An Approximate Solution of Dirac Equation for Second Pöschl-Teller like Scalar and Vector Potentials with a Coulomb Tensor Interaction

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Using the formalism of supersymmetric quantum mechanics (SUSYQM), we report approximate analytical solutions of relativistic symmetries of the Dirac equation in the presence of a second Pöschl-Teller like scalar and vector potentials. It has been shown from numerical results obtained that the degeneracy between spin and pseudo-spin doublets can be removed by tensor interaction, which is consistent with our present knowledge of the subject.

1. Introduction

In recent years [1-10], the problem of exact solutions of the Dirac equation for a number of special potentials has been a line of great interest. Several authors, by using different methods, have investigated the solution of the Dirac equation with spin and pseudospin symmetry and a tensor Coulomb interaction. These investigations include the Tietz potential [11] Pöschl-Teller and hyperbolical potential [12], pseudo-harmonic potential [13], Yukawa potential [14], Manning-Rosen potential [15,16], Deng-Fan potential [17] and Rosen-Morse potential [18]. The methods include the Nikiforov-Uvarov method [1,8,11], the asymptotic iteration method [3,6] and supersymmetry of quantum mechanics [2,7,11,12,17].

Up till now, to our knowledge, no work on the second Pöschl-Teller potential [19,20] exists. It is therefore, the priority purpose of the present work is to give approximate analytic solutions of the Dirac equation for this potential and a coulomb tensor interaction by super-symmetry approach.

The concept of super-symmetry was discovered in 1971 and was first investigated in High Energy Physics is an attempt to obtain a unified description of all basic interactions (fundamental forces) in nature. It offers a possible way to understand the space-time internal symmetries of the S-matrix. In this approach, the solutions of relativistic or non-relativistic equations for a given potential is brought into a well-known form of Schrödinger-like equation possessing known solutions via the methodology of the supersymmetry. Then, by using the idea of shape invariance, the exact solution can be obtained.

The scheme of our presentation is as follows. In Sec. 2, we provide the readers with a quite compact introduction to the SUSYQM. In Sec. 3, we review the spin and pseudospin symmetry limits of the Dirac equation. Thereafter, using a suitable approximation, we bring the problem into a rather more familiar form from which we obtain an approximate analytical solution of the problem. In the last section, some remarks and numerical results are given.

2. Supersymmetry

In this section we discuss the super-symmetry method in a simpler form. The partner Hamiltonian [21] is given as

$$H_{\pm} = \frac{p^2}{2m} + V_{\pm}(x) \tag{1}$$

Where

$$V_{\pm}(x) = \Phi^2(x) \pm \Phi'(x) \tag{2}$$

For good SUSY $(E_0 = 0)$, the ground state of the system is obtained via

$$\phi_0^-(x) = Ce^{-U} \tag{3}$$

Where, C is the normalization constant and the super-potential is given as

$$U(x) = \int_{x_0}^{x} dz\Phi(x) \tag{4}$$

If the SUSY shape invariant condition

$$V_+(a_0, x) = V_-(a_1, x) + R(a_1) \tag{5}$$

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holds, then the partner Hamiltonians are the shape-invariant. In Eqn. (5), a_1 is a new set of parameters determined from the old set a_0 via the mapping, F: $a_0 \rightarrow a_1 = F(a_0)$ and $R(a_1)$ does not include, x. In such a case, the problem is simplified to a high degree and everything of interest is calculated from [21]

$$E_n = \sum_{s=1}^{n} R(a_s) \tag{6a}$$

$$\phi_n^-(a_s, x) = C\exp[-\int_0^x dz\Phi(a_n, z)] \tag{6b}$$

Where

$$A_s^+ = \frac{\partial}{\partial x} + \Phi(a_s, x) \tag{7}$$

Thus, the shape invariant condition determines the spectrum of the bound states of the Hamiltonian

$$H_s = -\frac{\partial^2}{\partial x^2} + V_-(a_s, x) + E_s \tag{8}$$

Where the energy eigen-functions of

$$H_s\phi_{n-s}^-(a_s, x) = E_n\phi_{n-s}^-(a_s, x), \qquad n \geq s \tag{9}$$

are related by [22,23]

$$\phi_{n-s}^-(a_s, x) = \frac{A^+}{(E_n - E_s)^{\frac{1}{2}}}\phi_{n-(s+1)}^-(a_{s+1}, x) \tag{10}$$

