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BOUND STATE AND SCATTERING PHASE SHIFT OF THE SCHRÖDINGER EQUATION WITH MODIFIED TRIGONOMETRY SCARF TYPE POTENTIAL

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ABSTRACT

The approximate bound state of the nonrelativistic Schrödinger equation was obtained with the modified trigonometric scarf type potential in the framework of asymptotic iteration method for any arbitrary angular momentum quantum number ℓ using a suitable approximate scheme to the centrifugal term. The effect of the screening parameter and potential depth on the eigenvalue was studied numerically. Finally, the scattering phase shift of the nonrelativistic Schrödinger equation with the potential under consideration was calculated.

Keywords: Bound state solution; Schrodinger equation; Phase shift.

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1. INTRODUCTION

The study of the exact solutions and approximate solutions of both the relativistic and nonrelativistic wave equations are important research area in quantum mechanics due to the fact that the resulting wave function comprises of all the necessary information that can describe any quantum system wholistically. Series of studies have been carried out over the years with different physical potential term [1-15] using various traditional methodologies such as supersymmetry shape invariance and solvable potential [2, 4, 10, 16], asymptotic iteration method [17-19], conventional and parametric Nikiforov-Uvarov method [5, 11, 13], factorization method [14], shifted 1/N expansion method [20], exact/proper quantization method [7-9], Formula method for bound state problem [21] and others. However, complete information about the quantum systems can only be obtained by investigating scattering state solutions of the relativistic and nonrelativistic wave equations with quantum mechanical potential terms. Thus, the understandings of various knowledge about fine scale systems have been gained by the examination of scattering and bound state of such systems. Therefore, the study of scattering state becomes an interesting area in both the relativistic and nonrelativistic regimes. Recently, Onyeaju et al., [22], studied scattering and bound states of the Klein Gordon particles with Hylleraas potential within effective mass formalism, Oyewumi and Oluwadare [23], investigated relativistic energies and scattering phase shifts for the fermionic particles scattered by Hyperbolical potential with the Pseudo(spin) symmetry, Oluwadare and Oyewumi [24, 25], obtained scattering states solutions of the Klein-Gordon equation with three physically solvable potential models and scattering state solution of the Duffin-Kemmer-Petiau equation with the varshni potential model. Ikot et al., [26], in their study, investigated scattering state of Klein-Gordon particles by q-parameter Hyperbolic Pöschl-Teller potential, Yazarloo et al., [27], obtained the relativistic scattering states of the Hellmann potential. In this study, we intend to investigate the bound state solution of the Schrödinger equation in the framework of asymptotic Iteration method (AIM) and the scattering phase shift of the modified Trigonometry scarf type Potential which has not been reported yet. The modified Trigonometry scarf type potential is given as

$$V(r)=\frac{-4V_0e^{-2\alpha r}}{\left(be^{2\alpha r}\right)\left(1-e^{-2\alpha r}\right)^2}, \tag{1}$$

Where V_0 is the depth of the potential and b characterized the range of the potential.

2. METHOD

In this section, we briefly outline the methodology of asymptotic iteration. AIM is proposed to solve the homogenous linear second-order differential equation of the form [17, 18]

$$y_n''(x)=\lambda_0(x)y_n'(x)+S_0(x)y_n(x), \tag{2}$$

Where $\lambda_0(x)\neq 0,$ the prime denotes the derivative with respect to x and n is the radial quantum number. The variables $S_0(x)$ and $\lambda_0(x)$ are sufficiently differentiable. The energy equation of any Schrödinger-like equation is obtained by transforming the equation into the form of Eq. (1) and then obtains the values of $\lambda_k(x)$ and $S_k(x)$ with $k>0$ as follows:

$$\lambda_k(x)=\lambda_{k-1}'(x)+S_{k-1}(x)+\lambda_0(x)\lambda_{k-1}(x), \tag{3}$$

$$S_k(x)=S_{k-1}'(x)+S_0(x)\lambda_{k-1}(x). \tag{4}$$

With Eqs. (3) And (4), one can obtain the quantization condition

$$\delta_k(x)=\begin{vmatrix}\lambda_k(x) & S_k(x)\\ \lambda_{k-1}(x) & S_{k-1}(x)\end{vmatrix}=0, \qquad k=1,2,-,-,-, \tag{5}$$

The energy eigenvalues are then obtained from Eq. (5) if the problem is exactly solvable. A comprehensive/detail of the methodology can be found in Ref. [17] and [18].

