Probabilistic Neural Networks for Multi-user Detection in Code Divisional Multiple Access Communication Channels

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Abstract:- A Probabilistic Neural Network (PNN) is proposed and applied here for implementation of a Maximum Likelihood (ML) detector and classifer.The network is trained using the algorithm based on Parzen probability density function estimation theory for detection of signals in CDMA multi-user communications Gaussian channel. And, by viewing these multi-user detector's problem as a nonlinear classification decision problem, we apply this algorithm, which has the abilities of arbitrary nonlinear transformations, adaptive learning and tracking to implement this decision optimally and adaptively. The performance of the proposed neural networks detector is evaluated via extensive computer simulations and compared with other detectors and neural classifiers' schemes in a multi-user environment. The neural detector is shown to exhibits some desirable properties and significantly outperforms the conventional matched filter detector.

1. Introduction

In a situation where the receiver is interested in the data transmitted by only one of the K active users, the receiver needs to demodulate a single-user signal in the presence of K-1 interfering users. It has been shown in [13] that, the optimum decentralized receiver is a one shot detector where the detection of each symbol is based upon the received process during its corresponding interval, (e.g., $b_k^{(i)}$, is processed only in the interval $[iT+\tau_k,(i+1)T+\tau_k]$). In a K active users asynchronous CDMA Gaussian channel with a set of preassigned signature waveforms, $s_k(t)$, single users' detection can be represented as a binary hypothesis-testing problem, $\mathbf{H_1}$ **and** $\mathbf{H_0}$:

$\mathbf{H_1}$: $r(t) = S_i + n_i$

$$:\bar{r}(t)=+S_1(t)+\sum_{K=2}^{K}\left[b_k^{(-1)}S_k(t-\tau_k+T)+b_k^{(0)}S_k(t-\tau_k)\right]+n_t \quad (11a)$$

$\mathbf{H_0}$: $r(t) = -S_i + n_i$

$$:\bar{r}(t)=-S_1(t)+\sum_{K=2}^{K}\left[b_k^{(-1)}S_k(t-\tau_k+T)+b_k^{(0)}S_k(t-\tau_k)\right]+n_t, \quad (11b)$$

where r(t) are the samples of the received continuous waveforms. The samples of the desired signal is S_i , and the samples of the interference from the other k-users and Gaussian noise are n_i. The desired information symbol is assumed without loss of generality, to be the first bit in the packet of the first user $b_1^{(0)}$, and where the overlapping bits of the k-th user with $b_1^{(0)}$ are denoted by $\{b_k^{(0)},b_k^{(-1)}\},k=2,3,..,K$. In a multi-user environment, asynchronous arrival of interfering signals increases the complexity of the optimum receiver due to the fact that two adjacent bits of each interfering user overlaps with the desired bit. Since there is no time synchronism among transmitters, there are 2K-2 bits overlapping with the first user during the decision interval. Therefore, the 2K-1 orthonomal basis functions are constructed from the product of full desired user's waveform and time shifted partial code waveform of the other users.

2. The Neural Networks Implementation

The overall diagram of the neural network receiver considered for the detection problem described in equation (1) and the conversion of the continuous time received signal into a discrete time vector signal is shown in figure 1. It consists of a correlator, a sampler, a buffer and a PNN detector. Since multilayer perceptrons allow only for discrete time inputs, the continuous time hypothesis testing problem in (1) is converted to its corresponding discrete version by integrating over the chip interval, T_c, sampling at the chip rate, buffered the samples, and then sampling at the symbol rate, $(\frac{1}{T_b})$. The length of the signature sequence, s_k, is N. Thus, the vector of chip rate samples taken during the i-th symbol interval and the Gaussian noise process are of dimension N × 1.

2.1 Hypothesis testing and ML detection

As we shall be concentrating on the detection of a single user in a multi-user Gaussian channel, the detection and classification problem of interest is modeled as a binary hypothesis-testing problem [13]. Essentially, there exist two relevant possibilities: our active k-user's transmit +1 or a -1.

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This user's bits are embedded in noise that consists of Multiple Access Interference (MAI) and Gaussian noise. When all the system parameters are known, we can characterized the statistics of the noise. We consider Bayesian hypothesis, which minimizes the average cost incurred by the decision rule[8]. And, knowing fully well that neural networks which has three or multi-layers perceptron (MLP) structure could implement arbitrary complex nonlinear functions by the use of proper learning algorithm, provided that the number of neurons in the hidden layers is adequate. We therefore apply the Probabilistic Neural Network algorithm that is based on Parzen probability density function estimation theory to implement the nonlinear decision function so as to realized a CDMA multi-user detector (MUD). The optimum MUD for any user k is the problem of nonlinear decision and classification which has two states as given in equation (1).