3. Dirac Equation Including a Tensor Coupling

The Dirac equation with tensor is given by [24,25]

$$[\vec{\alpha}\cdot\vec{p} + \beta(M + S(r)) - i\beta\vec{\alpha}\cdot\hat{r}U(r)]\Psi(r)$$

$$= [E - V(r)]\Psi(r) \tag{11}$$

Where, $V(r)$, $S(r)$ and $U(r)$ are vector, scalar and tensor potentials, respectively. Also, E, M and $\vec{p}$ denote the relativistic energy, Fermion mass and momentum operator, respectively. α and β matrices are given as

$$\alpha = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} I & O \\ 0 & -I \end{pmatrix} \tag{12}$$

Where, I is 2×2 unitary matrix and the spin matrices are

$$\sigma_1 = \begin{pmatrix} O & I \\ I & O \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} O & -i \\ i & O \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} I & O \\ O & -I \end{pmatrix} \tag{13}$$

The total angular momentum operator, $\vec{j}$, and the spin-orbit coupling operator $k = (\vec{\sigma}\cdot\vec{L} + 1)$ with $\vec{L}$ being the orbital angular momentum of spherical nucleons, commute with the Dirac Hamiltonian. The eigenvalues of spin-orbit coupling operator $k = \left(j + \frac{1}{2}\right) > 0$ and $k = -\left(j + \frac{1}{2}\right) < 0$, for the unaligned spin $j = \ell - \frac{1}{2}$ and the aligned spin $j = \ell - \frac{1}{2}$, respectively. The set (H^2, K, J^2, J_2) is taken as the complete set of the conservative quantities. Therefore, we can write the spinors as [24,25]

$$\Psi_{nk}(r) = \begin{pmatrix} f_{nk}(r) \\ g_{nk}(r) \end{pmatrix} = \begin{pmatrix} \frac{F_{nk}(\vec{r})}{r} Y_{j_m}^{\ell}(\theta,\varphi) \\ i\frac{G_{nk}(\vec{r})}{r} Y_{j_m}^{\bar{\ell}}(\theta,\varphi) \end{pmatrix} \tag{14}$$

Where, $f_{nk}(r)$ and $g_{nk}(r)$ are the upper and lower components of the Dirac spinors. $Y_{j_m}^{\ell}(\theta,\varphi)$ and $Y_{j_m}^{\bar{\ell}}(\theta,\varphi)$ respectively denote the spin and pseudo-spin spherical harmonics and *m* is the projection of the angular momentum on the z-axis. By using the following relations

$$(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) = \vec{A}\cdot\vec{B} + i\vec{\sigma}\cdot(\vec{A}\times\vec{B}) \tag{15a}$$

$$(\vec{\sigma}\cdot\vec{p}) = \vec{\sigma}\cdot\hat{r}\left(\hat{r}\cdot\vec{p} + i\frac{\vec{\sigma}\cdot\vec{L}}{r}\right) \tag{15b}$$

and relations

$$(\vec{\sigma}\cdot\vec{L})Y_{j_m}^{\ell}(\theta,\varphi) = (k-1)Y_{j_m}^{\bar{\ell}}(\theta,\varphi) \tag{16a}$$

$$(\vec{\sigma}\cdot\vec{L})Y_{j_m}^{\ell}(\theta,\varphi) = -(k-1)Y_{j_m}^{\ell}(\theta,\varphi) \tag{16b}$$

