	0.685	0.808	0.975	0.996	0.648	0.679	0.792	0.947	0.964	0.630	0.594	0.637	0.690	0.666
VMO	0.552	0.504	0.871	0.971	0.993	0.618	0.752	0.852	0.944	0.961	0.576	0.649	0.688	0.692	0.665
VMR	1.169	1.235	0.929	1.002	0.998	0.746	0.815	0.908	0.973	0.966	0.700	0.682	0.713	0.707	0.667
VMSR	0.559	0.511	0.744	0.935	0.989	0.528	0.604	0.730	0.909	0.958	0.512	0.549	0.610	0.672	0.663
VMRSR	0.797	0.948	0.808	0.975	0.996	0.648	0.679	0.792	0.947	0.964	0.630	0.594	0.637	0.690	0.666
AMO	0.676	0.563	0.775	0.942	0.990	0.525	0.635	0.760	0.916	0.958	0.516	0.580	0.637	0.678	0.663
AMR	0.664	0.729	0.628	0.753	0.958	0.792	0.697	0.625	0.734	0.927	0.799	0.680	0.570	0.559	0.643
AMSR	0.615	0.518	0.687	0.912	0.987	0.512	0.554	0.675	0.887	0.955	0.503	0.518	0.578	0.660	0.662
AMRSR	0.592	0.620	0.584	0.811	0.973	0.684	0.589	0.579	0.790	0.942	0.684	0.566	0.515	0.596	0.653
HMO	0.855	0.884	0.413	0.413	0.414	0.460	0.418	0.409	0.410	0.410	0.460	0.434	0.420	0.398	0.382
HMR	1.010	1.139	0.853	0.988	0.997	0.765	0.762	0.838	0.959	0.965	0.756	0.670	0.671	0.698	0.667
HMSR	0.717	0.672	0.484	0.644	0.843	0.503	0.454	0.478	0.632	0.965	0.496	0.454	0.460	0.524	0.596
HMRSR	0.731	0.891	0.743	0.956	0.995	0.668	0.643	0.731	0.929	0.963	0.662	0.582	0.600	0.678	0.665
GMO	0.780	0.763	0.613	0.805	0.943	0.481	0.520	0.603	0.785	0.915	0.484	0.509	0.549	0.614	0.644
GMR	0.830	0.983	0.701	0.898	0.990	0.772	0.699	0.694	0.873	0.959	0.776	0.658	0.591	0.643	0.663
GMSR	0.664	0.577	0.604	0.842	0.970	0.504	0.506	0.595	0.821	0.939	0.498	0.488	0.531	0.625	0.653
GMRSR	0.644	0.770	0.642	0.885	0.987	0.674	0.602	0.634	0.860	0.956	0.673	0.566	0.544	0.637	0.661
MO	0.793	0.802	0.796	0.945	0.990	0.545	0.664	0.780	0.919	0.958	0.534	0.603	0.653	0.682	0.664
MR	0.821	1.006	0.669	0.748	0.952	0.821	0.739	0.667	0.730	0.922	0.827	0.721	0.608	0.554	0.639
MSR	0.680	0.620	0.698	0.916	0.987	0.509	0.562	0.685	0.891	0.956	0.501	0.526	0.587	0.663	0.662
MRSR	0.633	0.784	0.593	0.804	0.971	0.702	0.607	0.587	0.783	0.940	0.702	0.583	0.522	0.590	0.651
	$N = 20 \quad \hat{k}_{\text{AL2}} \quad p = 3$														
MSEOLS	0386	0.928	1.889	10.031	106.098	0.482	0.956	1.947	10.338	109.346	0.952	1.888	3.845	20.420	215.986
FMO	0.696	0.569	0.472	0.390	0.378	0.694	0.569	0.475	0.397	0.386	0.674	0.599	0.556	0.528	0.525
FMR	0.715	0.688	0.761	0.923	0.937	0.721	0.704	0.790	0.969	0.986	0.749	0.765	1.024	1.520	1.582
FMSR	0.692	0.560	0.465	0.458	0.657	0.692	0.561	0.469	0.470	0.684	0.687	0.596	0.554	0.663	1.050
FMRSR	0.697	0.618	0.646	0.866	0.932	0.700	0.627	0.667	0.909	0.981	0.722	0.674	0.829	1.413	1.576
VMO	0.692	0.570	0.647	0.839	0.922	0.562	0.576	0.668	0.879	0.971	0.609	0.715	0.921	1.358	1.556
VMR	0.969	1.176	0.761	0.923	0.937	0.721	0.704	0.790	0.969	0.986	0.749	0.765	1.024	1.520	1.582
VMSR	0.691	0.568	0.561	0.784	0.915	0.604	0.537	0.577	0.820	0.963	0.626	0.619	0.756	1.254	1.542
VMRSR	0.666	0.785	0.646	0.866	0.932	0.700	0.627	0.667	0.909	0.981	0.722	0.674	0.829	1.413	1.576
AMO	0.815	0.719	0.559	0.781	0.914	0.569	0.520	0.574	0.816	0.961	0.597	0.623	0.774	1.252	1.540
AMR	0.625	0.613	0.608	0.654	0.888	0.821	0.719	0.615	0.683	0.934	0.831	0.728	0.652	0.976	1.491
