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Approximate Solutions of the Non-Relativistic Schrödinger Equation with An Interaction of Coulomb and Hulthèn Potentials

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Abstract:
By using the Pekeris approximation type, the Schrödinger equation is solved for the interaction of Coulomb and Hulthèn potentials within the framework of supersymmetric approach and Nikiforov-Uvarov method. The energy levels are obtained with the corresponding wave functions in terms of hypergeometric functions.

Keywords:
Schrödinger Equation; Hulthèn Potential; Interaction; Supersymmetric Method

1. INTRODUCTION

The experimental verifications of the Schrödinger equation that were theoretically discussed long ago, have recently aroused interest in Physics. The two most important ingredients in many studies are the wave functions and energy eigenvalues of the corresponding Schrödinger for which we do not know exact solutions in many cases. However, in some instances, the solutions are known in terms of the familiar mathematical functions [1]. The analytic solutions of the wave equations with some physical potentials are only possible for $\ell = 0$. For $\ell \neq 0$ state, the Pekeris approximation type [2–4] have been used to obtain an approximate solutions. To obtain the bound state energy eigenvalues for any ℓ state, various methods such as Asymptotic iteration method [5–9], Nikiforov-Uvarov method [10–14], exact quantization rule [15, 16], shifted 1/N expansion method [17], supersymmetric method [18, 19] have been used.

In the present work, we attempt to investigate the bound state solutions of the radial Schrödinger equation with the interaction of Coulomb potential and Hulthèn potential using both supersymmetric and Nikiforov-Uvarov methods. Hulthèn potential is one of the important molecular potentials used in different areas of Physics such as nuclear and particle, atomic and condensed matter Physics and chemical Physics to describe the interaction between two atoms. As a result of its applications, several works have been done on this potential. For instance, Agboola [20, 21] solved the Schrödinger equation with Hulthèn plus ringed-shaped potential. Bayrak and Boztosun [5], applied the asymptotic iteraction method to obtain a solution with Hulthèn potential. Saad [22] investigated the potential within the framework of the Klein-Gordon equation in D-dimensional space. Haouat and Chetouani [23], solved the problem for both Klein-Gordon and Dirac equation by an approximate technique. Ikhdair and Sever [24, 25] solved the Klein-Gordon equation with a position-dependent mass. Hall [26], in an instructive paper, discussed the Yukawa and Hulthèn potentials together. Zarrinkamar et al. [27] obtained analytical treatment of the two-body spinless Salpeter equation with the Hulthèn potential.

The organization of the work is as follows: In the next section, we obtain the bound state solutions. In section 3, we obtain numerical results while in the final section, we give the concluding remark.

2. BOUND STATE SOLUTIONS USING SUPERSYMMETRIC APPROACH

To study any quantum system, we solve the original Schrödinger equation [28, 29]:

$$\left(\frac{P^2}{2m}-E_{n\ell}+V(r)\right)\psi_{n\ell m}(r)=0, \tag{1}$$

where the potential $V(r)$ is taking as Coulomb potential minus Hulthèn potential written in the form:

$$V(r)=V_C(r)-V_H(r)=-\frac{A}{r}+\frac{V_0e^{-\delta r}}{1-e^{-\delta r}}. \tag{2}$$

Setting the wave function $\psi_{n\ell m}(r)=R_{n\ell}(r)Y_{\ell m}(\theta,\phi)r^{-1}$ to obtain the following radial Schrödinger equation:

$$\frac{d^2R_{n\ell}(r)}{dr^2}+\left[\frac{2\mu}{\hbar^2}\left(E_{n\ell}-V(r)\right)-\frac{\ell(\ell+1)}{r^2}\right]R_{n\ell}(r). \tag{3}$$

