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SOME NEW ADJUSTED RIDGE ESTIMATORS OF LINEAR REGRESSION MODEL

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ABSTRACT

The ridge estimator for handling multicollinearity problem in linear regression model requires the use the biasing parameter. In this paper, some new adjusted ridge parameters which do not require the biasing parameter are proposed. The performances of the proposed Adjusted Ridge Estimators are compared with a recently proposed Adjusted Ridge Estimator, Generalized Ridge Regression Estimator (GRRE), Ordinary Ridge Regression Estimator (ORRE) and Ordinary Least Square estimator (OLSE) via Monte Carlo study by counting the number of times each estimator has smallest Mean Square Error (MSE) in ten thousand (10,000) replications. The proposed Adjusted Ridge Estimator is most efficient especially when multicollinearity is severe and the error variance is high.

Keywords: Adjusted Ridge estimator, Generalized Ridge Estimator, Ordinary Ridge Estimator

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1. INTRODUCTION

In linear regression model, one of the crucial conditions for the application of Ordinary Least Squares (OLS) estimator to estimate the model parameters is that the explanatory variables are not strongly or perfectly correlated. With strong correlations or linear relationship among the explanatory variables, multicollinearity problem arises. In practice, this problem is inherent in most economic functions due to the nature of economic variables moving together over time (Kuotsoyiannis, 2003). It is therefore common in time series data, even though, it might occur in cross-sectional data as well. For example in time series data, exchange rate and inflation rate tends to increase together as time increases.

There is no conclusive evidence as regards the degree of multicollinearity that will affect the parameter estimates when collinearity is present (Kuotsoyiannis, 2003). The seriousness of this problem is a function of the degree of intercorrelation. The use of OLS estimator in this situation produces unstable and imprecise parameter estimates, questionable predictions and invalid statistical inferences about the model parameters (Kuotsoyiannis, 2003).

Various corrective measures have been discussed in literature to handle the problem of multicollinaerity. Brown *et al.* (1973) suggested that the data disaggregation will reduce the level of multicollinearity. Gujarati (1995) suggested that increasing the sample sizes will reduce the degree of multicollinearity. However, in practice, these are not often feasible but the possibility cannot be overlooked. A simple way to handle this problem is to drop one of the collinear variables. Johnston (1972) stated that deleting a relevant variable may lead to specification bias. Biased estimation techniques such as principal component regression (Massey, 1965), Stein estimator (Stein, 1956), ridge regression (Hoerl and Kennard, 1970), Liu estimator (Liu, 1993) had been suggested in literature to handle this problem. In this study, ridge regression estimator proposed by Hoerl and Kennard (1970) is considered which requires the addition of a positive constant, k, is added to the diagonal elements of the matrix $X'X$ to ill-condition the matrix so as to reduce multicollinearity which in turn makes the mean squared error (MSE) for ridge regression to be smaller than the MSE of OLS. Different estimators for k have been suggested by different authors at different period of times. These include Hoerl and Kennard (1970), Hoerl *et al.* (1975), McDonald and Galarneau (1975), Lawless and Wang (1976), Hocking *et al.* (1976), Dempster *et al.* (1977), Wichern and Churchill (1978), Gibbons (1981), Nordberg (1982), Saleh and Kibria (1993), Haq and Kibria (1996), Singh and Tracy (1999), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi*et al.* (2006), Alkhamisi and Shukur (2008), Muniz and Kibria (2009), Dorugade and Kashid (2010), Mansson *et al.* (2010), Khalaf (2013), Ghadhan and Mohamed (2014), Dorugade (2014), Lukman and Ayinde (2016), Adnan *et al.* (2016) and Dorugade (2016).

Dorugade (2016) modified the Ordinary Ridge Regression estimator and provided an estimator which avoids computing the ridge parameter, k. The modified estimator is referred to as adjusted ridge parameter. It is most efficient when multicollinearity is severe. This study also provides some adjusted estimators following the idea of Dorugade (2016). The performances of the proposed estimators with some existing ones were compared. This article is organized as follows. Section 2 discusses the linear regression model and their estimators including the proposed ones. Simulation study is provided in Section 3 and its result discussed in Section 4. Finally, a concluding remark is made in section 5.

2.0. MODEL AND ESTIMATORS

Consider the multiple linear regression model:

$$y = X\beta + u \quad (1)$$

Where y is an n×1 vector of response variable, X is an n×p full rank matrix of known regressors variables augmented with a column of ones. β is p×1 vector of the unknown regression coefficients and u is the nx1 vector of error terms such that $u \sim (0, \sigma^2 I_n)$ and I_n is an nxn identity matrix.

The OLS estimator of β is defined as:

$\hat{\beta} = (X'X)^{-1}X'y$ (2)

Assume that the response variable y is centered and the regressors X's are standardized. Let Λ and T be the matrices of eigen values and eigen vectors of $X'X$ respectively such that $T'X'XT = \Lambda$ = diagonal ($\lambda_1, \lambda_2, \ldots\lambda_p$), where λ_1 represents the i[th] eigenvalue of $X'X$ and $T'T = TT' = I_p$. The equivalent model for equation (1) is

$y = Z\alpha + u$ (3)

Where $Z = XT$ such that $Z'Z = \Lambda$ and $\alpha = T'\beta$.

The OLS estimator of α is defined as:

$\hat{\alpha} = (Z'Z)^{-1} Z'Y = \Lambda^{-1} Z'Y$ (4)

The relationship between the OLS estimator of β and α is given as:

$\hat{\beta} = T\hat{\alpha}$. (5)

To circumvent the problem of multicollinearity and improve the OLS estimator, Hoerl and Kennard (1970) suggested the ridge estimator as an alternative method of parameter estimation by adding a biasing parameter, k, to the diagonal elements of the $X'X$ matrix in equation (2). The Generalized ridge regression estimator (GRRE) was suggested by them require varying ridge parameter to each regressors in the diagonal elements of $X'X$ matrix. Stephen and Christopher (2011), among many others, claimed that the estimator yields a smaller MSE when compared with OLS in the presence of multicollinearity.