$$(\vec{\sigma}\cdot\hat{r})Y^{\bar{\ell}}_{j_m}(\theta,\varphi) = -Y^{\ell}_{j_m}(\theta,\varphi) \quad (16c)$$

$$(\vec{\sigma}\cdot\hat{r})Y^{\ell}_{j_m}(\theta,\varphi) = -Y^{\bar{\ell}}_{j_m}(\theta,\varphi) \quad (16d)$$

we have the following coupled differential equations [24,25],

$$\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right)F_{nk}(r) = (M+E_{nk}+\Delta(r))G_{nk}(r) \quad (17a)$$

$$\left(\frac{d}{dr}-\frac{k}{r}+U(r)\right)G_{nk}(r) = (M-E_{nk}+\sum(r))F_{nk}(r) \quad (17b)$$

Where

$$\sum(r) = V(r)+S(r) \quad (18a)$$

$$\Delta(r) = V(r)-S(r) \quad (18b)$$

From Eqns. (17a) and (17b), we obtain

$$\left(\frac{d^2}{dr^2}-\frac{\kappa(\kappa+1)}{r^2}+\frac{2\kappa}{r}U(r)-\frac{dU(r)}{dr}-U^2(r)+\frac{\frac{d\Delta(r)}{dr}}{M+E_{nk}-\Delta(r)}\left(\frac{d}{dr}+\frac{\kappa}{r}-U(r)\right)\right)F_{nk}(r) =$$
$$(M+E_{nk}-\Delta(r))(M-E_{nk}+\Sigma(r))F_{nk}(r) \quad (19)$$

$$\left(\frac{d^2}{dr^2}-\frac{\kappa(\kappa+1)}{r^2}+\frac{2\kappa}{r}U(r)+\frac{dU(r)}{dr}-U^2(r)+\frac{\frac{d\sum(r)}{dr}}{M-E_{nk}+\Sigma(r)}\left(\frac{d}{dr}-\frac{\kappa}{r}+U(r)\right)\right)G_{nk}(r) =$$
$$(M+E_{nk}-\Delta(r))(M-E_{nk}+\Sigma(r))G_{nk}(r) \quad (20)$$

Where, $\kappa(\kappa-1) = \bar{\ell}(\bar{\ell}+1)$ and $\kappa(\kappa+1) = \ell(\ell+1)$.

3.1. The pseudo-spin symmetry limit

For pseudo-spin symmetry, we have $\frac{d\sum(r)}{dr} = 0$ and $\sum(r) = C_{ps} =$ constant. Here, the difference potential is taken as the second Pöschl-Teller potential, which is plotted as variation of r for different values of α and the tensor is taken as the Coulomb-like potential, i.e.,

$$\Delta(r) = \frac{V_1 - V_2\cosh(\alpha r)}{\sinh^2(\alpha r)} \quad (21)$$

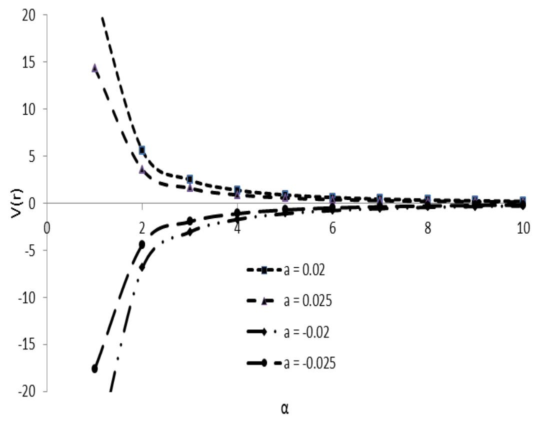

Fig.1: The second Pöschl-Teller like potential.