The radial wave function $R_{n\ell}(r)$ satisfying Eq. (3), should be normalizable and finite near $r=0$ and $r\to\infty$for the bound-state solutions. The wave Eq. (3) with the interaction of Coulomb and Hulthèn potentials cannot be solved analytically when $\ell\neq 0$because of the centrifugal term$\frac{\ell(\ell+1)}{r^2}$. Therefore, to solve Eq. (3) analytically, we must use an approximation scheme to deal with the centrifugal term. It is found that the following

$$\frac{1}{r^2}=\frac{\delta^2}{\left(1-e^{-\delta r}\right)^2} \tag{4}$$

is a good approximation to the centrifugal term in a short potential range. This approximation is valid when $\delta << 1$. Substituting Eqs. (2) and (4) into Eq. (3), we obtain a differential equation of the form

$$\left[\frac{d^2}{dr^2}+\frac{2\mu E_{n,\ell}+2\mu A\delta}{\hbar^2}-\ell(\ell+1)\delta^2-\frac{\left(\frac{2\mu\delta\left(A-\frac{V_0}{\delta}\right)}{\hbar^2}+\ell(\ell+1)\delta^2\right)e^{-\delta r}}{1-e^{-\delta r}}-\frac{\ell(\ell+1)\delta^2e^{-\delta r}}{\left(1-e^{-\delta r}\right)^2}\right]R_{n,\ell}(r)=0. \tag{5}$$

For bound state, the ground state wave function can be written in the form

$$U_{0\ell}(r)=\exp\left(-\int W(r)\,dr\right), \tag{6}$$

where $W(r)$ is called the superpotential in supersymmetric quantum mechanics [30, 31] which satisfy Eq. (5). Substituting Eq. (6) into Eq. (5), we obtain the following equation for $W(r)$:

$$\frac{d^2R_{n,\ell}(r)}{dr^2}=W^2(r)-\frac{dW(r)}{dr} \tag{7}$$

where we take the superpotential $W(r)$ as

$$W(r) = B_0 + \frac{B_1}{1-e^{-\delta r}}. \tag{8}$$

Substituting Eq. (8) into Eq. (7) lead us to the following relations:

$$B_0^2 = -\frac{2\mu E_{n\ell}}{\hbar^2} - \frac{2\mu A\delta}{\hbar^2} + \ell(\ell+1)\delta^2, \tag{9a}$$

$$B_1 = -\delta(\ell+1). \tag{9b}$$

$$B_0 = \frac{2\mu(\delta A - V_0) - B_1^2\hbar^2 - \ell(\ell+1)\delta^2\hbar^2}{2B_1\hbar^2}. \tag{9c}$$

Using the superpotential given in Eq. (8), we construct the supersymmetric partner potentials in the following form:

$$U_+(r) = W^2(r) + W'(r) = B_0^2 + \frac{2B_0B_1}{1-e^{-\delta r}} + \frac{B_1^2\left(1-e^{-\delta r}\right)}{\left(1-e^{-\delta r}\right)^2} - \frac{B_1(B_1+\delta)e^{-\delta r}}{\left(1-e^{-\delta r}\right)^2}, \tag{10}$$

$$U_-(r) = W^2(r) - W'(r) = B_0^2 + \frac{2B_0B_1}{1-e^{-\delta r}} + \frac{B_1^2\left(1-e^{-\delta r}\right)}{\left(1-e^{-\delta r}\right)^2} - \frac{B_1(B_1-\delta)e^{-\delta r}}{\left(1-e^{-\delta r}\right)^2}. \tag{11}$$