The GRRE of α is defined by Dorugade (2016) as:

$$\hat{\alpha}_{GR} = (I - K(\Lambda + K)^{-1})\hat{\alpha} \quad (6)$$

Where $K = \text{diag}(k_1, k_2 \ldots k_p), k_i \geq 0$, i = 1, 2,…,p. Hence, GRRE for β is

$$\hat{\beta}_{GR} = T\hat{\alpha}_{GR} \quad (7)$$

The mean square error of GRRE is

$$\text{MSE}(\hat{\alpha}_{GR}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k_i)^2} + \sum_{i=1}^{p} \frac{k_i^2 \hat{\alpha}_i^2}{(\lambda_i + k_i)^2} \quad (8)$$

GRRE reduces to Ordinary Ridge Regression Estimator (ORRE) by adding a fixed ridge parameter to the diagonal elements of the $X'X$ matrix. That is when $k_1 = k_2 \ldots = k_p = k$ and $k \geq 0$. The mean square error of ORRE becomes

$$\text{MSE}(\hat{\alpha}_{ORR}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\hat{\alpha}_i^2}{(\lambda_i + k)^2} \quad (9)$$

The MSE of ORR becomes MSE of OLS when k=0. The MSE of OLS is given as:

$$\text{MSE}(\hat{\alpha}_{LS}) = \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \quad (10)$$

Some of the well-known methods of estimating the ridge parameter are presented as follows. Hoerl and Kennard (1970) for the GRRE proposed

$$K_{HK_i} = \frac{\sigma^2}{\alpha_i^2} \quad (11)$$

They also suggested estimating ridge parameter k by taking the maximum (Fixed Maximum) of α_i^2. This is provided as:

$$\hat{K}_{HK}^{FM} = \frac{\hat{\sigma}^2}{Max(\hat{\alpha}_i{}^2)} \quad (12)$$

The non-linear function of the ridge parameter k is a major drawback of the ridge regression estimator (Liu, 1993; Dorugade, 2016). This makes it very difficult to choose a value for k even though different authors have suggested different techniques of estimation. Dorugade (2016) suggested a modification of the Ordinary Ridge Regression estimator where calculating the ridge parameter k can be avoided. The proposed estimator was obtained by adding the diagonal elements of the correlation coefficient between X and Y. The adjusted ridge regression estimator (ARRE) suggested by Dorugade (2016) is given as:

$$\hat{\alpha}_{AR1} = (I - C(\Lambda + K)^{-1})\hat{\alpha} \quad (13)$$

Or

$$\hat{\alpha}_{AR1} = (\Lambda + C)^{-1} Z'Y \quad (14)$$

Where C= $diag\left[\left(|Z'Y|\right)^{0.5}\right]$

The ARRE of β is

$$\hat{\beta}_{AR1} = T\hat{\alpha}_{AR1} \quad (15)$$

The MSE of ARRE is given as:

$$MSE(\hat{\alpha}_{AR1}) = \sum_{i=1}^{p} \frac{\hat{\sigma}^2 \lambda_i + (\hat{\alpha}_i c_i)^2}{(\lambda_i + c_i)^2} \quad (16)$$

Where $\hat{\alpha}_i$, i=1,2,…p is the i[th] element of OLS estimator of α and $\hat{\sigma}^2 = \frac{Y'Y - \hat{\alpha}'Z'Y}{n-p}$. It was established that the MSE ($\hat{\alpha}_{ORR}$) ≥ $MSE(\hat{\alpha}_{AR1})$ if and only if $(\lambda_i + c_i)^2 k^2 \geq (\lambda_i + k)^2 c_i^2$.

2.1. Proposed Adjusted Ridge Regression Estimators

Following Dorugade (2016), the following estimators are proposed:

$$\hat{\alpha}_{ARi} = (\Lambda + C_i)^{-1})Z'Y \quad (17)$$

Where C_i= $diag\left[\left(|Z'Y|\right)^{1/k_i}\right]$ for i=1,2,3…8.

The value of k_i is defined as follows: k_1=2, k_2=p, k_3=n, k_4=np, $k_5=p^p$, $k_6=n^p$, $k_7=(np)^p$, $k_8=(np)^{p+5}$

It should be noted that with k_1=2, the estimator becomes the adjusted ridge estimator proposed by Dorugade (2016) and that the k_i increases as i increases. Thus, $C_i \to I_n$ as $k_i \to \infty$; This is often the case for i= 4, 5, 6, 7 and 8. More so, the addition of 5 to p in k_8 is a further attempt to force C_8 to an identity matrix.

3.0. SIMULATION STUDY

The performances of the proposed adjusted ridge estimators are compared with adjusted ridge estimator proposed by Dorugade (2016), GRRE, ORRE by Hoerland Kennard (1970) and OLS estimator via Monte Carlo study. Time series processor (TSP) software was used to write the programme for the simulations study. The mean square error of the estimators was compared at varying degree of multicollinearity, sample sizes and error variances as recently done by Dorugade (2016).

To generate the varying degree of multicollinearity among the regresors, the procedure adopted by McDonald and Galarneau (1975), Wichern and Churchill (1978), Gibbons (1981) and Dorugade (2016) was used.

$$X_{ti} = (1-\rho^2)^{\frac{1}{2}} Z_{ti} + \rho Z_{tp},\ t=1, 2, 3\ldots n.\ i=1, 2, p. \quad (18)$$

Where Z_{ti} are independent standard pseudo-random numbers, ρ represents the correlation between any two regressors taken as 0.6, 0.8, 0.9, 0.95, 0.99, 0.999 and 0.9999; p is the number of regressors taken to be either three (3) or seven (7). The dependent variable for the n observations are determined using the model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + U_i \qquad t = 1,2,\ldots,n\,; \quad (19)$$

Where$U_i \sim N(0, \sigma^2)$. The true values of the parameter when p=3 are β_0=14, β_1=5, β_2=2 and β_3=6. When p=7, the values of β were β_1=0.4, β_2=0.1, β_3=0.6, β_4=0.2, β_5=0.25, β_6=0.3, β_7=0.53. Sample sizes were varied between 10, 20, 30, 40, 50 and 100. Ten thousand simulations are run for different values of σ^2= 0.25, 1, 9, 25 and 100. The mean square error of the estimators at each replication was computed using the following equation:

$$MSE(\hat{\beta}) = \sum_{i=1}^{p} \left(\hat{\beta}_i - \beta_i\right)^2 \quad (20)$$