$$U(r) = -\frac{H}{r}, \qquad r \geq R_c \tag{22}$$

$$H = \frac{Z_a Z_b e^2}{4\pi\varepsilon_0} \tag{23}$$

Where, R_c is the Coulomb radius, Z_a and Z_b denote the charges of the projection particle *a* and the target nucleus *b*, respectively. Under this symmetry, we obtain the coefficient of the inverse square term by using Eqns. (21), (22) and (23) as

$$\delta_1 = k(k-1) + 2kH - H + H^2 = (k+H)(k+H-1) \tag{24}$$

Substituting the approximation [12]

$$\frac{1}{r^2} \approx \frac{\alpha^2}{\sinh^2(\alpha r)} \tag{25}$$

into Eqn. (20) gives

$$\left(\frac{d^2}{dr^2} - \csc h^2(\alpha r)[V_1(E_{nk} - M - C_{ps}) + \alpha^2\delta]\right.$$

$$\left. - \cosh(\alpha r)\csc h^2(\alpha r) V_2[M - E_{nk} + C_{ps}]\right)$$

$$\times G_{nk}(r) = (M + E_{nk})(M - E_{nk} + C_{ps})G_{nk}(r) \tag{26}$$

Where, $k = -\bar{\ell}$ and $k = \bar{\ell} + 1$ for $k < 0$ and $k > 0$, respectively.

3.2. The spin symmetry limit

For spin symmetry, $\frac{d\Delta(r)}{dr} = 0$ and $\Delta(r) = C_s = \text{constant}$ [24,25]. Under this symmetry, we considered the sum potential as the second Pöschl-Teller potential given by

$$\Sigma(r) = \frac{V_1 - V_2\cosh(\alpha r)}{\sinh^2(\alpha r)} \tag{27a}$$

and consequently

$$\delta_2 = k(k+1) + 2kH + H + H^2 = (k+H)(k+H+1) \tag{27b}$$

Substituting Eqns. (27a) and (27b) into Eqn. (19) yields

$$\left(\frac{d^2}{dr^2} - \csc h^2(\alpha r)[V_1(M + E_{nk} - C_s) + \alpha^2\delta_2]\right.$$

$$\left. - \cosh(\alpha r)\csc h^2(\alpha r) V_2[C_s - M - E_{nk}]\right)$$

$$\times F_{nk}(r) = (M - E_{nk})(M + E_{nk} - C_s)F_{nk}(r) \tag{28}$$

Where, $k = \ell$ and $k = -\ell - 1$ for $k < 0$ and $k > 0$, respectively.

4. Approximate Relativistic Bound-States of a Particle in the Field of the Second Pöschl-Teller like Field Plus Tensor Interaction

In this section, we present the approximate bound state solutions of the Dirac equation with the second Pöschl-Teller like potential in the presence of a tensor potential, using the SUSYQM.

4.1. Pseudospin symmetry bound state solutions

The Schrödinger-like equation we obtained in the previous section under this symmetry can be written as

$$\frac{d^2 G_{nk}(r)}{dr^2} = (V_{eff}(r) - \tilde{E}_{ps,nk})G_{nk}(r) \tag{29}$$

Where, we have introduced the following parameters

$$V_{eff} = V_a \csc h^2(\alpha r) + V_b \cosh(\alpha r)\csc h^2(\alpha r) \tag{30a}$$

$$V_a = \alpha^2\delta_1 + V_1(E_{ps,nk} - M - C_p) \tag{31b}$$

$$V_b = V_2(M - E_{ps,nk} + C_p) \tag{31c}$$

$$\tilde{E}_{ps,nk} = (M + E_{ps,nk})(M - E_{ps,nk} + C_p) \tag{31d}$$

For mathematical simplicity, the super-potential related to the ground state function is as follows

$$R(r, P_1, V_a) + P_2 - \frac{P_1\cosh(\alpha r)}{\sinh^2(\alpha r)} \tag{32a}$$