3. BOUND STATE SOLUTIONS.

To obtain energy equation for the nonrelativistic wave equation of any quantum physical system, we solve the original Schrödinger equation given in well-known textbooks [28, 29]

$$\left[\frac{p^2}{2\mu}+V(r)\right]\psi_{n,\ell,m}(r)=E_{n,\ell}\psi_{n,\ell,m}(r), \tag{6}$$

Where, $V(r)$ is the interacting potential, $E_{n,\ell}$ is the nonrelativistic energy and $\psi_{n,\ell,m}(r)$ is the wave function. Setting the wave function $\psi_{n,\ell,m}(r)=U_{n,\ell}(r)r^{-1}Y_{\ell,m}(\theta,\phi)$, we obtain the following radial Schrödinger equation with a centrifugal term

$$\left[\frac{d^2}{dr^2}+\frac{2\mu E_{n,\ell}-2\mu V(r)}{\hbar^2}-\frac{\ell(\ell+1)}{r^2}\right]U_{n,\ell}(r)=0. \tag{7}$$

It is noted that Eq. (7) cannot be solved for $\ell=0$ due to the centrifugal barrier. Thus, we must apply approximation scheme to deal with the centrifugal barrier [30]. For a short potential range, the following approximation

$$\frac{1}{r^2}\approx\frac{4\alpha^2e^{-2\alpha r}}{\left(1-e^{-2\alpha r}\right)^2}, \tag{8}$$

Is a good approximation scheme to the centrifugal barrier for $\alpha r \ll 1$. Substituting Eqs. (1) And (8) into Eq. (7) and by defining a variable of the form $y=e^{-2\alpha r}$, the second order differential equation in Eq. (7) becomes

$$\left[\frac{d^2}{dy^2}+\frac{1}{y}\frac{d}{dy}+\frac{Ay^2+By+C}{y^2(1-y)^2}\right]U_{n\ell}(y)=0, \tag{9}$$

Where

$$A=\frac{\mu E_{n\ell}}{2\alpha^2\hbar^2}+\frac{2\mu V_0}{b\alpha^2\hbar^2}, \tag{10}$$

$$B=\frac{-\mu E_{n\ell}}{\alpha^2\hbar^2}-\ell(\ell+1), \tag{11}$$

$$C=\frac{\mu E_{n\ell}}{2\alpha^2\hbar^2}. \tag{12}$$

In order to solve Eq. (10) using AIM, we need to transform the equation into the form of Eq. (2). This makes us to write the physical wave function of the form

$$U_{n\ell}(y)=y^{\upsilon}(1-y)^{\gamma}f(y), \tag{13}$$

Where

$$\upsilon = \sqrt{-\frac{\mu E_{n\ell}}{2\alpha^2\hbar^2}}, \tag{14}$$

$$\gamma = \frac{1}{2} + \frac{1}{2}\sqrt{\left(1+2\ell\right)^2 - \frac{2\mu V_0}{b\alpha^2\hbar^2}}. \tag{15}$$

Inserting Eq. (13) into Eq. (9), we have a second order homogeneous linear differential equation as follow

$$f''(y) + \left[\frac{(2\upsilon+1)-(2\upsilon+\gamma+1)y}{y(1-y)}\right] f'(y) - \left[\frac{(2\upsilon+\gamma)^2+B}{y(1-y)}\right] f(y) = 0, \tag{16}$$

Whose solution can be found by using asymptotic iteration method? Comparing Eq. (16) with Eq. (2), we deduced the following

$$\lambda_0 = \frac{(2\upsilon+\gamma+1)y-(2\upsilon+1)}{y(1-y)}, \tag{17}$$

$$S_0 = \frac{(2\upsilon+\gamma)^2+B}{y(1-y)}. \tag{18}$$

Using Eqs. (3), (4) and the quantization condition of Eq. (5), we can write the following relations:

$$S_0\lambda_1 = \lambda_0 S_1 \Leftrightarrow \upsilon+\gamma = 0+\sqrt{-A}, \tag{19}$$

$$S_1\lambda_2 = \lambda_1 S_2 \Leftrightarrow \upsilon+\gamma = -1+\sqrt{-A}, \tag{20}$$

$$S_2\lambda_3 = \lambda_2 S_3 \Leftrightarrow \upsilon+\gamma = -2+\sqrt{-A}, \tag{21}$$

$$S_3\lambda_4 = \lambda_3 S_4 \Leftrightarrow \upsilon+\gamma = -3+\sqrt{-A}, \tag{22}$$