AMSR	0.756	0.637	0.528	0.749	0.909	0.623	0.530	0.541	0.782	0.956	0.638	0.599	0.695	1.189	1.531
AMRSR	0.638	0.559	0.554	0.682	0.896	0.761	0.644	0.563	0.712	0.943	0.770	0.665	0.633	1.048	1.507
HMO	0.906	0.915	0.430	0.407	0.411	0.597	0.489	0.435	0.416	0.421	0.604	0.554	0.550	0.580	0.598
HMR	0.803	1.023	0.696	0.881	0.935	0.787	0.711	0.716	0.925	0.984	0.804	0.725	0.822	1.444	1.581
HMSR	0.818	0.768	0.464	0.541	0.745	0.642	0.524	0.471	0.558	0.778	0.649	0.578	0.585	0.816	1.214
HMRSR	0.646	0.714	0.609	0.825	0.930	0.741	0.640	0.624	0.865	0.979	0.755	0.667	0.730	1.331	1.572
GMO	0.871	0.849	0.487	0.637	0.837	0.579	0.497	0.497	0.660	0.878	0.597	0.582	0.656	0.994	1.394
GMR	0.686	0.836	0.629	0.772	0.923	0.805	0.707	0.641	0.809	0.972	0.818	0.717	0.695	1.217	1.561
GMSR	0.790	0.709	0.498	0.678	0.880	0.631	0.526	0.508	0.706	0.925	0.642	0.587	0.644	1.060	1.474
GMRSR	0.630	0.621	0.571	0.746	0.916	0.751	0.639	0.582	0.781	0.964	0.763	0.663	0.665	1.172	1.546
MO	0.889	0.884	0.599	0.812	0.918	0.557	0.537	0.615	0.848	0.966	0.595	0.662	0.849	1.311	1.549
MR	0.713	0.906	0.643	0.627	0.873	0.843	0.754	0.648	0.653	0.919	0.851	0.758	0.670	0.902	1.464
MSR	0.806	0.743	0.540	0.767	0.912	0.611	0.527	0.553	0.801	0.960	0.629	0.604	0.724	1.224	1.537
MRSR	0.619	0.634	0.564	0.664	0.891	0.774	0.661	0.572	0.693	0.937	0.782	0.678	0.632	1.008	1.497

Appendix 1

(Continued 1)

Method	$\sigma = 0.5$					$\sigma = 1$					$\sigma = 5$				
	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999
	$N = 30$ $\hat{k}_{AL2}$ $p = 3$														
MSEOLS	0.211	0.804	1.700	9.536	104.240	0.396	0.826	1.746	9.794	107.059	0.732	1.526	3.226	18.094	197.792
FMO	0.690	0.560	0.470	0.400	0.389	0.685	0.555	0.466	0.396	0.385	0.570	0.433	0.352	0.295	0.286
FMR	0.714	0.704	0.832	1.034	1.050	0.714	0.697	0.815	1.010	1.026	0.759	0.643	0.582	0.584	0.591
FMSR	0.689	0.558	0.479	0.519	0.752	0.686	0.555	0.475	0.512	0.737	0.619	0.467	0.370	0.347	0.451
FMRSR	0.698	0.630	0.705	0.973	1.046	0.698	0.626	0.693	0.951	1.021	0.716	0.575	0.502	0.550	0.588
VMO	0.688	0.575	0.772	0.970	1.038	0.570	0.635	0.757	0.948	1.015	0.457	0.430	0.466	0.551	0.585
VMR	1.119	1.364	0.832	1.034	1.050	0.714	0.697	0.815	1.010	1.026	0.759	0.643	0.582	0.584	0.591
VMSR	0.689	0.575	0.645	0.903	1.029	0.596	0.556	0.634	0.883	1.006	0.541	0.433	0.421	0.516	0.580
VMRSR	0.700	0.899	0.705	0.973	1.046	0.698	0.626	0.693	0.951	1.021	0.716	0.575	0.502	0.550	0.588
AMO	0.813	0.708	0.661	0.912	1.030	0.565	0.554	0.649	0.892	1.007	0.474	0.403	0.420	0.524	0.581
AMR	0.614	0.622	0.625	0.716	0.982	0.832	0.729	0.622	0.701	0.960	0.846	0.732	0.581	0.444	0.552
AMSR	0.754	0.632	0.597	0.866	1.023	0.616	0.540	0.587	0.847	0.999	0.562	0.435	0.404	0.499	0.576
AMRSR	0.637	0.572	0.575	0.763	1.000	0.766	0.645	0.570	0.746	0.977	0.768	0.622	0.485	0.454	0.562
HMO	0.927	0.936	0.456	0.447	0.450	0.588	0.490	0.452	0.442	0.446	0.491	0.380	0.335	0.318	0.319
HMR	0.884	1.176	0.747	0.992	1.048	0.792	0.709	0.736	0.969	1.024	0.824	0.704	0.597	0.566	0.590
HMSR	0.837	0.792	0.497	0.621	0.844	0.636	0.524	0.491	0.610	0.827	0.575	0.434	0.367	0.393	0.495