Where

$$P_1 = \frac{\alpha \pm \sqrt{\alpha^2 + 4V_a}}{2} \quad (32b)$$

$$P_2 = -\frac{V_b}{2P_1} \quad (33c)$$

With Eqns. (32), we can obtain the partner potential as

$$V_+(r, P_1, V_b) = R^2 + \frac{dR}{dr}$$

$$= \frac{V_b^2}{4P_1^2} + P_1^2 \operatorname{csc} h^2(\alpha r) + P_1^2 \operatorname{csc} h^4(\alpha r)$$

$$-2P_1P_2 \cosh(\alpha r) \operatorname{csc} h^2(\alpha r)$$

$$+ \operatorname{csc} h(\alpha r)[2 \operatorname{csc} h^2(\alpha r) + 1](P_1 + \alpha)$$

$$-P_1^2 \operatorname{csc} h(\alpha r)[2 \operatorname{csc} h^2 + 1] \quad (33)$$

Similarly,

$$V_-(r, P_1, V_b) = R^2 + \frac{dR}{dr}$$

$$= \frac{V_b^2}{4P_1^2} + P_1^2 \operatorname{csc} h^2(\alpha r) + P_1^2 \operatorname{csc} h^4(\alpha r)$$

$$-2P_1P_2 \cosh(\alpha r) \operatorname{csc} h^2(\alpha r)$$

$$+ P_1 \operatorname{csc} h(\alpha r)[2 \operatorname{csc} h^2(\alpha r) + 1](P_1 + \alpha)$$

$$-P_1^2 \operatorname{csc} h(\alpha r)[2 \operatorname{csc} h^2(\alpha r) + 1] \quad (34)$$

From Eqns. (33) and (34) it can be deduced that

$$V_+(r, P_1, V_b) = V_-(r, P_1 - \alpha, V_a) + R(r, P_1 - \alpha, V_a) \quad (35)$$

Where

$$R(r, P_1 - \alpha, V_a) = \frac{V_b^2}{4P_1^2} - \frac{V_b^2}{4(P_1 - \alpha)^2} \quad (36)$$

Hence, the shape invariance condition is satisfied and then we can find

$$\tilde{E}_{ps,nk} = \sum_{k=1}^{n} R(r, P_1 - k\alpha, V_b)$$

$$= \frac{V_b^2}{4P_1^2} - \frac{V_b^2}{4(P_1 - n\alpha)^2}$$

$$\tilde{E}_{ps,nk} = \tilde{E}^-_{ps,nk} + \tilde{E}_{ps,0k} = -\frac{V_b^2}{4(P_1 - n\alpha)^2} \quad (37)$$

On substituting for the relations in the above equation, we can easily find

$$(M + E_{ps,nk})(M - E_{ps,nk} + C_p) + \left(\frac{V_2(M + C_p - E_{ps,nk})}{\alpha(2n+1) + \sqrt{\alpha^2 + 4[\alpha^2(k+H)(k+H-1) + V_1(E_{ps,nk} - M - C_p]}} \right)^2 = 0 \quad (38)$$

4.2. Exact solutions of spin symmetry limit

In this symmetry limit, our Schrödinger-like equation takes the following form,

$$\frac{d^2F_{nk}(r)}{dr^2} = (\overline{V}_{eff}(r) - \tilde{E}_{s,nk})F_{nk}(r) \quad (39)$$

With

$$\overline{V}_{eff} = \overline{V}_a \operatorname{csc} h^2(\alpha r) + \overline{V}_b \cosh(\alpha r) \operatorname{csc} h^2(\alpha r) \quad (40a)$$

$$\overline{V}_a = \alpha^2 \delta_2 + V_1(M + E_{s,nk} - C_s) \quad (40b)$$

$$\overline{V}_b = V_2(C_s - M - E_{s,nk}) \quad (40c)$$

$$\tilde{E}_{s,nk} = (M - E_{s,nk})(M + E_{s,nk} + C_s) \quad (40d)$$

We have decided to use the same variables so as to avoid repetition of algebra. It is clear that Eqn. (39) is similar to Eqn. (29); therefore, substituting for $\tilde{E}_{s,nk}, \overline{V}_b$ and P_1 in Eqn. (37), the relativistic energy spectrum turns out as

$$(M - E_{s,nk})(M + E_{s,nk} - C_s) + \left(\frac{V_2 (C_s - M - E_{s,nk})}{\alpha(1-2n) + \sqrt{\alpha^2 + 4[\alpha^2 (k+H)(k+H+1) + V_1 (M + E_{s,nk} - C_s)]}} \right)^2 = 0 \tag{41}$$