When the preceding expressions are generalized, the energy eigenvalues of the nonrelativistic Schrödinger equation is obtained as

$$E_{n\ell} = \left[-\frac{\alpha^2\hbar^2}{2\mu}\left(\frac{\frac{2\mu V_0}{b\alpha^2\hbar^2} + \left(n+\frac{1}{2}+\frac{1}{2}\sqrt{\left(1+2\ell\right)^2-\frac{8\mu V_0}{b\alpha^2\hbar^2}}\right)^2}{n+\frac{1}{2}+\frac{1}{2}\sqrt{\left(1+2\ell\right)^2-\frac{8\mu V_0}{b\alpha^2\hbar^2}}}\right)^2\right]. \tag{23}$$

4. SCATTERING PHASE SHIFT

In this section, we calculate the scattering phase shift of the potential under consideration. First, we define $z = 1-e^{-2\alpha r}$, and after substituting Eqs. (1) And (8) into Eq. (7), we have

$$\left[(1-z)^2\frac{d^2}{dz^2} - (1-z)\frac{d}{dz} + \frac{-Pz^2+Qy-R}{z^2}\right]U_{n\ell}(z) = 0, \tag{24}$$

Where

$$P=\frac{2\mu V_0}{b\alpha^2\hbar^2}-\frac{k^2}{4\alpha^2}-\ell(\ell+1), \tag{25}$$

$$Q=-\frac{4\mu V_0}{b\alpha^2\hbar^2}-\ell(\ell+1), \tag{26}$$

$$R=\frac{2\mu V_0}{b\alpha^2\hbar^2}, \tag{27}$$

$$k=\sqrt{\frac{2\mu E_{n\ell}}{\hbar^2}-4\ell(\ell+1)\alpha^2}. \tag{28}$$

k Is the asymptotic wave number. Now introducing a wave function of the form

$$U_{n\ell}(z)=z^{\gamma}(1-z)^{-\frac{ik}{2\alpha}}f(z), \tag{29}$$

And substituting it into Eq. (24), the following hypergeometric equation is obtained

$$z(1-z)f''(z)+\left[2\gamma-\left(2\gamma+1-\frac{ik}{\alpha}\right)z\right]f'(z)+\left[\left(\gamma-\frac{ik}{\alpha}\right)^2+P\right]f(z)=0. \tag{30}$$

Thus, the radial wave function for any arbitrary $\ell-$ wave scattering states is given as

$$U_{n,\ell}(r)=N_{n,\ell}\left(1-e^{-2\alpha r}\right)^{\gamma}e^{ikr}\times{}_2F_1(\rho;\beta;\sigma;1-e^{-2\alpha r}), \tag{31}$$

Where

$$\rho=\gamma-\frac{ik}{2\alpha}-\sqrt{P}, \tag{32}$$

$$\beta=\gamma-\frac{ik}{2\alpha}+\sqrt{P}, \tag{33}$$

$$\sigma=2\gamma. \tag{34}$$

In order to completely determine the scattering phase shift by analyzing the asymptotic behavior of the wave function, we consider a recurrence relation of the hyper geometric function [25, 27]

$${}_2F_1(\rho;\beta;\sigma;z)=\frac{\Gamma(\sigma)\Gamma(\sigma-\rho-\beta)}{\Gamma(\sigma-\rho)\Gamma(\sigma-\beta)}{}_2F_1(\rho;\beta;1+\rho+\beta-\sigma;1-z)+$$

$$(1-z)^{\sigma-\rho-\beta}\frac{\Gamma(\sigma)\Gamma(\rho+\beta-\sigma)}{\Gamma(\rho)\Gamma(\beta)}{}_2F_1(\sigma-\rho;\sigma-\beta;\sigma-\rho-\beta+1;1-z). \tag{35}$$

Now, using the property

$${}_2F_1(\rho;\beta;\sigma;0)=1, \tag{36}$$

As $r\to\infty$ and Eq. (35), we have

$$ {}_2F_1(\rho;\beta;\sigma;1-e^{-2\alpha r}) = \Gamma(\sigma)\left|\frac{\Gamma(\sigma-\rho-\beta)}{\Gamma(\sigma-\rho)\Gamma(\sigma-\beta)} + \frac{\Gamma(\sigma-\rho-\beta)}{\Gamma(\rho^*)\Gamma(\beta^*)}e^{-2\alpha(\sigma-\rho-\beta)r}\right|, \tag{37} $$