Where the estimator $\hat{\beta}_i$ provides the ith estimate of β; and β_i is the true value of the parameter previously mentioned. The number of times at which each estimator has the smallest mean square error, $MSE(\hat{\beta})$, in ten thousand replication is counted. A sample of this is provided in Table 1 and 2 when n=10 and 20 respectively. Furthermore, for ease of results presentation on the performance of the estimators, the following classifications were done. The sample size (n) was classified into small (10 and 20), moderate (30, 40 and 50) and high (100). The multicollinearity level were classified into moderate (ρ=0.6 and 0.8), high (ρ=0.9 and 0.95) and severe (ρ=0.99, 0.999 and 0.9999). Also, the error variances were classified into low (σ^2=0.25 and 1), moderate (σ^2=9 and 25) and high (σ^2=100). The number of times each estimator has the highest number of smallest MSE on the basis of the classifications are counted. Thus, the higher the number the better the estimator. Moreover, each combination of classification has its expected number of counts. For instance, the expected number of counts with small sample (10 and 20), moderate multicollinearity (ρ=0.6 and 0.8) and low level of error variance (σ^2=0.25 and 1) is eight (2×2×2=8).

4.0. RESULTS AND DISCUSSION

The results from the simulation study are presented pictorially in Figure 1 to 10. From Figure 1 to 10, it was observed that as the sample sizes, level of autocorrelation and standard error increases the performances of some estimators are affected. Moreover, AR8 and GRR perform well and occasionally AR1. The results of the highest number of smallest MSE on the basis of the different classification is provided in Table 3. From Table 3, the generalized ridge regression (GRR) estimator performs consistently well when the multicollinearity level is moderate at the different level of error variance. The proposed estimator, AR8, performs especially when the multicollinearity level is moderate and severe coupled with both moderate and high level of variance. Irrespective of the sample sizes. The performance is not satisfactory when the error variance is small but the adjusted ridge estimator, AR1, proposed by Dorugade

(2016) perform consistently well under this condition. Ocassionally, the proposed estimator, AR2, competes favourably with AR1 especially when the number of regressors increased. GRR and AR8 perform equally especially when the sample size is low and there is high variance. It was also observed that AR7 and AR8 perform equally especially when the sample sizes and the number of regressors increases. It was observed that increasing the multicollinearity level and error variances improves the performance of adjusted ridge regression estimator over the existing ones.

Table 1: Number of times each estimator has smallest MSE in ten thousand replications when n=10

ρ	Estimator	p=3					p=7				
		σ^2									
		0.25	1	9	25	100	0.25	1	9	25	100
0.6	OLS	4540	4590	3939	3109	1494	4320	2594	373	57	0
	GRR	4187	2451	850	1026	2694	2727	1798	1646	1710	1662
	ORR	831	868	486	354	281	701	721	116	15	55
	AR1	441	1859	1929	1526	814	1624	2450	1585	386	18
	AR2	1	223	1138	910	577	28	36	233	193	19
	AR3	0	8	698	638	420	33	20	31	31	4
	AR4	0	1	264	293	202	83	48	28	26	3
	AR5	0	0	71	76	56	8	14	3	2	0
	AR6	0	0	13	11	6	0	0	0	0	0
	AR7	0	0	0	1	0	201	602	892	1029	593
	AR8	0	0	612	2056	3456	387	2042	5514	6950	7945
0.8	OLS	4737	4474	2337	1310	573	1975	674	16	0	0
	GRR	3972	2214	1694	2446	3513	1920	1158	761	829	1025
	ORR	664	706	577	392	349	834	483	27	8	22
	AR1	598	1613	1249	597	162	2814	2621	250	33	1
	AR2	29	921	1216	654	212	1599	1040	235	52	3
	AR3	0	66	906	605	223	320	591	82	21	0
	AR4	0	5	412	332	133	155	506	106	19	1
	AR5	0	0	104	87	44	14	54	14	1	0
	AR6	0	0	13	16	3	0	0	0	0	0
	AR7	0	0	0	0	0	214	1094	982	420	137
	AR8	0	1	1492	3561	4788	206	2075	7883	8796	8901
0.9	OLS	3967	2650	817	423	423	488	77	0	0	0
	GRR	3812	3285	2354	2549	2549	1071	592	304	418	581
	ORR	1473	1172	526	319	319	389	120	6	2	11
	AR1	517	1006	480	155	155	5141	2000	44	4	0
	AR2	231	1502	869	335	335	2178	2897	181	26	0
	AR3	0	318	910	415	415	233	532	84	14	2
	AR4	0	32	527	225	225	197	460	115	9	1
	AR5	0	12	138	68	68	20	51	21	0	0
	AR6	0	0	22	11	11	0	0	0	0	0
	AR7	0	0	2	0	0	137	1141	853	239	51

	AR8	0	23	3355	5500	5500	186	2427	8717	9386	9389
0.95	OLS	1519	1164	287	146	67	65	7	0	0	0
	GRR	5967	3916	1971	1788	1459	625	309	131	181	254
	ORR	650	564	330	251	202	163	20	2	3	5
	AR1	1222	925	289	78	45	6866	2061	22	3	0
	AR2	642	2882	1031	381	137	1561	2933	188	23	1
	AR3	0	518	1562	625	208	452	834	136	17	2
	AR4	0	21	907	471	129	194	855	172	19	1
	AR5	0	1	217	157	51	14	103	22	3	0
	AR6	0	2	35	29	9	0	0	0	0	0
	AR7	0	0	2	0	0	25	1038	683	162	70
	AR8	0	7	3369	6074	7693	42	2107	8886	9663	9712
0.99	OLS	1234	502	82	48	22	6	1	0	0	0
	GRR	4273	2669	1189	749	310	339	126	14	10	17
	ORR	1299	839	339	360	290	72	8	2	2	0
	AR1	2639	3036	764	260	80	6045	1651	33	4	0
	AR2	505	1946	2120	1073	315	2989	3960	462	87	7
	AR3	4	122	1174	896	317	210	816	290	62	3
	AR4	3	25	388	407	185	106	600	389	101	6
	AR5	0	2	63	96	48	10	62	52	9	2
	AR6	0	1	15	11	10	0	0	0	0	0
	AR7	0	0	0	1	1	102	981	1400	757	510
	AR8	43	858	3866	6099	8422	161	2138	7676	9145	9583
0.999	OLS	344	115	29	14	2	0	0	0	0	0
	GRR	2650	1478	370	227	57	76	16	1	2	2
	ORR	670	513	451	295	183	9	3	1	1	0
	AR1	4339	4544	3989	3229	1922	5235	2935	450	112	8
	AR2	53	75	628	1140	1193	1722	2352	2261	1279	244
	AR3	44	61	85	255	538	148	248	458	405	150
	AR4	14	35	35	61	194	116	205	327	369	155
	AR5	7	5	4	13	46	10	14	48	36	21
	AR6	0	0	0	4	10	0	0	0	0	0
	AR7	0	0	0	0	0	1291	1647	2251	2474	2311
	AR8	1879	3174	4409	4762	5855	1747	2835	4352	5457	7205