HMRSR	0.660	0.810	0.653	0.931	1.043	0.743	0.638	0.644	0.909	1.019	0.754	0.609	0.503	0.529	0.586
GMO	0.882	0.859	0.557	0.757	0.957	0.572	0.514	0.549	0.742	0.935	0.480	0.388	0.378	0.456	0.547
GMR	0.693	0.915	0.655	0.863	1.033	0.814	0.711	0.649	0.843	1.009	0.836	0.716	0.577	0.505	0.579
GMSR	0.797	0.713	0.551	0.788	0.992	0.625	0.530	0.543	0.771	0.970	0.568	0.433	0.387	0.465	0.561
GMRSR	0.628	0.663	0.600	0.839	1.025	0.755	0.639	0.594	0.820	1.002	0.762	0.615	0.489	0.487	0.576
MO	0.880	0.865	0.691	0.932	1.033	0.556	0.568	0.679	0.912	1.009	0.459	0.405	0.435	0.536	0.583
MR	0.684	0.934	0.663	0.703	0.969	0.848	0.761	0.660	0.688	0.947	0.861	0.763	0.622	0.445	0.543
MSR	0.800	0.728	0.606	0.878	1.025	0.607	0.537	0.596	0.858	1.002	0.551	0.427	0.407	0.506	0.578
MRSR	0.616	0.648	0.587	0.752	0.995	0.776	0.662	0.582	0.736	0.972	0.778	0.640	0.500	0.449	0.559
	$N = 40$ $\hat{k}_{AL2}$ $p = 3$														
MSEOLS	0.818	0.695	0.571	0.430	0.407	0.816	0.692	0.567	0.427	0.404	0.746	0.598	0.465	0.334	0.314
FMO	0.817	0.712	0.691	0.960	1.031	0.818	0.711	0.684	0.942	1.013	0.857	0.744	0.624	0.598	0.643
FMR	0.818	0.693	0.565	0.465	0.656	0.817	0.691	0.562	0.461	0.647	0.781	0.635	0.488	0.347	0.438
FMSR	0.817	0.699	0.623	0.855	1.022	0.818	0.698	0.619	0.839	1.004	0.836	0.709	0.569	0.536	0.636
FMRSR	0.817	0.691	0.645	0.886	1.011	0.639	0.579	0.635	0.870	0.993	0.580	0.464	0.446	0.555	0.631
VMO	0.857	1.202	0.691	0.960	1.031	0.818	0.711	0.684	0.942	1.013	0.857	0.744	0.624	0.598	0.643
VMR	0.818	0.692	0.558	0.782	0.994	0.722	0.594	0.552	0.768	0.976	0.698	0.540	0.440	0.496	0.620
VMSR	0.703	0.717	0.623	0.855	1.022	0.818	0.698	0.619	0.839	1.004	0.836	0.709	0.569	0.536	0.636
VMRSR	0.903	0.835	0.572	0.814	0.999	0.676	0.564	0.565	0.800	0.981	0.622	0.475	0.420	0.516	0.623
AMO	0.714	0.607	0.741	0.580	0.900	0.917	0.851	0.741	0.573	0.883	0.924	0.858	0.741	0.447	0.558
AMR	0.866	0.768	0.545	0.739	0.984	0.748	0.611	0.541	0.726	0.966	0.722	0.561	0.444	0.474	0.614
AMSR	0.764	0.629	0.653	0.608	0.936	0.875	0.778	0.652	0.599	0.919	0.878	0.777	0.631	0.427	0.582
AMRSR	0.951	0.954	0.496	0.444	0.448	0.723	0.585	0.492	0.441	0.444	0.657	0.499	0.395	0.334	0.336
HMO	0.739	0.989	0.707	0.875	1.027	0.889	0.800	0.705	0.858	1.009	0.908	0.825	0.698	0.564	0.640
HMR	0.902	0.867	0.529	0.533	0.763	0.772	0.635	0.526	0.526	0.751	0.739	0.582	0.447	0.376	0.497
HMSR	0.724	0.663	0.640	0.786	1.015	0.857	0.749	0.637	0.771	0.997	0.867	0.757	0.610	0.504	0.631
HMRSR	0.933	0.917	0.524	0.662	0.904	0.696	0.567	0.519	0.653	0.889	0.637	0.483	0.403	0.441	0.573
GMO	0.698	0.759	0.716	0.699	0.992	0.905	0.829	0.715	0.687	0.974	0.917	0.844	0.719	0.488	0.614
GMR	0.887	0.828	0.535	0.661	0.939	0.759	0.620	0.531	0.651	0.923	0.730	0.570	0.444	0.436	0.589
GMSR	0.738	0.620	0.643	0.677	0.980	0.867	0.765	0.642	0.666	0.963	0.873	0.768	0.620	0.456	0.608
GMRSR	0.946	0.939	0.613	0.864	1.007	0.653	0.565	0.605	0.849	0.990	0.595	0.461	0.434	0.545	0.629
MO	0.701	0.869	0.782	0.576	0.863	0.929	0.876	0.782	0.570	0.847	0.935	0.881	0.780	0.470	0.535
MR	0.898	0.852	0.549	0.767	0.991	0.734	0.597	0.544	0.754	0.973	0.708	0.544	0.438	0.489	0.618