Where we have used the following identities

$$ (\rho+\beta-\sigma)^* = \sigma-\rho-\beta = \frac{ik}{\alpha}, \tag{38} $$

$$ \beta^* = \sigma-\rho = \gamma + \frac{ik}{2\alpha} + \sqrt{P}, \tag{37} $$

$$ \rho^* = \sigma-\beta = \gamma + \frac{ik}{2\alpha} - \sqrt{P}. \tag{40} $$

Thus, as $r \to \infty$, the asymptotic form of the wave function is obtain as

$$ U_{n,\ell}(r) = N_{n,\ell}\Gamma(\sigma)\frac{\Gamma(\sigma-\rho-\beta)}{\Gamma(\sigma-\rho)\Gamma(\sigma-\beta)}\times\sin\left(kr+\frac{\pi}{2}+\delta\right). \tag{41} $$

Taking the boundary condition into consideration:

$r \to \infty \Rightarrow U_{n,\ell}(\infty) \to 2\sin\left(kr - \frac{\ell\pi}{2} + \delta_\ell\right)$, we obtain the phase shift as

$$ \delta_\ell = 1.571(\ell+1) + \arg\left[\Gamma\left(\frac{ik}{\alpha}\right)\right] - \arg\left[\Gamma\left(\gamma+\frac{ik}{2\alpha}+\sqrt{P}\right)\right] - \arg\left[\Gamma\left(\gamma+\frac{ik}{2\alpha}-\sqrt{P}\right)\right]. \tag{42} $$

Table 1: Bound state energy eigenvalues of the trigonometry scarf type potential at ground state as a function of the screening parameter with $b=-1,\ \mu=\hbar=1$.

ℓ	α	$E_{0,\ell}(V_0=0.1)$	$E_{0,\ell}(V_0=0.2)$	$E_{0,\ell}(V_0=0.3)$
1	0.10	-0.019373129	-0.018816093	-0.018316896
	0.15	-0.043589539	-0.042353621	-0.041213016
	0.20	-0.077492514	-0.075264373	-0.073267584
	0.25	-0.121082053	-0.117600583	-0.114480601
2	0.10	-0.044220071	-0.043478196	-0.042771490
	0.15	-0.099495159	-0.097825941	-0.096235652
	0.20	-0.176880283	-0.173912785	-0.171085960
	0.25	-0.276375443	-0.271738726	-0.267321812
3	0.10	-0.079154603	-0.078332032	-0.077531332
	0.15	-0.178097856	-0.176247071	-0.174445496
	0.20	-0.316618411	-0.313328127	-0.310125327
	0.25	-0.494716268	-0.489575198	-0.484570823
4	0.10	-0.124118768	-0.123252577	-0.122401031
	0.15	-0.279267227	-0.277318298	-0.275402321
	0.20	-0.496475071	-0.493010307	-0.489604125
	0.25	-0.775742298	-0.770328605	-0.765006446
5	0.10	-0.179096281	-0.178203177	-0.177320496

	0.15	-0.402966633	-0.400957149	-0.398971115
	0.20	-0.716385126	-0.712812709	-0.709281983
	0.25	-1.119351759	-1.113769858	-1.108253098

Table 2: Bound state energy eigenvalues of the trigonometry scarf type potential as a function of the angular quantum number with $b=-1$, $\alpha=0.5$ and $\mu=\hbar=1$.

n	ℓ	$E_{n,\ell}(V_0=0.1)$	$E_{n,\ell}(V_0=0.2)$	$E_{n,\ell}(V_0=0.3)$
0	0	-0.125000000	-0.125000000	-0.125000000
1	0	-0.540118664	-0.568595000	-0.590904422
	1	-1.125000000	-0.125000000	-1.125000000
2	0	-1.207260055	-1.268220746	-1.317564473
	1	-2.016082289	-2.031100037	-2.045186861
	2	-3.125000000	-3.125000000	-3.125000000
3	0	-2.124755271	-2.219026093	-2.296511497
	1	-3.157286517	-3.187657398	-3.216341797
	2	-4.509861462	-4.519455720	-4.528796699
	3	-6.125000000	-6.125000000	-6.125000000
4	0	-3.292359946	-3.420214324	-3.526229489
	1	-4.548539449	-4.594399927	-4.637893716
	2	-6.144746541	-6.164003447	-6.182795132
	3	-8.007090294	-8.014077448	-8.020964345
	4	-10.12500000	-10.12500000	-10.12500000
5	0	-4.710009347	-4.871563357	-5.006278497
	1	-6.189815593	-6.251231481	-6.309638074
	2	-8.029644258	-8.058600600	-8.086902349
	3	-10.13918800	-10.15318412	-10.16699354
	4	-12.50553026	-12.51101051	-12.51644163
	5	-15.12500000	-15.12500000	-15.12500000