Table 2: Number of times each estimator has smallest MSE in ten thousand replications when n=20

ρ	Estimator	p=3					p=7				
		σ^2									
		0.25	1	9	25	100	0.25	1	9	25	100
0.6	OLS	3172	3147	2203	1981	1797	3184	1701	136	24	1
	GRR	4501	3687	3831	4402	4627	1878	2135	5899	7423	8627
	ORR	2155	1657	468	214	410	1215	684	75	22	58
	AR1	146	787	922	706	378	755	546	86	18	1
	AR2	23	337	287	243	199	1403	731	91	10	0

	AR3	3	171	179	121	119	665	451	69	12	0
	AR4	0	34	49	24	30	228	223	44	3	1
	AR5	0	5	16	9	7	26	38	4	0	1
	AR6	0	2	7	2	7	0	0	0	0	0
	AR7	0	0	0	0	0	646	3491	3596	2488	1311
	AR8	0	173	2038	2298	2426	646	3491	3596	2488	1311
	OLS	662	864	1406	1409	882	1207	356	7	1	0
	GRR	9177	8107	5313	4438	5458	5064	3714	5740	6684	7085
	ORR	98	71	58	142	341	337	259	11	1	7
	AR1	25	209	697	665	221	130	153	7	0	0
	AR2	23	141	108	213	128	422	250	15	2	0
0.8	AR3	11	140	78	144	118	514	204	16	2	0
	AR4	2	38	36	27	32	401	128	13	3	0
	AR5	0	9	5	10	8	46	19	1	0	0
	AR6	0	3	5	0	1	0	0	0	0	0
	AR7	0	0	0	0	0	1879	4917	4190	3307	2908
	AR8	2	418	2294	2952	2811	1879	4917	4190	3307	2908
	OLS	2929	2188	1344	807	361	1307	211	2	1	0
	GRR	4677	4935	3099	3173	4909	2259	2292	3704	4328	4638
	ORR	2150	1131	412	355	352	397	123	0	0	6
	AR1	27	194	590	385	127	1388	339	2	0	0
	AR2	155	345	365	220	79	2435	735	11	0	0
0.9	AR3	55	501	304	296	70	966	479	9	0	0
	AR4	5	122	85	83	21	299	274	4	1	0
	AR5	0	41	24	25	5	34	45	0	0	0
	AR6	0	8	6	6	3	0	1	0	0	0
	AR7	0	0	0	0	0	915	5501	6268	5670	5356
	AR8	2	535	3771	4650	4073	915	5501	6268	5670	5356
	OLS	3453	2505	867	398	166	1242	170	1	0	0
	GRR	3909	3502	2083	2454	3460	2094	1737	1789	2132	2299
	ORR	1819	1190	566	363	306	290	45	0	0	8
	AR1	216	373	422	238	72	2359	472	1	0	0
	AR2	590	1424	675	275	66	2866	1292	11	0	0
0.95	AR3	12	599	691	370	91	394	728	13	0	0
	AR4	0	66	160	94	29	169	366	5	2	0
	AR5	0	21	51	15	6	18	44	2	0	0
	AR6	0	10	14	6	2	0	0	0	0	0
	AR7	0	0	0	0	0	568	5146	8178	7866	7693
	AR8	1	310	4471	5787	5802	568	5146	8178	7866	7693
0.99	OLS	2913	1508	257	112	29	427	30	0	0	0
	GRR	2756	1930	1389	1149	863	2079	835	157	113	116
	ORR	1640	1128	347	288	185	245	15	2	1	2
	AR1	2492	3203	681	219	93	3397	455	0	0	0
	AR2	182	1214	1688	741	188	3121	2669	61	3	0
	AR3	15	575	990	693	232	512	1347	74	8	0

	AR4	0	51	183	154	57	101	635	42	4	0
	AR5	0	13	57	36	12	9	68	6	1	0
	AR6	0	3	13	14	5	0	0	0	0	0
	AR7	0	0	0	0	0	109	3946	9658	9870	9882
	AR8	2	375	4395	6594	8336	109	3946	9658	9870	9882
0.999	OLS	902	285	54	23	6	4	0	0	0	0
	GRR	2667	1625	473	317	128	463	122	7	1	0
	ORR	1073	602	478	254	115	24	2	1	2	0
	AR1	3712	3629	2568	1969	791	3822	787	10	0	0
	AR2	376	916	1244	1223	1116	3697	4482	1121	242	10
	AR3	26	82	447	544	613	159	435	684	216	26
	AR4	5	9	52	95	101	47	142	300	146	15
	AR5	1	1	4	21	34	3	15	31	19	3
	AR6	0	0	3	10	11	0	0	0	0	0
	AR7	0	0	0	0	0	1781	4015	7846	9374	9946
	AR8	1238	2851	4677	5544	7085	1781	4015	7846	9374	9946

Table 3: Estimator and the number of times each has highest number of smallest MSE **based on the each classification**