MSR	0.719	0.615	0.674	0.591	0.922	0.884	0.796	0.673	0.583	0.905	0.887	0.793	0.653	0.425	0.572
MRSR	0.818	0.695	0.571	0.430	0.407	0.816	0.692	0.567	0.427	0.404	0.746	0.598	0.465	0.334	0.314

Appendix 1

(Continued 2)

Method	$\sigma=0.5$					$\sigma=1$					$\sigma=5$				
	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999
	$N=50$ $\hat{k}_{AL2}$ $p=3$														
MSEOLS	1.908	0.333	0.686	3.711	39.738	0.168	0.338	0.696	3.768	40.352	0.249	0.502	1.035	5.604	60.002
FMO	0.860	0.756	0.686	0.445	0.410	0.859	0.753	0.627	0.443	0.408	0.810	0.680	0.541	2.044	0.335
FMR	0.868	0.771	0.711	0.953	1.051	0.869	0.772	0.709	0.939	1.037	0.895	0.804	0.696	0.667	0.718
FMSR	0.863	0.756	0.625	0.454	0.617	0.862	0.755	0.623	0.451	0.611	0.838	0.714	0.567	0.369	0.455
FMRSR	0.867	0.762	0.658	0.822	1.039	0.867	0.762	0.656	0.811	1.024	0.880	0.777	0.643	0.581	0.708
VMO	0.859	0.751	0.634	0.875	1.026	0.710	0.614	0.628	0.863	1.012	0.665	0.531	0.489	0.606	0.702
VMR	0.944	1.298	0.711	0.953	1.051	0.869	0.772	0.709	0.939	1.037	0.895	0.804	0.696	0.667	0.718
VMSR	0.862	0.754	0.570	0.745	1.003	0.785	0.653	0.567	0.735	0.989	0.769	0.619	0.496	0.527	0.686
VMRSR	0.765	0.732	0.658	0.822	1.039	0.867	0.762	0.656	0.811	1.024	0.880	0.777	0.643	0.581	0.708
AMO	0.929	0.877	0.574	0.793	1.012	0.740	0.612	0.570	0.783	0.998	0.700	0.547	0.465	0.557	0.692
AMR	0.778	0.651	0.806	0.570	34.610	0.943	0.895	0.806	0.566	0.859	0.947	0.900	0.810	0.501	0.598
AMSR	0.901	0.824	0.570	0.700	0.991	0.805	0.672	0.567	0.691	0.978	0.787	0.640	0.505	0.501	0.679
AMRSR	0.822	0.692	0.720	0.569	0.923	0.911	0.836	0.719	0.564	0.910	0.913	0.837	0.712	0.458	0.632
HMO	0.965	0.966	0.532	0.438	0.439	0.781	0.645	0.529	0.435	0.436	0.732	0.577	0.451	0.353	0.352
HMR	0.798	1.045	0.757	0.847	1.046	0.923	0.854	0.757	0.836	1.031	0.935	0.873	0.769	0.625	0.713
HMSR	0.930	0.902	0.574	0.502	0.734	0.825	0.700	0.572	0.498	0.725	0.803	0.662	0.515	0.392	0.527
HMRSR	0.783	0.688	0.693	0.742	1.028	0.897	0.809	0.692	0.733	1.014	0.904	0.818	0.688	0.544	0.701
GMO	0.953	0.941	0.540	0.632	0.901	0.757	0.621	0.537	0.625	0.889	0.714	0.558	0.451	0.465	0.627
GMR	0.760	0.789	0.779	0.659	0.995	0.935	0.878	0.780	0.653	0.980	0.942	0.889	0.789	0.534	0.677
GMSR	0.919	0.873	0.568	0.619	0.937	0.814	0.684	0.566	0.612	0.924	0.794	0.650	0.508	0.456	0.645
GMRSR	0.797	0.665	0.706	0.630	0.981	0.905	0.824	0.705	0.623	0.967	0.909	0.829	0.700	0.487	0.669
MO	0.968	0.965	0.616	0.859	1.024	0.716	0.608	0.611	0.848	1.010	0.671	0.530	0.481	0.598	0.700
MR	0.772	0.968	0.843	0.587	0.817	0.954	0.916	0.843	0.584	0.806	0.956	0.918	0.843	0.536	0.564
MSR	0.932	0.901	0.567	0.735	1.001	0.790	0.655	0.564	0.726	0.987	0.773	0.621	0.495	0.522	0.685
MRSR	0.775	0.660	0.742	0.558	0.902	0.919	0.851	0.742	0.553	0.890	0.921	0.851	0.734	0.460	0.619
	$N=10$ $\hat{k}_{AL5}$ $p=7$														
MSEOLS	18.560	41.405	91.811	549.321	6229.414	19.622	43.773	37.340	580.746	6585.78	52.21	116.5	258.26	1545.2	17522.9
FMO	0.380	0.369	0.362	0.353	0.348	0.402	0.392	0.385	0.376	0.371	0.679	0.671	0.666	0.658	0.653