5. CONCLUSIONS

The nonrelativistic bound states and scattering states of the Schrödinger equation in the presence of trigonometry scarf type potential has been analyzed. As a result of the centrifugal barrier, we have used a proper approximation scheme and the approximate bound states and scattering states were obtained. Some numerical results were generated for the bound state as shown in Tables 1 and 2. In Table 1, we presented numerical values of the ground state for the energy equation given in equation (23) as a function of the screening parameter. Our result reviewed that the eigenvalue decreases as the screening parameter and angular quantum number respectively increases. However, as the potential depth increases, the eigenvalue increases. In Table 2, it can be seen that at constant value of the screening parameter $(\alpha=0.5)$, the eigenvalue decreases as the quantum number n increases. Our result also reviewed here that when $n=\ell$, an increase in the value of the potential depth has no effect on the eigenvalue. However, for all $n=\ell$, the eigenvalues are the same to any decimal place. For a free particle, the phase shifts would be zero. One could therefore say that the phase shift measures how far the asymptotic solution of the scattering problem is displaced at the origin from the asymptotic free solution.

AUTHORS' DECLARATION.

There is no conflict of interest within the authors.

REFERENCES

[1] G.F. Wei and S.H. Dong, "Pseudospin symmetry in the relativistic Manninig-Rosen potential including a Pekeris-type approximation to the pseudo-centrifugal term", *Physics Letters B,* vol. 686, pp. 288-292, 2010.

[2] C.-S. Jia, P. Guo, Y.-F. Diao, L.-Z. Yi and X.-J. Xie, "Solutions of Dirac equations with the Poschl-Teller potential", *The Euopean Physical Journal A,* Vol. 34, pp. 41-48, 2007.

[3] C.-S. Jia, Y. Sun and Y. Li, "Complexified Poschl-Teller II potentialmodel", *Physics Letters A,* Vol. 305, pp. 231-238, 2002.

[4] H. Hassanabadi, S. Zarrinkamar and H. Rahimov, "Approximate solutions of D-Dimensional Klein-Gordon equation with Hulthen-type potential via SUSYQM", *Communication in Theoretical Physics,*Vol. 56, pp. 423-428, 2011.

[5] M.Hamzavi and S.M. Ikhdair, "Any J-state solution of the Duffin-Kemmer-Petiau equation for a vector deformed Woods-Saxon potential", *Few-Body System,*

[6] H. Hassanabadi, H.B. Yazarloo and L.-L. LU, "Approximate analytical solutions to the generalized Poschl-Teller potential in D Dimensions", *Chinese Physics Letter,* Vol. 29 No: 2, 020303, 2012.

[7] Z.-Q. Ma, A.G. –Cisneros, B.-W., Xu and S.-H. Dong, "Energy spectrum of the trigonometric Rosen-Morse potential using improved quantization rule", *Physics Letters A*, vol. 371, pp. 180-184, 2007.

[8] S.-H. Dong and A.G. –Cisneros, "Energy spectra of the hyperbolic and second Poschl-Teller like potentials and solved by new exact quantization rule", *Annals of Physics,* vol. 323, pp. 1136-1149, 2008.

[9] X.-Y. Gu, S.-H. Dong and Z.-Q. Ma, "Energy spectra for modified Rosen-Morse potential solved by the new exact quantization rule", vol. 42, 035303 (8pp) 2009.

[10] S. Zarrinkamar, A.A. Rajabi and H. Hassanabadi, "Dirac equation for the harmonic scalar and vector potentials and linear plus Coulomb like tensor potential: the SUSY approach", *Annals of Physics*, vol. 325, pp. 2522-2528, 2010.

[11] C.A. Onate and J.O.A. Idiodi, "Eigensolutions of the Schrodinger equation with some physical potentials", *Chinese Journal of Physics,* vol. 53, (No.7), pp. 120001–10, 2015.

[12] C.A. Onate, "Approximate solutions of the non-relativistic Schrodinger equation with Poschl-Teller potential:, *Chinese Journal of Physics,* vol.53, (No.3), 060002-1, 2015.