Eqs. (10) and (11) are shape invariant and thus satisfied the shape invariance condition. Therefore, the two partner potentials are related by:

$$U_+(r,a_0) = U_-(r,a_1) + R(a_1), \tag{12}$$

where a_1 is a function of a_0, i.e. $a_1 = f(a_0) = a_0 - \delta$ and consequently, $a_n = a_0 - n\delta$ and the residual term $R(a_1)$ is independent of the variable r. According to [32], the shape invariance holds via mapping of the form: $B_0 \to B_0 - \delta$,where $B_0 = a_0$. If all desirable results are obtained, then, one can obtain the following relations:

$$R(a_1) = \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_0^2 - \ell(\ell+1)\delta^2}{2a_0}\right)^2 - \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_1^2 - \ell(\ell+1)\delta^2}{2a_1}\right)^2, \tag{13}$$

$$R(a_2) = \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_1^2 - \ell(\ell+1)\delta^2}{2a_1}\right)^2 - \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_2^2 - \ell(\ell+1)\delta^2}{2a_2}\right)^2, \tag{14}$$

$$R(a_n) = \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_{n-1}^2 - \ell(\ell+1)\delta^2}{2a_{n-1}}\right)^2 - \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_n^2 - \ell(\ell+1)\delta^2}{2a_n}\right)^2. \tag{15}$$

Based on the concept of shape invariance approach and formalism [32, 33], we can determine the energy equation of the $U_-(r)$ potential by using the formalism:

$$\bar{E}_{n\ell} = \bar{E}_{n\ell}^{(-)} + \bar{E}_{0\ell}, \tag{16}$$

$$\bar{E}_{0\ell} = 0, \tag{17}$$

$$\bar{E}_{n\ell}^{(-)} = \sum_{\kappa=1}^{n} R(a_\kappa) = -\left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_n^2 - \ell(\ell+1)\delta^2}{2a_n}\right)^2, \tag{18}$$

$$-\frac{2\mu E_{n\ell}}{\hbar^2} - \frac{2\mu A\delta}{\hbar^2} + \ell(\ell+1)\delta^2 = \left(\frac{\frac{2\mu(\delta A - V_0)}{\hbar^2} - a_n^2 - \ell(\ell+1)\delta^2}{2a_n}\right)^2. \tag{19}$$

Substituting for a_n into Eq. (19), we obtain the energy eigenvalue equation as
:

$$E_{n\ell} = \frac{\ell(\ell+1)\delta^2\hbar^2}{2\mu} - \delta A - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(A - \frac{V_0}{\delta}\right) - \delta(\ell+n+1)^2 - \ell(\ell+1)\delta}{2(\ell+n+1)}\right]^2. \tag{20}$$

3. THE EIGENFUNCTION

In order to obtain unnormalized wave function, we define a new variable of the form $s = e^{-\delta r}$. Substituting this, into Eq. (5), we obtain equation of the form:

$$\frac{d^2R_{n\ell}(s)}{ds^2} + \frac{1}{s}\frac{dR_{n\ell}(s)}{ds} + \left[\frac{\mathbb{N} + \beta s + Cs^2}{[s(1-s)]^2}\right]R_{n\ell}(r) = 0, \tag{21}$$

where

$$\mathbb{N} = -\frac{2\mu E_{n\ell}}{\hbar^2\delta^2} + \frac{4\mu A}{\delta} + \ell(\ell+1), \tag{22}$$

$$\beta = -4\ell(\ell+1) - \frac{4\mu[2A + (A - V_0)]}{\hbar^2\delta^2}, \tag{23}$$

$$C = \frac{4\mu[A + (A - V_0)]}{\hbar^2\delta} - \frac{2\mu E_{n\ell}}{\hbar^2\delta^2} + 2\ell(\ell+1). \tag{24}$$

Analyzing the asymptotic behavior of (21) at origin when $r \to 0 (s \to 1)$and at infinity when $r \to \infty (s \to 0)$, Eq. (21) has solution

$$U_{n\ell}(r) = (1-s)^a s^\alpha, \tag{25}$$

where

$$a = -\frac{2\mu E_{n\ell}}{\hbar^2\delta^2} - \frac{2\mu A}{\hbar^2\delta} - \ell(\ell+1), \alpha = -(\ell+1)\delta. \tag{26}$$

by taking trial wave function of the form $R_{n\ell}(s) = (1-s)^a s^\alpha$ and inserting it into Eq. (21), one obtain

$$f''(s) + f'(s)\left[\frac{(2\alpha+1) - s(2\alpha+2a+1)}{s(1-s)}\right] - f(s)\left[\frac{(\alpha+a)^2 + \mathbb{N}}{s(1-s)}\right] = 0. \tag{27}$$