We denote the probability density, $p(x_m/H_1)$, of the observation of the desired signal in noise given that hypothesis (H_1) is true by;

$$p(x_m/H_1)=\frac{1}{n}\sum_{i=1}^{n}\prod_{m=1}^{M}\frac{1}{\sqrt{2\pi\sigma_n}}\exp\left[-\frac{(x_m-x_{im})^2}{2\sigma_n^2}\right] \quad (2)$$

and similarly, the probability density, $p(x_m/H_o)$, of the observation given that hypothesis (H_0) is true by;

$$p(x_m/H_0)=\prod_{m=1}^{M}\frac{1}{\sqrt{2\pi\sigma_n}}\exp\left[-\frac{x_m^2}{2\sigma_n^2}\right] \quad (3)$$

From equations (2) and (3), We have the resulting likelihood ratio for a binary decision problem as;

$$\lambda(x_m)=\frac{p(x_m/H_1)}{p(x_m/H_0)}\cong\frac{1}{n}\sum_{i=1}^{n}\exp\left[\frac{1}{\sigma_n^2}\sum_{m=1}^{M}x_m x_{im}-\frac{E_s}{2\sigma_n^2}\right] \quad (4)$$

The received discrete time signal vectors into the neural network is $\mathbf{x_m}$ and the i-th samples (+1or-1) of the m-th signal is $\mathbf{x_{im}}$ .The training samples number is **n**. The Gaussian noise variance of '**n**' is σ^2, and $E_s=\frac{1}{n}\sum_{i=1}^{n}x_{im}^2$ is the signal energy. **M** is the number of samples in one bit, for i = 1,2,...n, and m = 1,2,...M. The Bayesian decision rule compares the likelihood ratio to a threshold, β, which is determined by a cost function and prior probabilities of the desired signal vector. For unit cost for correct decision and a zero cost for an incorrect decision, and for equal priors, the desired threshold in the work is taken as, β = 1. The decision rule is:

H_1 is true : $p(x_m/H_1) > p(x_m/H_o)$ (5a)

H_0 is true : $p(x_m/H_1) < p(x_m/H_o)$ (5b)

We also assume that each user's bit is independent of the other users' bits and each data bit is equally likely to be +1, -1. Thus, it can be seen that in other to make a decision one has to calculate $p(x_m/H_1)$ and $p(x_m/H_o)$using equations (2) and (3). The likelihood ratio in equation (4) is now used for the construction of the Receiver Operating Characteristics (ROC) curves for this detection problem. An ROC curve for a binary hypotheses problem is a plot of the probability of detected signals, $\mathbf{P_D}$, versus the probability of un-detected signals, $\mathbf{P_{No.D}}$, calculated from equation (6) by simulation for different values of varying threshold, β = {1,-1}. That is,

$$P_D = \int_{\beta}^{\alpha} p(\lambda / H_1)d\lambda \quad (6a)$$

$$P_{No.D} = \int_{\beta}^{\alpha} p(\lambda / H_0)d\lambda \quad (6b)$$

Equation (6) also forms the two performance measures for the detection problem. From (4), it is seen that the neural detector consists of M biased correlators, the output of which are exponentiated and added to form the likelihood ratio. The exponential nonlinearities are as a result of the knowledge that the MAI and additive noise has a Gaussian distribution.Thus, providing the system with noise information.