The detailed behavior of the system and the degeneracy removing role of the tensor term are well observed in Tables 1 and 2. In addition the variation of the relativistic energy spectrum as a function of α is given in Figs. 2 and 3.

Table 1: The bound state energy eigenvalues (fm^{-1}) of the p-spin symmetry second Pöschl-Teller like potential for various values of n and κ with $\alpha = 0.01, M = 1 fm^{-1}, V_1 = 0.001 fm^{-1}, V_2 = 0.01 fm^{-1}$ and $C_{ps} = -10 fm^{-1}$.

ℓ	n	k	(ℓ, f)	$E_{ps,nk}(H=0)$	$E_{ps,nk}(H=0.5)$	$E_{ps,nk}(H=1)$
1	1	-1	$1s_{\frac{1}{2}}$	-1.000080595	-1.000044248	-1.000020417
2	1	-2	$1p_{\frac{3}{2}}$	-1.000168538	-1.000124395	-1.000080595
3	1	-3	$1d_{\frac{5}{2}}$	-1.000218865	-1.000203821	-1.000168538
4	1	-4	$1f_{\frac{7}{2}}$	-1.000130820	-1.000199959	-1.000218865
1	2	-1	$2s_{\frac{1}{2}}$	-1.000051881	-1.000019746	-1.000003511
2	2	-2	$2p_{\frac{3}{2}}$	-1.000197359	-1.000107533	-1.000051881
3	2	-3	$2d_{\frac{5}{2}}$	-1.000537328	-1.000355054	-1.000197359
4	2	-4	$2f_{\frac{7}{2}}$	-1.001217317	-1.000823875	-1.000537328
1	1	2	$0d_{\frac{3}{2}}$	-1.000080595	-1.000124395	-1.000168538
2	1	3	$0f_{\frac{5}{2}}$	-1.000168538	-1.000203821	-1.000218865
3	1	4	$0g_{\frac{7}{2}}$	-1.000218865	-1.000199959	-1.000130820
4	1	5	$0h_{\frac{9}{2}}$	-1.000130820	-0.999992215	-0.999761434
1	2	2	$1d_{\frac{3}{2}}$	-1.000051881	-1.000107533	-1.000197359
2	2	3	$1f_{\frac{5}{2}}$	-1.000197359	-1.000335054	-1.000537328
3	2	4	$1g_{\frac{7}{2}}$	-1.000537328	-1.000823875	-1.001217317
4	2	5	$1h_{\frac{9}{2}}$	-1.001217317	-1.001743113	-1.002429432

Table 2: The bound state energy eigenvalues (fm^{-1}) of the spin symmetry second Pöschl-Teller like potential for various values of n and κ with $\alpha = 0.01, M = 1\,fm^{-1}, V_1 = 0.001\,fm^{-1}, V_2 = 0.01\,fm^{-1}, C_s = 10$ and $C_p = -10\,fm^{-1}$.