Eq. (27) is a differential equation satisfied by the hypergeometric function. Thus, its solution is obtain as

$$f(s) = F_1(-n, n + 2\alpha + 2a; 2\alpha + 1, s). \tag{28}$$

Replacing the function $f(s)$ with the hypergeometric function and write the complete radial wave function as

$$R_{n\ell}(s) = Ns^\alpha(1-s)^a F_1(-n, n + 2\alpha + 2a; 2\alpha + 1, s), \tag{29}$$

where N is the normalization constant.

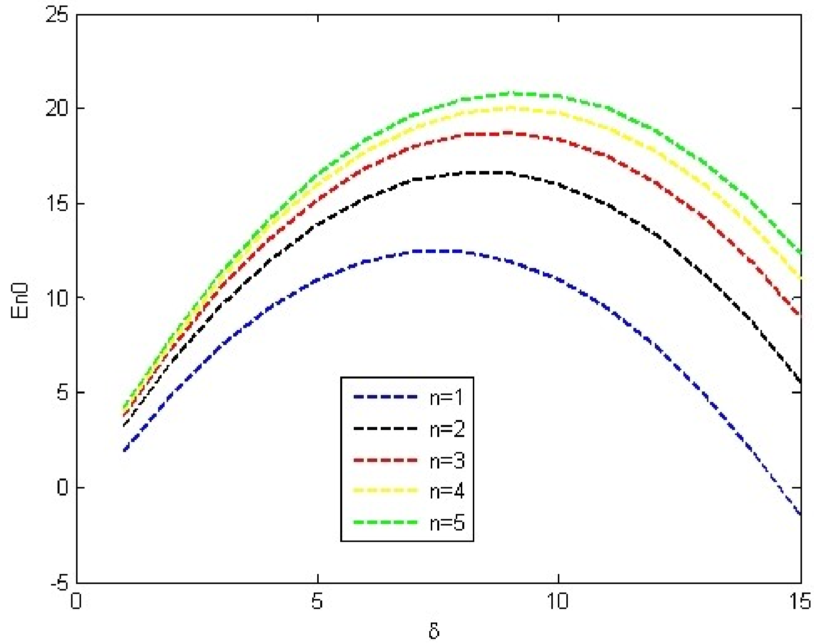

Figure 1. Variation of the energy spectrum as a function of the screening parameter δ with $A = 1, V_0 = 2\mu = \hbar = 1$.

4. BOUND STATE SOLTIONS USING NIKIFOROV-UVAROV METHOD

In other to test the accuracy of our result, we solve the same problem using Nikiforov-Uvarov (NU) method and compare it with the result obtained using SUSY. With Eqs. (2) and (4), Eq. (3) becomes

$$\frac{d^2R_{n\ell}(r)}{dr^2} + \left[\frac{2\mu E_{n\ell}}{\hbar^2} + \frac{2\mu A}{\hbar^2}\frac{\delta}{1-e^{-\delta r}} - \frac{2\mu}{\hbar^2}\frac{V_0}{e^{\delta r}-1} - \frac{\ell(\ell+1)\delta^2}{\left(1-e^{-\delta r}\right)^2}\right]R_{n\ell}(r) = 0. \tag{30}$$

