

L^p -HARDY–RELLICH AND UNCERTAINTY PRINCIPLE INEQUALITIES ON THE SPHERE

ABIMBOLA ABOLARINWA^{1*} and TIMOTHY APATA²

Communicated by D. M. Pellegrino

ABSTRACT. In this paper, we study the Hardy–Rellich type inequalities and uncertainty principle on the geodesic sphere. Firstly, we derive L^p -Hardy inequalities via divergence theorem, which are in turn used to establish the L^p -Rellich inequalities. We also establish Heisenberg uncertainty principle on the sphere via the Hardy–Rellich type inequalities. The best constants appearing in the inequalities are shown to be sharp.

REFERENCES

1. E. Berchio, L. D’Ambrosio, D. Ganguly, and G. Grillo, *Improved L^p -Poincaré inequalities on the hyperbolic space*, Nonlinear Anal. **157** (2017), 146–166.
2. H. Brezis and J. L. Vázquez, *Blow-up solutions of some nonlinear elliptic problems*, Rev. Mat. Univ. Comp. Madrid **10** (1997), no. 2, 443–469.
3. G. Carron, *Inégalités de Hardy sur les variétés riemanniennes non-compactes*, J. Math. Pures Appl., **76** (1997), no. 10, 883–891.
4. I. Chavel, *Eigenvalues in Riemannian geometry*. New York: Academic Press, 1984.
5. I. Chavel, *Riemannian geometry: a modern introduction*, second edition Cambridge Tracts in Mathematics, 108. Cambridge University Press, Cambridge, 2006.
6. F. Dai and Y. Xu, *The Hardy-Rellich inequality and uncertainty principle inequalities on the sphere*, <http://arxiv.org/abs/1212.3887v3> [math.CA], 2014.
7. E. B. Davies and A. M. Hinz, *Explicit constants for Rellich inequalities in $L^p(\Omega)$* , Math. Z. **227** (1998), no. 3, 511–523.

Copyright 2018 by the Tusi Mathematical Research Group.

Date: Received: Jan 01, 2018; Accepted: Apr. 15, 2018.

*Corresponding author .

2010 *Mathematics Subject Classification.* Primary 26D10; Secondary 46E30, 53C21.

Key words and phrases. Hardy inequalities, Rellich inequalities, uncertainty principle, geodesic sphere, compact manifold.

8. M. M. Fall and F. Mahmoudi, *Weighted Hardy inequality with higher dimensional singularity on the boundary*, Calc. Var. Partial Differential Equations **50** (2014), no. 3-4, 779–798.
9. G. B. Folland and A. Sitaram, *The uncertainty principle: a mathematical survey*, J. Fourier Anal. Appl. **3** (1997), no. 3, 207–233.
10. S. Gallot, D. Hulin, and J. Lafontaine, *Riemannian geometry*, Universitext. Springer-Verlag, Berlin, 1987.
11. N. Garofalo and E. Lanconelli, *Frequency functions on the Heisenberg group, the uncertainty principle and unique continuation*, Ann. Inst. Fourier (Grenoble) **40** (1990), no. 2, 313–356.
12. S. S. Goh and T. N. Goodman, *Uncertainty principles and asymptotic behavior*, Appl. Comput. Harmon. Anal. **16** (2004), no. 1, 69–89.
13. I. Kombe, *Hardy, Rellich and uncertainty principle inequalities on Carnot groups*, arXiv:math/0611850, 2006.
14. I. Kombe and M. Özaydin, *Improved Hardy and Rellich inequalities on Riemannian manifolds*, Trans. Amer. Math. Soc. **361** (2009), no. 12, 6191–6203.
15. I. Kombe and M. Özaydin, *Hardy-Poincaré, Rellich and uncertainty principle inequalities on Riemannian manifolds*, Trans. Amer. Math. Soc. **365** (2013), no. 10, 5035–5050.
16. I. Kombe and A. Yener, *Weighted Hardy and Rellich type inequalities on Riemannian manifolds*, Math. Nachr. **289** (2016), no. 8-9, 994–1004.
17. P. Lindqvist, *On the equation $\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda|u|^{p-2}u = 0$* , Proc. Amer. Math. Soc. **109** (1990), no. 1, 157–164.
18. F. J. Narcowich and J. D. Ward, *Nonstationary wavelets on the m -sphere for scattered data*, Appl. Comput. Harmon. Anal. **3** (1996), no. 4, 324–336.
19. A. Sitaram, M. Sundari, and S. Thangavelu, *Uncertainty principles on certain Lie groups*, Proc. Indian Acad. Sci. Math. Sci. **105** (1995), no. 2, 135–151.
20. X. Sun and F. Pan, *Hardy type inequalities on the sphere*, J. Ineq. Appl. **2017**, Paper No. 148, 8 pp.
21. Y. Xiao, *Some Hardy inequalities on the sphere*, J. Math. Inequal. **10** (2016), no. 3, 793–805.
22. Q. Yang, *Best constants in the Hardy–Rellich type inequalities on the Heisenberg group*, J. Math. Anal. Appl. **342** (2008), no. 1, 423–431.
23. Q. Yang, D. Su, and Y. Kong, *Hardy inequalities on Riemannian manifolds with negative curvature*, Commun. Contemp. Math. **16** (2014), no. 2, 1350043, 24 pp.

¹DEPARTMENT OF PHYSICAL SCIENCES LANDMARK UNIVERSITY, P. M. B. 1001, OMU-ARAN, KWARA STATE, NIGERIA.

E-mail address: A.Abolarinwa1@gmail.com

²DEPARTMENT OF MATHEMATICS AND STATISTICS, OSUN STATE COLLEGE OF TECHNOLOGY, P. M. B. 1011, ESA-OKE, OSUN STATE, NIGERIA.

E-mail address: rocktm24@gmail.com