FMR	0.874	0.881	0.883	0.883	0.881	1.017	1.028	1.033	1.035	1.035	2.631	2.687	2.716	2.745	2.759
FMSR	0.493	0.550	0.609	0.721	0.809	0.550	0.624	0.701	0.842	0.951	1.244	1.519	1.782	2.242	2.559
FMRSR	0.829	0.857	0.871	0.881	0.881	0.970	1.003	1.021	1.033	1.035	2.563	2.658	2.704	2.744	2.759
VMO	0.870	0.876	0.879	0.881	0.881	1.008	1.021	1.027	1.033	1.035	2.578	2.659	2.701	2.742	2.758
VMR	0.874	0.881	0.883	0.883	0.881	1.017	1.028	1.033	1.035	1.035	2.631	2.687	2.716	2.745	2.759
VMSR	0.815	0.846	0.863	0.878	0.881	0.950	0.989	1.011	1.030	1.035	2.482	2.607	2.674	2.737	2.758
VMRSR	0.829	0.857	0.871	0.881	0.881	0.970	1.003	1.021	1.033	1.035	2.563	2.658	2.704	2.744	2.759
AMO	0.805	0.841	0.860	0.877	0.881	0.931	0.979	1.005	1.028	1.034	2.402	2.566	2.651	2.731	2.757
AMR	0.719	0.744	0.779	0.837	0.873	0.831	0.876	0.922	0.989	1.027	2.051	2.333	2.512	2.689	2.751
AMSR	0.768	0.817	0.846	0.873	0.880	0.895	0.956	0.992	1.025	1.034	2.350	2.535	2.634	2.727	2.757
AMRSR	0.699	0.754	0.797	0.853	0.877	0.821	0.891	0.942	1.005	1.031	2.174	2.420	2.565	2.707	2.754
HMO	0.477	0.487	0.493	0.499	0.501	0.520	0.533	0.541	0.548	0.551	1.061	1.119	1.151	1.181	1.192
HMR	0.843	0.860	0.872	0.881	0.881	0.983	1.007	1.022	1.034	1.035	2.577	2.667	2.709	2.745	2.759
HMSR	0.610	0.670	0.719	0.793	0.844	0.699	0.776	0.838	0.931	1.035	1.754	2.022	2.221	2.499	2.663
HMRSR	0.794	0.834	0.858	0.879	0.881	0.932	0.979	1.008	1.031	1.035	2.488	2.623	2.689	2.742	2.759
GMO	0.696	0.756	0.799	0.851	0.875	0.792	0.873	0.928	0.997	1.028	1.979	2.260	2.442	2.652	2.741
GMR	0.792	0.811	0.832	0.864	0.879	0.925	0.955	0.980	1.017	1.033	2.411	2.568	2.654	2.733	2.758
GMSR	0.712	0.776	0.817	0.863	0.879	0.826	0.907	0.958	1.013	1.032	2.152	2.403	2.550	2.699	2.752
GMRSR	0.747	0.792	0.825	0.866	0.880	0.879	0.934	0.973	1.018	1.034	2.354	2.535	2.635	2.728	2.757
MO	0.697	0.763	0.807	0.858	0.878	0.792	0.879	0.937	1.005	1.031	1.959	2.268	2.462	2.671	2.748
MR	0.811	0.819	0.832	0.859	0.877	0.943	0.962	0.980	1.012	1.031	2.419	2.566	2.648	2.728	2.757
MSR	0.713	0.780	0.822	0.866	0.879	0.827	0.911	0.963	1.016	1.033	2.144	2.406	2.557	2.704	2.754
MRSR	0.750	0.791	0.822	0.862	0.879	0.882	0.933	0.970	1.015	1.033	2.357	2.533	2.631	2.726	2.757

Appendix 1

(Continued 3)

Method	$\sigma = 0.5$					$\sigma = 1$					$\sigma = 5$				
	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999
	$N = 20$ $\hat{k}_{\text{AL5}}$ $p = 7$														
MSEOLS	1.712	3.501	7.280	39.768	427.216	1.756	3.591	7.469	40.799	438.293	3.248	6.643	13.817	75.474	810.794
FMO	0.589	0.510	0.476	0.455	0.447	0.601	0.532	0.502	0.482	0.474	0.782	0.767	0.760	0.754	0.750
FMR	0.674	0.681	0.738	0.847	0.855	0.734	0.776	0.855	0.972	0.984	1.782	2.352	2.649	2.882	2.965
FMSR	0.585	0.498	0.462	0.497	0.634	0.599	0.523	0.498	0.559	0.731	0.789	0.822	0.924	1.395	2.177
FMRSR	0.628	0.606	0.656	0.811	0.853	0.669	0.684	0.761	0.936	0.981	1.357	1.965	2.416	2.844	2.963
VMO	0.622	0.709	0.773	0.838	0.852	0.696	0.804	0.880	0.960	0.980	1.787	2.230	2.518	2.834	2.959
VMR	0.674	0.681	0.738	0.847	0.855	0.734	0.776	0.855	0.972	0.984	1.782	2.352	2.649	2.882	2.965