ℓ	n	k	(ℓ, f)	$E_{s,nk}\,(H=0)$	$E_{s,nk}\,(H=0.5)$	$E_{s,nk}\,(H=1)$
0	0	-1	$0s_{1/2}$	9.976961054	9.976924109	9.976961054
0	1	-1	$1s_{1/2}$	9.972871721	9.972824491	9.972871721
0	2	-1	$2s_{1/2}$	9.967586518	9.967524791	9.967586518
0	3	-1	$3s_{1/2}$	9.960587741	9.960504896	9.960587741
1	0	-2	$0p_{3/2}$	9.977252360	9.977071174	9.976961054
1	1	-2	$1p_{3/2}$	9.973243779	9.973012439	9.972871721
1	2	-2	$2p_{3/2}$	9.968072245	9.967770339	9.967586518
1	3	-2	$3p_{3/2}$	9.961238771	9.960834301	9.960587741
2	0	-3	$0d_{5/2}$	9.977813179	9.977501208	9.977252360
2	1	-3	$1d_{5/2}$	9.973958334	9.973561125	9.973243779
2	2	-3	$2d_{5/2}$	9.969002434	9.968485792	9.968072245
2	3	-3	$3d_{5/2}$	9.962481201	9.961791839	9.961238771
3	0	-4	$0f_{7/2}$	9.978603874	9.978182795	9.978182795
3	1	-4	$1f_{7/2}$	9.974961894	9.974428024	9.973958334
3	2	-4	$2f_{7/2}$	9.970302863	9.969611938	9.969002434
3	3	-4	$3f_{7/2}$	9.964208567	9.963292206	9.962481201
1	0	1	$0p_{1/2}$	9.977252360	9.977501208	9.977813179
1	1	1	$1p_{1/2}$	9.973243779	9.973561125	9.973958334
1	2	1	$2p_{1/2}$	9.968072245	9.968485792	9.969002434
1	3	1	$3p_{1/2}$	9.961238771	9.961791839	9.962481201
2	0	2	$0d_{3/2}$	9.977813179	9.978182795	9.978603874
2	1	2	$1d_{3/2}$	9.973958334	9.974428024	9.974961894
2	2	2	$2d_{3/2}$	9.969002434	9.969611938	9.970302863
2	3	2	$3d_{3/2}$	9.962481201	9.963292206	9.964208567
3	0	3	$0f_{5/2}$	9.978603874	9.979069768	9.979573594
3	1	3	$1f_{5/2}$	9.974961894	9.975551063	9.976186394
3	2	3	$2f_{5/2}$	9.970302863	9.971063037	9.971880028
3	3	3	$3f_{5/2}$	9.964208567	9.965213091	9.966288367

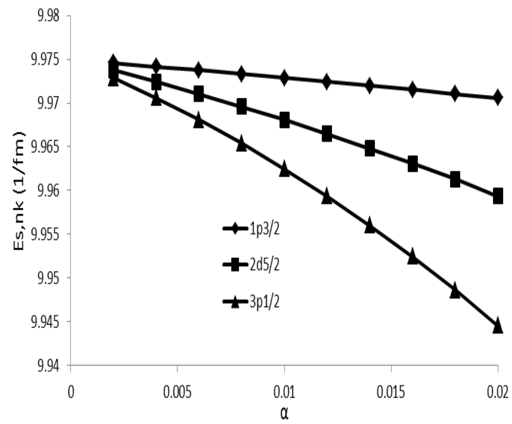

Fig.2.

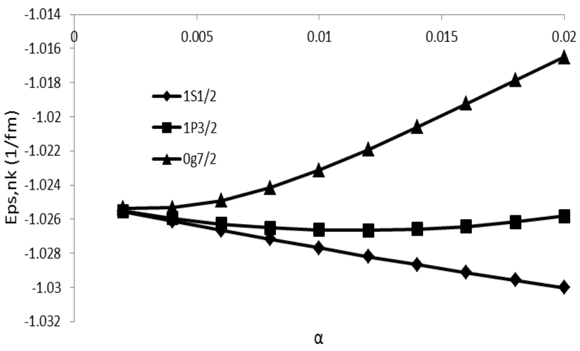

Fig.3.

5. Conclusion

By using the Pekeris approximation type, we have obtained approximate analytical solutions of the Dirac equation with Second Pöschl-Teller like potential under a tensor Coulomb interaction. We found that the presence of tensor removes the energy degeneracy in both the spin and pseudospin symmetries. The energy degeneracy for H = 0, 0.5 and 1.0 for some values of n and κ is given below.