For solving the above equation using NU method, let us consider the differential equation [34]:

$$\left\{\frac{d^2}{ds^2} + \frac{\alpha_1 - \alpha_2 s}{s(1-\alpha_3 s)}\frac{d}{ds} + \frac{\left[-\xi_1 s^2 + \xi_2 s - \xi_3\right]}{s^2(1-s)^2}\right\}\psi(s) = 0, \tag{31}$$

where,

$$\alpha_4 = \frac{1}{2}(1-\alpha_1), \alpha_5 = \frac{1}{2}(\alpha_2 - 2\alpha_3), \alpha_6 = \alpha_5^2 + \xi_1, \alpha_7 = 2\alpha_4\alpha_5 - \xi_2,$$

$$\alpha_8 = \alpha_4^2 + \xi_3, \alpha_9 = \alpha_3\alpha_7 + \alpha_3^2\alpha_8 + \alpha_6.$$

To obtain the solution of Eq. (30), we first introduce $s = e^{-\delta r}$to obtain

$$\frac{d^2R_{n\ell}(s)}{ds^2} + \frac{1-s}{s(1-s)}\frac{dR_{n\ell}(s)}{ds} + \left[\frac{-D(1-s)^2 + (P-Qs)(1-s) - \ell(\ell+1)\delta^2}{s^2(1-s)^2\delta^2}\right]R_{n\ell}(s) = 0, \tag{32}$$

where,

$$D = -\frac{2\mu E_{n\ell}}{\hbar^2}, P = \frac{2\mu\delta A}{\hbar^2}, Q = \frac{2\mu V_0}{\hbar^2}.$$

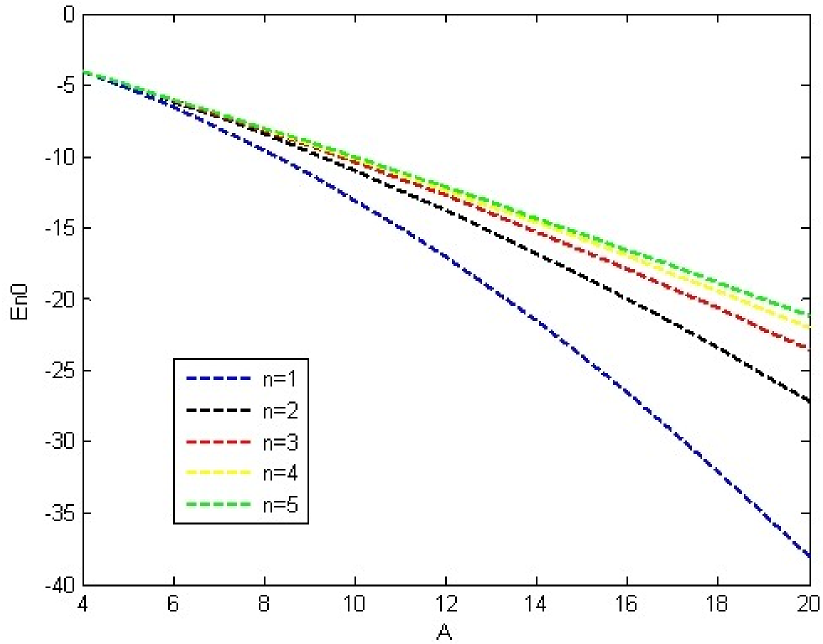

Figure 2. Variation of the energy spectrum as a function of the Potential strength with $V_0 = \hbar = \delta = 2\mu = 1$.

Comparing Eq. (31) with Eq. (32), we find the following relations:

$$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 0, \alpha_5 = -\frac{1}{2}, \xi_1 = -\frac{2\mu\left(E_{n\ell} + A\delta\right)}{\hbar^2\delta^2} + \ell(\ell+1),$$

$$\xi_2 = -\frac{2\mu(2E_{n\ell} + \delta A + V_0)}{\hbar^2\delta^2},, \xi_3 = -\frac{2\mu E_{n\ell}}{\hbar^2\delta^2} - \frac{2\mu V_0}{\hbar^2\delta^2}, \alpha_6 = \frac{1}{4} + \frac{2\mu(V_0 - E_{n\ell})}{\hbar^2\delta^2},,$$

$$\alpha_7 = \frac{2\mu(A\delta + V_0 - 2E_{n\ell})}{\hbar^2\delta^2}, \alpha_8 = \ell(\ell+1) - \frac{2\mu(A\delta + E_{n\ell})}{\hbar^2\delta^2}, \alpha_9 = \left(\ell + \frac{1}{2}\right)^2.$$