VMSR	0.535	0.578	0.652	0.792	0.847	0.585	0.656	0.750	0.912	0.975	1.320	1.820	2.236	2.752	2.950
VMRSR	0.628	0.606	0.656	0.811	0.853	0.669	0.684	0.761	0.936	0.981	1.357	1.965	2.416	2.844	2.963
AMO	0.549	0.636	0.717	0.820	0.850	0.606	0.716	0.815	0.939	0.978	1.368	1.870	2.268	2.758	2.950
AMR	0.855	0.776	0.675	0.588	0.752	0.858	0.786	0.703	0.683	0.880	0.904	0.992	1.275	2.252	2.855
AMSR	0.522	0.545	0.617	0.774	0.844	0.562	0.612	0.706	0.891	0.972	1.121	1.587	2.048	2.685	2.941
AMRSR	0.733	0.630	0.562	0.624	0.802	0.743	0.656	0.615	0.733	0.930	0.899	1.099	1.507	2.432	2.899
HMO	0.483	0.477	0.485	0.496	0.499	0.515	0.517	0.529	0.544	0.550	0.875	0.954	1.013	1.085	1.119
HMR	0.797	0.718	0.682	0.794	0.854	0.813	0.769	0.774	0.921	0.982	1.142	1.768	2.357	2.850	2.964
HMSR	0.517	0.486	0.505	0.607	0.735	0.544	0.530	0.565	0.695	0.848	0.900	1.098	1.359	1.984	2.570
HMRSR	0.699	0.616	0.607	0.760	0.850	0.718	0.666	0.694	0.884	0.978	1.043	1.547	2.127	2.787	2.961
GMO	0.522	0.588	0.664	0.786	0.842	0.569	0.657	0.750	0.898	0.969	1.174	1.595	1.995	2.607	2.915
GMR	0.836	0.751	0.657	0.633	0.804	0.841	0.769	0.703	0.744	0.933	0.943	1.161	1.606	2.529	2.924
GMSR	0.518	0.526	0.589	0.749	0.838	0.554	0.587	0.672	0.862	0.965	1.037	1.442	1.891	2.596	2.922
GMRSR	0.721	0.622	0.567	0.656	0.820	0.733	0.654	0.630	0.770	0.948	0.935	1.213	1.685	2.553	2.926
MO	0.548	0.637	0.718	0.820	0.850	0.602	0.714	0.812	0.937	0.978	1.306	1.785	2.189	2.723	2.944
MR	0.864	0.792	0.697	0.602	0.751	0.867	0.803	0.729	0.701	0.880	0.925	1.057	1.394	2.344	2.872
MSR	0.518	0.543	0.616	0.774	0.844	0.557	0.608	0.704	0.890	0.972	1.092	1.540	2.001	2.663	2.937
MRSR	0.740	0.640	0.571	0.627	0.802	0.750	0.666	0.626	0.737	0.930	0.916	1.141	1.568	2.468	2.905
	$N = 30$ $\hat{k}_{\text{AL5}}$ $p = 7$														
MSEOLS	1.124	2.347	4.961	27.782	303.180	1.152	2.405	5.085	28.472	310.719	2.050	4.280	9.047	50.66	552.848
FMO	0.667	0.573	0.527	0.500	0.489	0.664	0.572	0.528	0.501	0.490	0.658	0.594	0.559	0.522	0.501
FMR	0.709	0.730	0.833	1.019	1.048	0.711	0.731	0.835	1.021	1.048	0.715	0.722	0.867	1.050	1.043
FMSR	0.666	0.564	0.520	0.574	0.761	0.665	0.564	0.520	0.573	0.760	0.663	0.572	0.526	0.558	0.735
FMRSR	0.685	0.655	0.735	0.966	1.045	0.686	0.655	0.735	0.968	1.045	0.688	0.637	0.736	0.992	1.040
VMO	0.641	0.779	0.888	1.009	1.044	0.645	0.783	0.891	1.010	1.044	0.709	0.850	0.949	1.034	1.039
VMR	0.709	0.730	0.833	1.019	1.048	0.711	0.731	0.835	1.021	1.048	0.715	0.722	0.867	1.050	1.043
VMSR	0.572	0.624	0.734	0.943	1.036	0.573	0.626	0.736	0.944	1.036	0.591	0.649	0.764	0.964	1.031
VMRSR	0.685	0.655	0.735	0.966	1.045	0.686	0.655	0.735	0.968	1.045	0.688	0.637	0.736	0.992	1.040
AMO	0.583	0.704	0.826	0.987	1.041	0.586	0.707	0.828	0.988	1.042	0.633	0.762	0.879	1.012	1.036
AMR	0.901	0.834	0.739	0.676	0.902	0.901	0.834	0.738	0.673	0.901	0.901	0.825	0.714	0.605	0.879
AMSR	0.571	0.595	0.697	0.921	1.032	0.572	0.596	0.698	0.923	1.033	0.585	0.614	0.721	0.941	1.027
AMRSR	0.804	0.700	0.626	0.730	0.970	0.804	0.700	0.624	0.728	0.970	0.803	0.687	0.595	0.694	0.960
HMO	0.537	0.537	0.555	0.580	0.590	0.538	0.539	0.557	0.581	0.590	0.563	0.566	0.577	0.585	0.581