In the pseudospin symmetry, we have:

For H = 0

$$1s_{\frac{1}{2}}=0d_{\frac{3}{2}},1p_{\frac{3}{2}}=0f_{\frac{5}{2}},1d_{\frac{5}{2}}=0g_{\frac{7}{2}},1f_{\frac{7}{2}}=0h_{\frac{9}{2}},$$

$$2s_{\frac{1}{2}}=1d_{\frac{3}{2}},2p_{\frac{3}{2}}=1f_{\frac{5}{2}},2d_{\frac{5}{2}}=1g_{\frac{7}{2}},2f_{\frac{7}{2}}=1h_{\frac{9}{2}}$$

For H = 0.5

$$1p_{\frac{3}{2}}=0d_{\frac{3}{2}},1d_{\frac{5}{2}}=0f_{\frac{5}{2}},1f_{\frac{7}{2}}=0g_{\frac{7}{2}},2p_{\frac{3}{2}}=1d_{\frac{3}{2}},$$

$$2d_{\frac{5}{2}}=1f_{\frac{5}{2}},2f_{\frac{7}{2}}=1g_{\frac{7}{2}}$$

For H = 1

$$1d_{\frac{5}{2}}=0d_{\frac{3}{2}},1f_{\frac{7}{2}}=0f_{\frac{5}{2}},2d_{\frac{5}{2}}=1d_{\frac{3}{2}},2f_{\frac{7}{2}}=1f_{\frac{5}{2}}$$

The degenerate states in the spin symmetry limit for various H, as shown in Table 2, are as follows:

For H = 0

$$0p_{\frac{3}{2}}=0p_{\frac{1}{2}},1p_{\frac{3}{2}}=1p_{\frac{1}{2}},2p_{\frac{3}{2}}=2p_{\frac{1}{2}},3p_{\frac{3}{2}}=3p_{\frac{1}{2}},$$

$$0d_{\frac{5}{2}}=0d_{\frac{3}{2}},1d_{\frac{5}{2}}=1d_{\frac{3}{2}},2d_{\frac{5}{2}}=2d_{\frac{3}{2}},3d_{\frac{5}{2}}=3d_{\frac{3}{2}},$$

$$0f_{\frac{7}{2}}=0f_{\frac{5}{2}},1f_{\frac{7}{2}}=1f_{\frac{5}{2}},2f_{\frac{7}{2}}=2f_{\frac{5}{2}},3f_{\frac{7}{2}}=3f_{\frac{5}{2}}$$

For H = 0.5

$$0d_{\frac{5}{2}}=0p_{\frac{1}{2}},1d_{\frac{5}{2}}=1p_{\frac{1}{2}},2d_{\frac{5}{2}}=2p_{\frac{1}{2}},3d_{\frac{5}{2}}=3p_{\frac{1}{2}},$$

$$0f_{\frac{7}{2}}=0d_{\frac{3}{2}},1f_{\frac{7}{2}}=1d_{\frac{3}{2}},2f_{\frac{7}{2}}=2d_{\frac{3}{2}},3f_{\frac{7}{2}}=3d_{\frac{3}{2}}$$

For H = 1.0

$$0s_{\frac{1}{2}}=0p_{\frac{3}{2}},1s_{\frac{1}{2}}=1p_{\frac{3}{2}},2s_{\frac{1}{2}}=2p_{\frac{3}{2}},3s_{\frac{1}{2}}=3p_{\frac{3}{2}},$$

$$0f_{\frac{7}{2}}=0p_{\frac{1}{2}},1f_{\frac{7}{2}}=1p_{\frac{1}{2}},2f_{\frac{7}{2}}=2p_{\frac{1}{2}},3f_{\frac{7}{2}}=3p_{\frac{1}{2}}$$

Our results find application in both Hadron and nuclear physics.

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