σ^2	Small sample size			Moderate Sample size			High Sample size		
	p=3								
	Multicollinearity level								
	Moderate	High	Severe	Moderate	High	Severe	Moderate	High	Severe
Small	OLS(4) GRR(4)	OLS(1) GRR(7)	OLS (1) GRR (1) AR1(10)	OLS (6) GRR(6)	OLS (12)	OLS(3) AR1(13) AR2(1) AR8(1)	GRR(4)	OLS(2) GRR(2)	OLS(2) AR1(4)
Moderate	OLS (3) GRR(4) AR8(1)	AR8(8)	AR1(2) AR8(10)	OLS (1) GRR(11)	GRR (2) AR8 (10)	AR8 (18)	GRR (4)	GRR (4)	AR8 (6)
High	GRR (2) AR8(2)	GRR(1) AR8(3)	AR8(6)	GRR (6)	GRR (5) AR8 (1)	AR8(9)	GRR (2)	GRR (2)	AR8 (3)
p=7									
Small	OLS(3) GRR(1) AR1(2) AR8(2)	AR1(2) AR2(4) AR8(2)	AR1(7) AR2(3) AR8(2)	OLS(6) GRR(4) AR8(2)	GRR(4) AR2(3) AR8(5)	AR1(6) AR2(9) AR8(3)	OLS(2) GRR(1) AR8(1)	OLS(1) GRR(2) AR8(1)	AR1(2) AR2(4)
Moderate	GRR(4) AR8(4)	AR8(8)	AR8(12)	GRR(12)	GRR(7) AR8(5)	AR8(18)	GRR(3) AR8(1)	GRR(4)	AR6(2) AR8(4)
High	GRR(2) AR8(2)	AR8(4)	AR8(6)	GRR(6)	GRR(5) AR8(1)	AR8(9)	GRR(2)	GRR(2)	AR8(2) AR6(1)

NOTE: Estimator with the highest number of counts is bolded

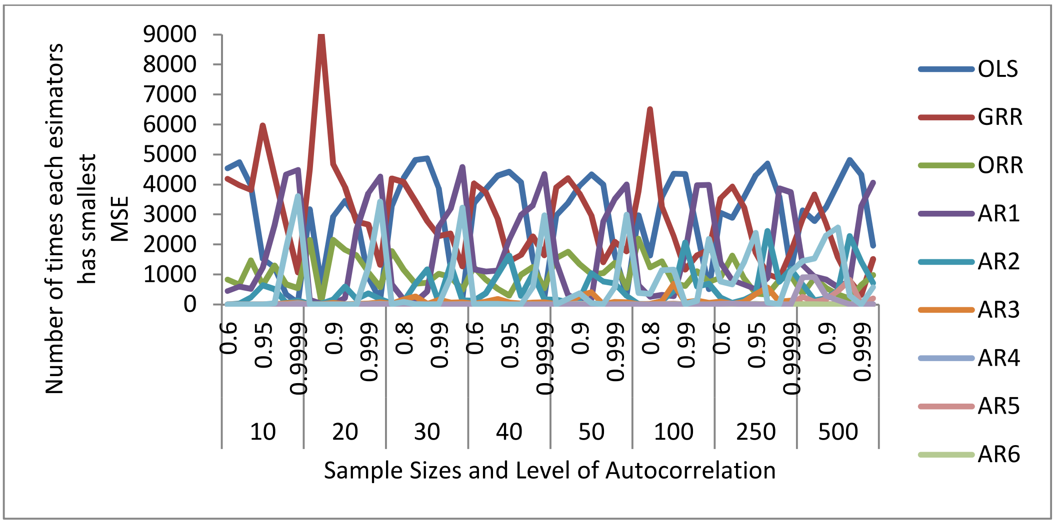

Figure 1: Number of times each estimator has smallest MSE in ten thousand replications when p=3 and $\sigma = 0.5$

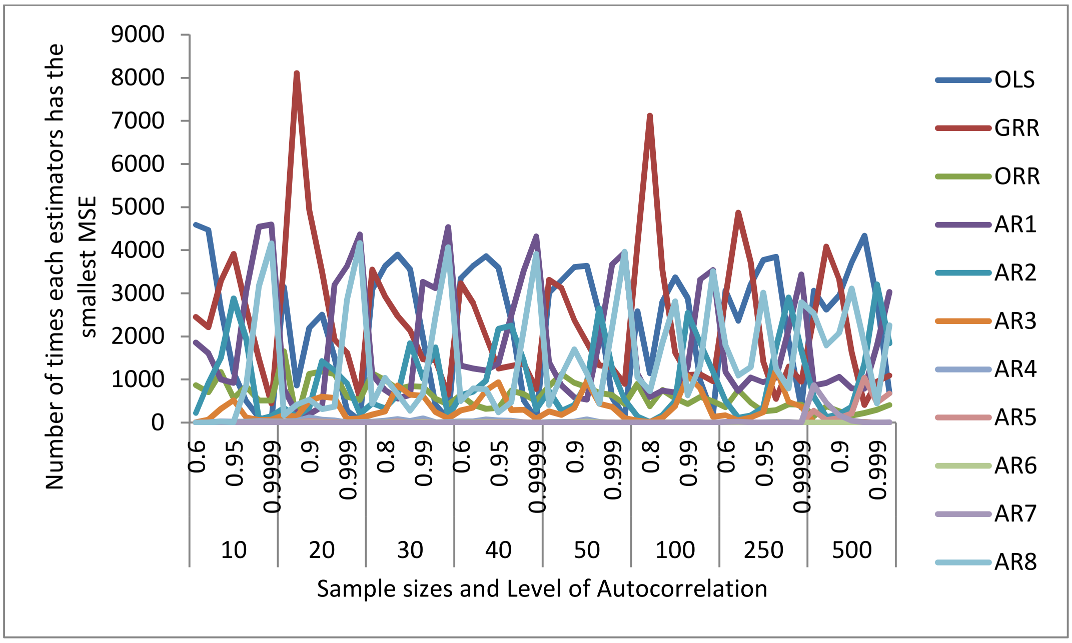

Figure 2: Number of times each estimator has smallest MSE in ten thousand replications when p=3 and $\sigma = 1$