HMR	0.853	0.774	0.761	0.941	1.046	0.854	0.773	0.758	0.943	1.046	0.852	0.731	0.690	0.964	1.041
HMSR	0.582	0.541	0.569	0.714	0.888	0.581	0.541	0.570	0.714	0.888	0.588	0.552	0.577	0.710	0.875
HMRSR	0.771	0.677	0.677	0.897	1.040	0.771	0.677	0.675	0.898	1.040	0.770	0.650	0.635	0.910	1.035
GMO	0.563	0.654	0.764	0.943	1.031	0.565	0.657	0.767	0.945	1.031	0.604	0.703	0.811	0.968	1.026
GMR	0.887	0.811	0.721	0.737	0.970	0.887	0.811	0.719	0.734	0.970	0.887	0.796	0.679	0.674	0.960
GMSR	0.573	0.578	0.666	0.889	1.023	0.574	0.578	0.667	0.891	1.024	0.584	0.593	0.684	0.906	1.018
GMRSR	0.793	0.690	0.631	0.768	0.994	0.794	0.689	0.629	0.767	0.995	0.793	0.674	0.594	0.742	0.987
MO	0.578	0.699	0.822	0.984	1.041	0.580	0.703	0.825	0.986	1.041	0.628	0.759	0.878	1.011	1.036
MR	0.903	0.842	0.757	0.699	0.904	0.903	0.842	0.755	0.695	0.903	0.903	0.831	0.725	0.618	0.879
MSR	0.568	0.590	0.692	0.920	1.032	0.568	0.591	0.694	0.921	1.032	0.582	0.611	0.718	0.940	1.027
MRSR	0.806	0.707	0.636	0.736	0.970	0.806	0.707	0.635	0.735	0.970	0.805	0.692	0.601	0.697	0.959

Appendix 1

(Continued 4)

Method	$\sigma = 0.5$					$\sigma = 1$					$\sigma = 5$				
	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999	0.8	0.9	0.95	0.99	0.999
	$N = 40$ $\hat{k}_{\text{AL5}}$ $p = 7$														
MSEOLS	0.612	1.225	2.508	13.382	141.652	0.625	1.252	2.562	13.669	144.689	1.047	2.098	4.294	22.908	0.431
FMO	0.793	0.684	0.600	0.542	0.530	0.790	0.680	0.595	0.538	0.527	0.690	0.567	0.492	0.443	0.683
FMR	0.813	0.707	0.675	0.939	1.021	0.815	0.707	0.669	0.923	1.005	0.867	0.752	0.608	0.608	0.448
FMSR	0.800	0.684	0.581	0.506	0.641	0.798	0.681	0.578	0.502	0.633	0.748	0.614	0.502	0.394	0.676
FMRSR	0.809	0.693	0.618	0.835	1.013	0.810	0.693	0.613	0.820	0.996	0.837	0.711	0.562	0.541	0.678
VMO	0.574	0.642	0.760	0.949	1.013	0.569	0.633	0.748	0.934	0.997	0.464	0.456	0.513	0.634	0.683
VMR	0.813	0.707	0.675	0.939	1.021	0.815	0.707	0.669	0.923	1.005	0.867	0.752	0.608	0.608	0.665
VMSR	0.645	0.560	0.584	0.828	0.995	0.643	0.556	0.576	0.814	0.979	0.607	0.476	0.433	0.549	0.676
VMRSR	0.809	0.693	0.618	0.835	1.013	0.810	0.693	0.613	0.820	0.996	0.837	0.711	0.562	0.541	0.676
AMO	0.570	0.597	0.707	0.924	1.010	0.566	0.590	0.697	0.910	0.993	0.475	0.439	0.485	0.619	0.483
AMR	0.958	0.927	0.865	0.630	0.735	0.958	0.927	0.865	0.627	0.723	0.962	0.931	0.868	0.583	0.662
AMSR	0.664	0.562	0.567	0.806	0.991	0.663	0.559	0.561	0.793	0.975	0.626	0.485	0.429	0.536	0.565
AMRSR	0.905	0.834	0.727	0.574	0.862	0.905	0.834	0.726	0.568	0.848	0.908	0.835	0.715	0.452	0.422
HMO	0.599	0.532	0.527	0.553	0.565	0.596	0.527	0.522	0.548	0.559	0.503	0.418	0.401	0.414	0.675
HMR	0.935	0.877	0.773	0.771	1.013	0.936	0.878	0.773	0.758	0.997	0.949	0.899	0.790	0.530	0.546
HMSR	0.699	0.581	0.526	0.599	0.802	0.697	0.578	0.521	0.592	0.789	0.652	0.508	0.424	0.427	0.663
HMRSR	0.884	0.795	0.677	0.715	0.998	0.885	0.795	0.676	0.703	0.982	0.896	0.807	0.665	0.484	0.666
GMO	0.575	0.570	0.655	0.872	0.995	0.571	0.564	0.646	0.858	0.979	0.483	0.429	0.459	0.587	0.548
GMR	0.951	0.915	0.840	0.614	0.845	0.952	0.915	0.841	0.609	0.830	0.958	0.923	0.848	0.538	0.652