[13] A.N. Ikot, H. Hassanabadi and T.M. Abbey, "Spin and pseudospin symmetries of Hellmann potential with three tensor interactions using Nikiforov-Uvarov method", *Coomunication in Theoretical Physics,* vol.64, pp.637-643, 2015.

[14] M.C. Onyeaju, J.O.A. Idiodi, A.N. Ikot, M. Solaimani and H. Hassanabadi, "Linear and nonlinear optical properties in spherical quantum dots: generalized Hulthen potential", *Few-Body Systems,* vol. 57, pp. 793-805, 2016.

[15] C.A. Onate, A.N. Ikot, M.C. Onyeaju, O. Ebomwonyi and J.O.A. Idiodi, "Effect of dissociation energy on Shannon and Renyi entropies", *Karbala International Journal of Modern Science,* vol.4, pp. 134-142, 2018.

[16] C.A. Onate, M.C. Onyeaju, A.N. Ikot and O. Ebomwonyi, "Eigen solutions and entropic system for Hellmann potential in the presence of the Schrodinger equation", *The European Physical Journal Plus,* vol.132, pp. 462-, 2017.

[17] B.J. Falaye, K.J. Oyewumi, T.T. Ibrahim, M.A. Punyasena and C.A. Onate, “Bound state solutions of the Manning-Rosen potential”, *Canadian Journal of Physics,* vol.91, pp. 98-104, 2013.

[18] O. Bayrak and I. Boztosun, “Bound state solutions of the Hulthen potential by using the asymptotic iteration method”, *Physica Scripta,* vol. 76, pp. 92-96, 2007.

[19] E. Ateser, H. Ciftci and M. Ugurlu, “A study of the Schrodinger equation with the linear potential by the asymptotic iteration method in 3D”, *Chinese Journal of Physics,* vol. 45 (No:3), pp.346-351, 2007.

[20] R.H. Hammed, “Approximate solution of Schrodinger equation with Manning-Rosen potential in two dimensions by using the shifted 1/N expansion method”, *Journal of Basrah Researches,* vol. 36, (No: 1.A), pp. 51-59, 2012.

[21] B.J. Falaye, S.M. Ikhdair and M. Hamzavi, "Formula method for bound state problems", *Few-Body System,* vol. 56, pp. 63-78, 2015

[22] M.C. Onyeaju, A.N. Ikot, E.O. Chukwuocha, H.P. Obong, S. Zare and H. Hassanabadi, "Scattering and bound states of Klein-Gordon particle with Hylleraas potential within effective mass function, *Few-Body System,* DOI: 10.1007/s00601-016-1122-0

[23] K.J. Onyewumi and O.J. Oluwadare, "Relativistic energies and scattering phase shift for the fermionic particles scattering by Hyperbolical potential with the Pseudo(spin) symmetry. *Advances in High Energy Physics,* vol. 2007, article (1) 1634717

[24] O.J. Oluwadare and K.J. Oyewumi, "Scattering states solutions of Klein-Gordon equation with three physically solvable potential models. *Chinese Journal of Physics*, vol. 55, pp. 2422-2435, 2017

[25] O.J. Oluwadare and K.J. Oyewumi, "Scattering state solutions of the Dufin-Kemmer-Petiau equation with the varshini potential model, *The European Physical Journal A,* vol. 53, pp. 29 (6papges), 2017.

[26] A.N. Ikot, H.P. Obong, I.O. Owate, M.C. Onyeaju and H. Hassanabadi, "Scattering state of Klein-Gordon particles by q-parameter Hyperbolic Poschl-Teller potential. *Advances in High Energy Physics,* vol. 2015,Article ID 632603

[27] B.H. Yazarloo, H. Mehraban and H. Hassanabadi, "Relativistic scattering states of the Hellmann potential, *Acta Physica Polonoca A,* vol. 127, pp. 684-688, 2015.

[28] L.I. Schiff 1968,"Quantum mechanics 3rd edition (NewYork: McGraw-Hill).

[29] L.D. Landua and E.M Lifshitz 1977,"Quantum mechanics: Non-Relativistic Theory 3rd edition (New York: Pergamon)

[30] C.A. Onate, O. Adebimpe, A.F. Lukman, I.J. Adama, J.O. Okoro and E.O. Davids. Approximate eigensolutions of the Attractive potential via parametric Nikiforov Uvarov method. *Heliyon* 4 (2018) e00977