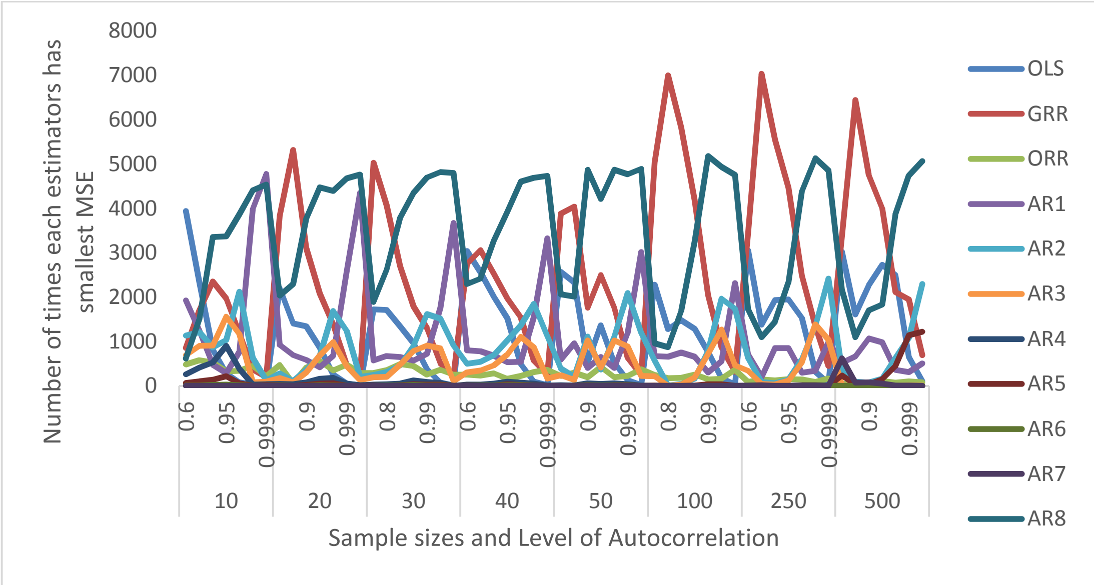

Figure 3: Number of times each estimator has smallest MSE in ten thousand replications when p=3 and $\boldsymbol{\sigma} = \mathbf{3}$

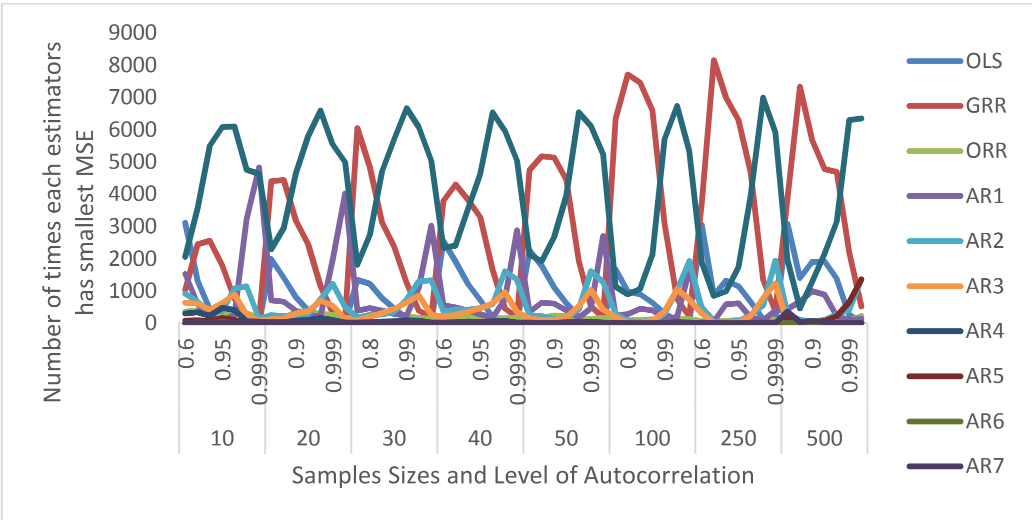

Figure 4: Number of times each estimator has smallest MSE in ten thousand replications when p=3

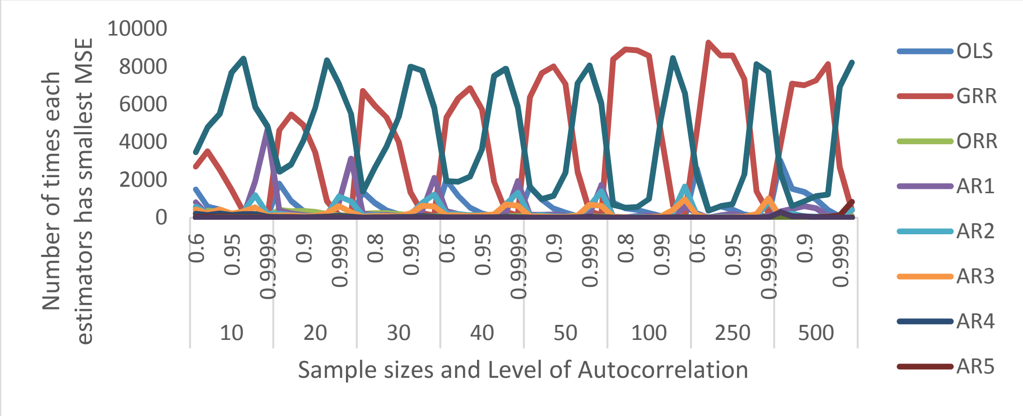

Figure 5: Number of times each estimator has smallest MSE in ten thousand replications when p=3 and $\sigma = 10$

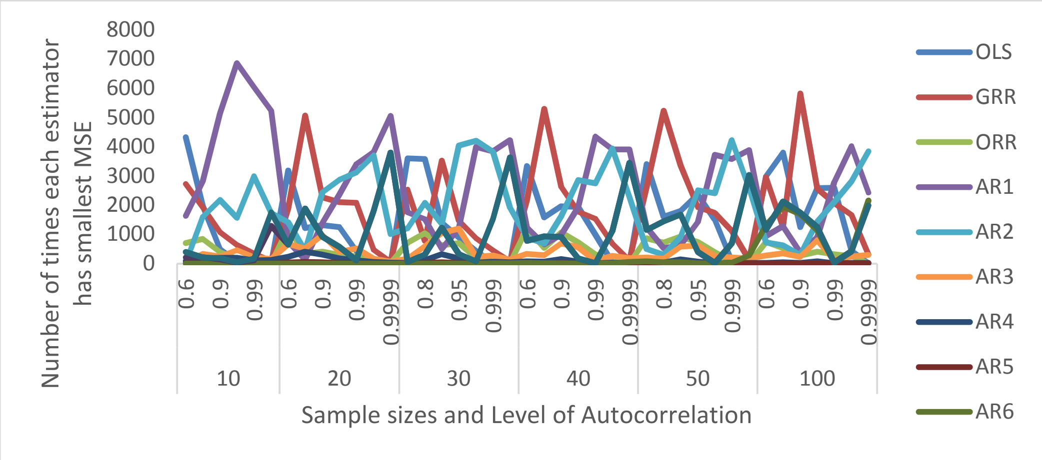

Figure 6: Number of times each estimator has smallest MSE in ten thousand replications when p=7 and $\sigma = 0.5$