GMSR	0.675	0.566	0.554	0.767	0.977	0.673	0.563	0.548	0.755	0.961	0.635	0.492	0.426	0.514	0.594
GMRSR	0.899	0.823	0.711	0.593	0.905	0.899	0.823	0.710	0.586	0.889	0.904	0.826	0.700	0.449	0.677
MO	0.568	0.613	0.729	0.935	1.011	0.563	0.605	0.719	0.920	0.995	0.467	0.445	0.499	0.627	0.475
MR	0.961	0.934	0.878	0.648	0.719	0.961	0.935	0.878	0.645	0.707	0.965	0.938	0.881	0.608	0.663
MSR	0.657	0.559	0.572	0.815	0.993	0.655	0.555	0.565	0.802	0.977	0.617	0.479	0.429	0.543	0.560
MRSR	0.909	0.842	0.737	0.575	0.855	0.909	0.842	0.736	0.569	0.840	0.912	0.842	0.726	0.457	0.431
	$N = 50$ $\hat{k}_{\text{AL5}}$ $p = 7$														
MSEOLS	0.520	1.082	2.279	12.668	137.520	0.528	1.099	2.315	12.868	139.689	0.785	1.634	3.441	19.128	207.649
FMO	0.795	0.669	0.571	0.498	0.485	0.793	0.666	0.569	0.496	0.483	0.732	0.609	0.528	9.036	0.459
FMR	0.805	0.720	0.724	0.920	1.009	0.806	0.721	0.721	0.912	1.001	0.844	0.736	0.676	0.791	0.875
FMSR	0.797	0.670	0.567	0.521	0.664	0.796	0.668	0.565	0.518	0.660	0.766	0.634	0.535	0.480	0.589
FMRSR	0.802	0.694	0.659	0.833	1.000	0.802	0.694	0.656	0.826	0.992	0.822	0.699	0.618	0.716	0.867
VMO	0.587	0.635	0.737	0.923	1.000	0.585	0.631	0.731	0.916	0.992	0.546	0.565	0.644	0.803	0.868
VMR	0.805	0.720	0.724	0.920	1.009	0.806	0.721	0.721	0.912	1.001	0.844	0.736	0.676	0.791	0.875
VMSR	0.660	0.584	0.611	0.816	0.982	0.659	0.582	0.608	0.810	0.974	0.643	0.547	0.547	0.708	0.851
VMRSR	0.802	0.694	0.659	0.833	1.000	0.802	0.694	0.656	0.826	0.992	0.822	0.699	0.618	0.716	0.867
AMO	0.593	0.594	0.684	0.894	0.995	0.591	0.591	0.679	0.887	0.988	0.555	0.537	0.604	0.780	0.864
AMR	0.947	0.905	0.828	0.639	107.012	0.947	0.905	0.828	0.637	0.772	0.951	0.908	0.827	0.596	0.668
AMSR	0.681	0.583	0.592	0.793	0.977	0.680	0.581	0.589	0.787	0.969	0.662	0.549	0.535	0.690	0.847
AMRSR	0.892	0.807	0.698	0.629	0.866	0.892	0.807	0.698	0.626	0.860	0.895	0.807	0.687	0.561	0.747
HMO	0.623	0.543	0.529	0.552	0.567	0.621	0.541	0.526	0.550	0.565	0.577	0.501	0.489	0.512	0.524
HMR	0.924	0.858	0.779	0.806	1.000	0.924	0.859	0.778	0.800	0.992	0.936	0.871	0.768	0.696	0.866
HMSR	0.709	0.589	0.542	0.619	0.801	0.708	0.587	0.540	0.615	0.796	0.683	0.557	0.502	0.554	0.703
HMRSR	0.873	0.776	0.683	0.750	0.985	0.874	0.776	0.682	0.745	0.977	0.882	0.781	0.663	0.648	0.852
GMO	0.600	0.575	0.641	0.842	0.979	0.598	0.572	0.637	0.836	0.971	0.561	0.524	0.572	0.738	0.850
GMR	0.940	0.892	0.806	0.661	0.858	0.940	0.892	0.806	0.658	0.852	0.946	0.897	0.806	0.599	0.737
GMSR	0.690	0.584	0.578	0.761	0.962	0.689	0.582	0.575	0.755	0.955	0.669	0.551	0.526	0.664	0.835
GMRSR	0.886	0.797	0.690	0.652	0.901	0.886	0.798	0.689	0.649	0.894	0.891	0.799	0.677	0.576	0.777
MO	0.588	0.602	0.698	0.904	0.997	0.586	0.598	0.693	0.897	0.989	0.549	0.543	0.617	0.789	0.866
MR	0.951	0.913	0.842	0.652	0.769	0.951	0.913	0.842	0.650	0.764	0.955	0.916	0.842	0.610	0.660
MSR	0.675	0.579	0.594	0.800	0.978	0.674	0.577	0.591	0.794	0.971	0.655	0.545	0.536	0.697	0.849
MRSR	0.896	0.815	0.707	0.630	0.861	0.896	0.815	0.707	0.627	0.855	0.899	0.814	0.697	0.562	0.742

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