Following the Nikiforov-Uvarov method [13], we obtain the bound state energy condition

$$\alpha_2 n - (2n+1)\alpha_5 + (2n+1)(\sqrt{\alpha_9} + \alpha_3\sqrt{\alpha_8} + n(n-1)\alpha_3$$

$$+\alpha_7 + 2\alpha_3\alpha_8 + 2\sqrt{\alpha_8\alpha_9} = 0, \tag{33}$$

which gives the energy eigenvalues of the system as

$$E_{n\ell} = \frac{\ell(\ell+1)\delta^2\hbar^2}{2\mu} - \delta A - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(A - \frac{V_0}{\delta}\right) - \delta(\ell+n+1)^2 - \ell(\ell+1)\delta}{2(\ell+n+1)}\right]^2. \tag{34}$$

Eq. (20) is identical to Eq. (34). Now, let us consider some special cases. As δ approaches zero and $V_0 = 0$, the potential given in Eq. (2) reduces to Coulomb potential and the energy equation becomes

$$E_{n\ell} = \frac{\ell(\ell+1)\delta^2\hbar^2}{2\mu} - \delta A - \frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}A - \delta(\ell+n+1)^2 - \ell(\ell+1)\delta}{2(\ell+n+1)}\right]^2. \tag{35}$$

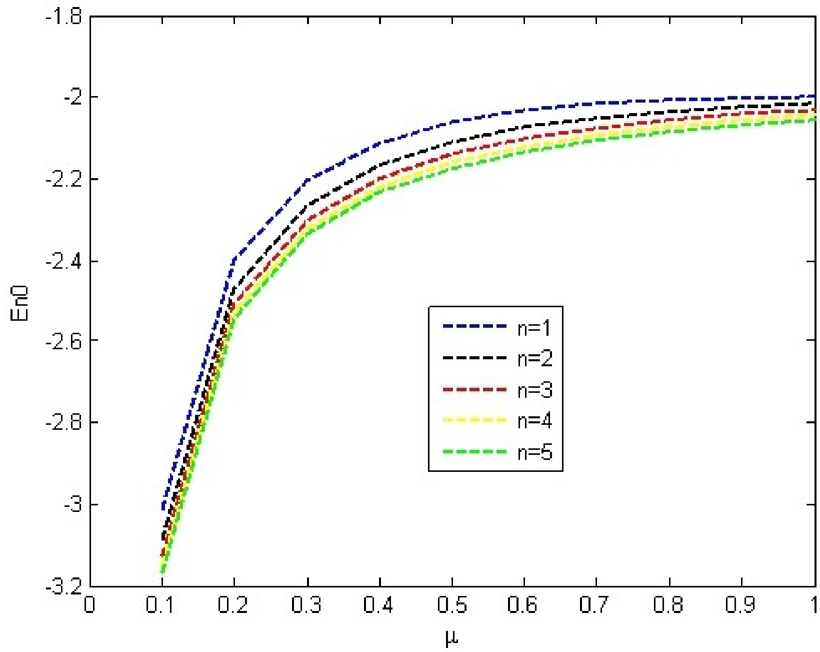

Figure 3. Variation of the energy spectrum as a function of the particle mass with $\delta = 1$, $A = 2$, $V_0 = 1$ and $\hbar = 1$.