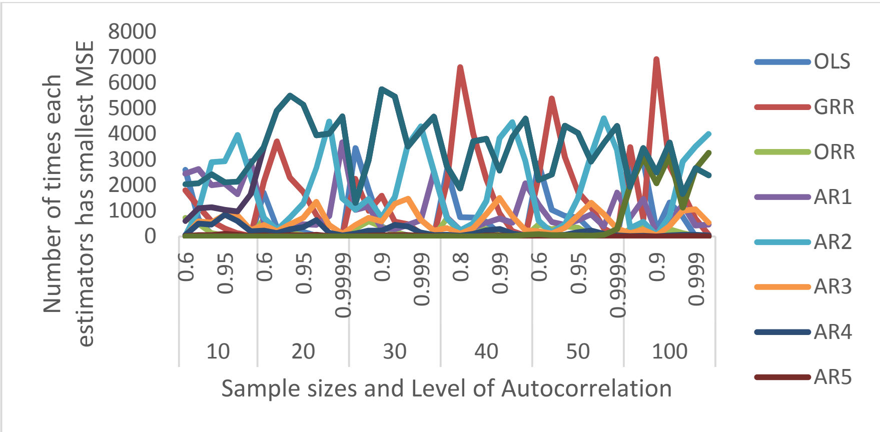

Figure 7: Number of times each estimator has smallest MSE in ten thousand replications when p=7 and $\sigma = 1$

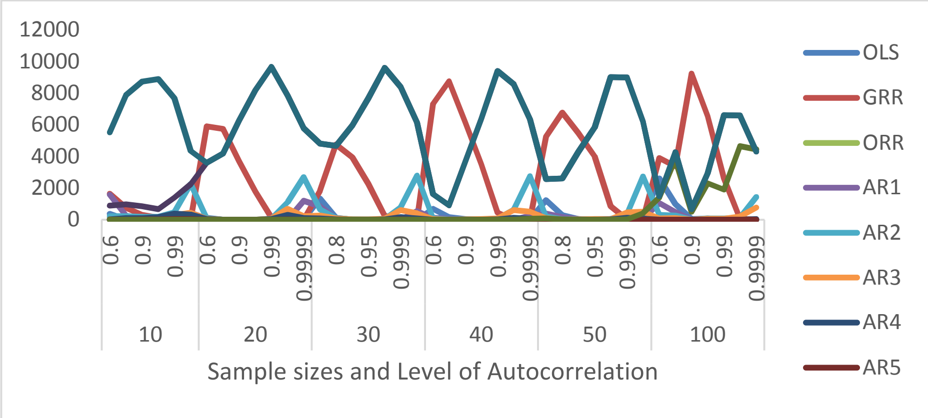

Figure 8: Number of times each estimator has smallest MSE in ten thousand replications when p=7 and $\boldsymbol{\sigma} = \mathbf{3}$

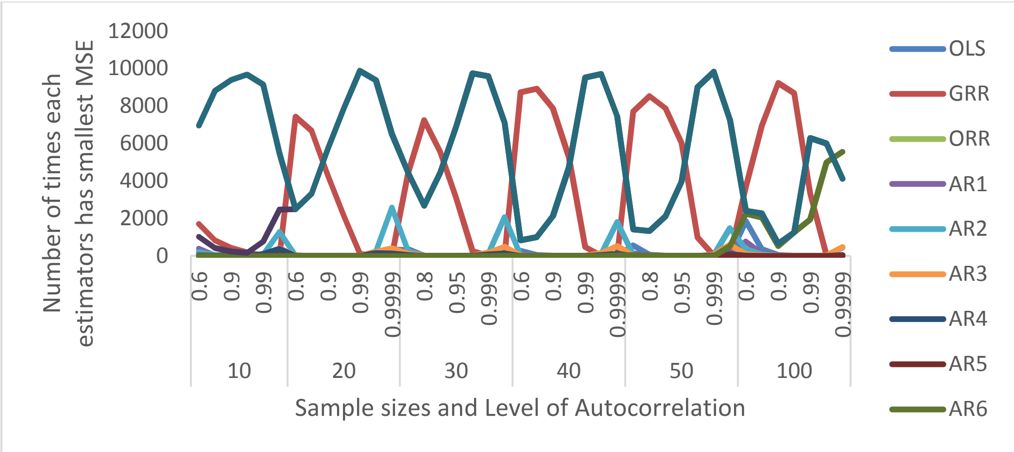

Figure 9: Number of times each estimator has smallest MSE in ten thousand replications when p=7 and $\boldsymbol{\sigma} = \mathbf{5}$

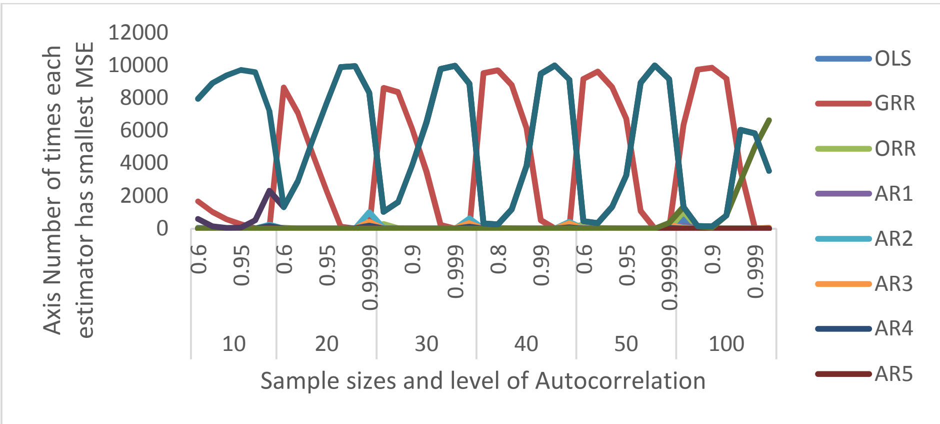

Figure 10: Number of times each estimator has smallest MSE in ten thousand replications when p=7 and $\boldsymbol{\sigma} = \mathbf{10}$

5.0. CONCLUSION

The proposed adjusted ridge estimator is superior to other existing estimators especially when the multicollinearity level is severe and error variance is moderate or high provided the sample size is small, moderate or high. Generalized ridge regression estimator performs well when there is moderate multicollinearity, moderate and high variances provided the sample size is moderate or high. Ocassionally, the proposed compete favourably with it. Adjusted ridge estimator by Dorugade (2016) performs consistently well when the multicollinearity level is severe and the error variance is small provided the sample size is small. Finally, under the condition that the multicollinearity level is severe, the ridge regression estimator that involves estimating the ridge parameter, k, can be avoided and replaced with the adjusted ridge regression estimator.