Table 1. Energy spectrum ($-E_{n\ell}$ in fm^{-1}) with $2\mu = \hbar = 1$.

state	n	ℓ	δ	$A = 6, V_0 = 2$	$A = 2, V_0 = 6$	$A = V_0 = 3$
2p	0	1	0.10 0.15 0.20	13.72250000 4.836736111 2.490000000	214.6225000 94.57562500 53.29000000	47.71000000 20.17562500 11.09000000
3p	1	1	0.10 0.15 0.20	6.733611111 2.841674383 1.867777778	97.02250000 43.67006944 25.33444444	22.03361111 9.811736111 5.801111111
3d	0	2	0.10 0.15 0.20	6.673611111 2.551118827 1.361111111	97.94027778 44.40173611 25.89444444	22.26250000 9.843402778 5.650000000
4p	2	1	0.10 0.15 0.20	4.300625000 2.172934028 1.702500000	55.87562500 25.88265625 15.60250000	13.06000000 6.213906250 4.002500000
4d	1	2	0.10 0.15 0.20	4.100625000 1.766684028 1.102500000	56.22562500 26.05140625 15.60251000	13.02250000 5.988906250 3.602500000
4f	0	3	0.10 0.15 0.20	3.810000000 1.178402778 0.240000000	56.76000000 26.32562500 15.64000000	12.97562500 5.672500000 3.040000000
5p	3	1	0.10	3.188900000	36.84490000	8.920900000
5d	2	2	0.10	2.924000000	36.93210000	8.760100000
5f	1	3	0.10	2.532900000	37.06890000	8.524900000
5g	0	4	0.10	2.022500000	37.26250000	8.222500000
6p	4	1	0.10	2.600277778	26.52250000	6.687777778
6d	3	2	0.10	2.300277778	26.46694444	6.460000000
6f	2	3	0.10	1.854444444	26.38777778	6.122500000
6g	1	4	0.10	1.267777778	26.29000000	5.680277778

Similarly, when $A=0$,the potential Eq. (2) reduces to Hulthèn and the energy equation turns to

$$E_{n\ell}=\frac{\ell(\ell+1)\delta^2\hbar^2}{2\mu}-\frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(-\frac{V_0}{\delta}\right)-\delta\left(\ell+n+1\right)^2-\ell\left(\ell+1\right)\delta}{2\left(\ell+n+1\right)}\right]^2.. \tag{36}$$

When$V_0=\delta b$, Eq. (34) turns to energy equation for the Hellmann potential which is identical to Eq. (24) of Ref. [13].

$$E_{n\ell}=\frac{\ell(\ell+1)\delta^2\hbar^2}{2\mu}-\frac{\hbar^2}{2\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(A-b\right)-\delta\left(\ell+n+1\right)^2-\ell\left(\ell+1\right)\delta}{2\left(\ell+n+1\right)}\right]^2. \tag{37}$$

In **Figure 1** - **Figure 3**, we have plotted energy with δ, A and μ. In **Figure 1**, the energy increases with δ and later decreases. In **Figure 2**, as A increases, the energy decreases (attractive). We numerically reported energy eigenvalues for Coulomb potential minus Hulthèn potential. It is deduced that as δ increases, the energy increases towards positive. The energy is more attractive (negative) when $A<V_0$ as shown in the **Table 1**.

5. CONCLUSION

In this work, we have solved the Schrödinger equation for the combination of Coulomb potential and Hulthèn potential in the framework of supersymmetric and Nikiforov-Uvarov methods by considering a suitable approximation scheme to get rid of the centrifugal barrier and obtained energy eigenvalues and the wave functions. It is seen that the energy equation obtained with the two methods are identical. This shows that the two methods are in excellent agreement. Some special cases of interest of the solution are obtained. These are eigenvalues for Coulomb potential, Hulthèn potential and Hellmann potential by putting $V_0=0$, $A=0$ and $V_0=\delta b$ respectively. In **Figure 1** - **Figure 3**, we make some plots to see the behavior of energy with screening parameter, potential strength and particle mass.

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