REFERENCES

[1] Adnan, K., Yasin, A. and Asir, G. (2016). Some new modifications of Kibria's and Dorugade's methods: An application to Turkish GDP data. Journal of the Association of Arab Universities for Basic and Applied Sciences, 20,89-99.

[2] Alkhamisi, M., Khalaf, G. and Shukur, G. (2006). Some modifications for choosing ridge parameters. Communications in Statistics - Theory and Methods, 35(11), 2005-2020.

[3] Alkhamisi, M. and Shukur, G. (2008). Developing ridge parameters for SUR model.Communications in Statistics - Theory and Methods, 37(4), 544-564.

[4] Brown, W.G., Farid, N. and Joe, B. S. (1973). The Oregon Big Game Resources: An Economic Evaluation. Oregon Agricultural Experiment Station Special Report #379, Corvallis.

[5] Dempster, A.P., Schatzoff, M. and Wermuth, N. (1977). A simulation study of alternatives to Ordinary Least Squares. Journal of the American Statistical Association, 72, 77-91.

[6] Dorugade, A. V. (2014). New ridge parameters for ridge regression. Journal of the Association of Arab Universities for Basic and Applied Sciences, 15, 94-99.

[7] Dorugade, A. V. (2016). Adjusted ridge estimator and comparison with Kibria's method in linear Regression. Journal of the Association of Arab Universities for Basic and Applied Sciences, 21, 96-102.

[8] Dorugade, A. V. and Kashid, D. N. (2010). Alternative method for choosing ridge parameter for regression. International Journal of Applied Mathematical Sciences, 4(9), 447-456.

[9] Gibbons, D. G. (1981). A simulation study of some ridge estimators. Journal of the American Statistical Association, 76, 131-139.

[10] Ghadban, K. and Mohamed, I. (2014). Ridge Regression and Ill-Conditioning. Journal of Modern Applied Statistical Methods, 13(2). 355-363.

[11] Gujarati, D. N. (1995). Basic Econometrics, McGraw-Hill, New York.

[12] Haq, M. S. and Kibria, B. M. G. (1996). A shrinkage estimator for the restricted linear regression model: Ridge Regression Approach. Journal of Applied Statistical Science,3(4), 301–316.

[13] Hocking, R., Speed, F. M. and Lynn, M. J. (1976). A class of biased estimators in linear regression. Technometrics, 18 (4), 425-437.

[14] Hoerl, A.E. and Kennard, R.W. (1970). Ridge regression: biased estimation for non-orthogonal problems. Technometrics, 12, 55-67.

[15] Hoerl, A. E., Kennard, R. W. and Baldwin, K. F. (1975). Ridge regression: Some simulation. Communications in Statistics, 4 (2), 105–123.

[16] Johnston, J. (1972). Econometric Methods, 2nd Ed. McGraw-Hill Book Co., Inc., New York.

[17] Koutsoyannis A. (2003). Theory of Econometrics. Palgrave publishers, 2nd Edition.

[18] Khalaf, G. and Shukur, G. (2005). Choosing ridge parameters for regression problems.Communications in Statistics - Theory and Methods, 34, 1177-1182.

[19] Khalaf, G. (2013). A Comparison between Biased and Unbiased Estimators. Journal of Modern Applied Statistical Methods, 12(2), 293-303.

[20] Kibria, B. M. G. (2003). Performance of some new ridge regression estimators.Communications in Statistics - Simulation and Computation, 32, 419-435.

[21] Lawless, J. F. and Wang, P. (1976). A simulation study of ridge and other regression estimators. Communications in Statistics A, 5, 307-323.

[22] Liu, K. (1993). A new class of biased estimate in linear regression. Communications in Statistics, 22(2), 393-402.

[23] Lukman, A. F. and Ayinde, K. (2016). Some Improved Classification-BasedRidge Parameter of Hoerl and Kennard Estimation Techniques. İSTATİSTİK, JOURNAL OF THE TURKISH STATISTICAL ASSOCIATION, 9(3), 93-106.

[24] Mansson, K., Shukur, G. and Kibria, B. M. G. (2010). A simulation study of some ridge regression estimators under different distributional assumptions.Communications in Statistics - Simulations and Computations, 39(8), 1639 –1670.

[25] Massey, W. F. (1965). Principal Components Regression in exploratory statistical research. J. Am. Stat. Assoc., 60, 234-256.

[26] McDonald, G. C. and Galarneau, D. I. (1975). A Monte Carlo evaluation of some ridge-type estimators. Journal of the American Statistical Association, 70, 407-416.

[27] Muniz, G. and Kibria, B. M. G. (2009). On some ridge regression estimators: An empirical comparison. Communications in Statistics-Simulation and Computation, 38, 621-630.

[28] Nordberg, L. (1982). A procedure for determination of a good ridge parameter in linear regression. Communications in Statistics A, 11, 285–309.

[29] Saleh, A. K. Md. E. and Kibria, B. M. G. (1993). Performances of some new preliminary test ridge regression estimators and their properties. Communications in Statistics - Theory and Methods, 22, 2747-2764.

[30] Singh, S. and Tracy, D. S. (1999). Ridge-regression using scrambled responses. Metrika, 41(2), 147–157.

[31] Stein, C. (1956). Inadmissibility of the usual estimator for the Mean of a multivarite normaldistribution. Third edition – Berkeley;California.

[32] Stephen, G. W. and Cristopher, J. P. (2001). Generalized ridge regression and a generalization of the Cp statistic. J. Appl. Stat. 28 (7), 911-922.

[33] Wichern, D. and Churchill, G. (1978). A Comparison of Ridge Estimators. Technometrics, 20, 301–311.