2.2 The Neural detector architecture

The neural network applied here for implementing the maximum likelihood detector and classifier is as described by equation (5), and the architecture realized is shown in Figure 3. It consists of two sets of networks (i.e., PPNN 1 and PPNN 0). One set is trained by the signal samples coming from hypothesis H_1 : b_k = 1, while the other is trained from samples coming from hypothesis H_0 : b_k = -1. It is a multi-layer feedforward network trained with the Probabilistic Neural Network algorithm that uses a sum of Gaussian distributions to estimate the probability density function (pdf) for a training data samples. The trained network is then used to classify new data samples based on the learned pdfs, and further to provide a probability factor associated with each class. The developed network though similar in structure to the Back Propagation Neural Network (BPNN) with three layers structure of: input, pattern (or hidden), and summation (or output) layers; but differs primarily by employing exponential function instead of sigmoid function as the activation function in the pattern layer [7,11-12]. Both network 1 and network 0 consists of n pattern units, and they estimates the pdf of each pattern unit. All the estimated pdfs are summed together to produced the combined conditional pdfs, $f(x/H_1)$and $f(x/H_0)$. Figure 2 shows detailed structure of a single pattern unit and the 'n' pattern units of a network. The input layer of each network consist of M nodes and it distributes the sampled signals to the pattern units. In the hidden (pattern) layer, the number of the pattern units correspond with the training samples number. The weight coefficients between inputs and in each of the pattern units of each network are set to corresponds to each of the training samples number from both hypotheses, $\mathbf{H_0}$ and $\mathbf{H_1}$, respectively. The input samples are distributionally processed, making the structure entirely plane and its hardware implementation easier. The output or decision device, which is a two-input neurons then compares the likelihood ratio functions outputted by each network 1 and network 0, and produces a binary signal output (-1 or 1) to declare that the input samples belongs to the hypothesis associated to the larger pdf. Hence, it has one node obtaining values in the range [1,-1] for the detection problem and M nodes for the classification problem.

As done in the case of the detection, the neural classifier for equal received powers of active users is based on cross-correlating transmitted sequences with the other interfering users' signals corrupted with Gaussian noise [5,6,8]. Instead of treating the MAI as the white Gaussian noise simply as the conventional detector, the proposed multi-user detector uses the PNN to estimates the combined conditional pdf of the signal samples to improve the system performance.

3. Discussion of simulation results

In the paper, we consider the performance of a feedforward multilayered perceptron referred to as Probabilistic Neural Network based maximum likelihood detector for the detection and classification of a single user in an asynchronous multi-user Gaussian channel. Shown in figures 4 are the ROC curves obtained for the neural multi-user detector for K = 6 active users with additive Gaussian noise. The K active users employ a spreading code of length N = 63. The training samples are generated for ratios of signal energy over noise variance of 10 and 1 with 10^3 samples. During simulations, the relative delays of the interfering users' signals at the receiver are assumed to be unknown constant during the training and data transmission periods. It is a one pass training algorithm, and one of the important consideration in employing neural networks receiver is the length of training periods necessary for near optimum performance. The probability of detection versus the probability of no-detection are plotted when the network is trained with and without noise (i.e., interfering users and Gaussian boise) added to the training samples during the training stage for different signal-energy-to-noise variance ratios. And, after the completion of the training process, a large number of patterns consisting of pure Gaussian noise samples and the MAI signals with added Gaussian noise was propagated through the neural detector. The values of the output nodes for each of the above samples was then compared to a threshold varying in the range [-1,1]. An output containing positive value samples which is larger than the threshold is interpreted as detection. Similarly, an output containing negative value samples exceeding the threshold is interpreted as no-detection. The number of detection and no-detection for every threshold value over the total number of propagated positive and negative value samples, yield the estimates of the pdf, which forms the PNN receiver operating characteristic (ROC) curves.

The effectiveness of the PNN detector in a near-far situation (i.e., the spreading code and the values of relative energies chosen to results in a worst case interference) is shown in the graph of the ROC curves in fig. 4. As can be seen, the distances between the curves obtained from simulations are not too far away from each other, which indicates no much performance degradation in the presence of interference. High detection probability were obtained for a quiet large number of active users in the system, but this tend to fall gradually as the signal-energy-to-noise variance ratios becomes smaller. The principal conclusion from our simulation results is that, the proposed PNN detector is capable of detecting user's signal in a multi-user communication Gaussian channel with a short spreading factor in the presence of strong interfering users. Because the weight coefficients between the inputs and in each of the pattern units is made to corresponds to each of the training samples number, the network computation and training algorithm becomes so simple and fast. There are indications that the neural detector is optimal when user's number is one, and near-optimal as the number of interfering signals become so large.

The classification results:- The classifier was tested on a large number of training samples corrupted with and without noise information for various E_s/σ^2. The performance of the PNN classifier was then evaluated on the basis of the number of misclassified patterns over the total number of transmitted sequences through the classifier. As can be seen from tables 1 and 2 below, the network classification error depends on the presence of total noise interference in the training stage, and classification error tends to change slightly with increase in number of active users. For example, with 6 active users in the channel, the classification errors when compared with that of 10 active users is much lower. Secondly, differences between classification errors of the network trained with and without noise interference tend to decrease for smaller signal energy to noise variance ratios of 10 and 1. Also, the network trained without noise interference for higher $E_s / \sigma^2 = 1$ gives lower classification errors than that of network trained with noise interference for lower variance ratio. The optimal (Bayesian) classifier which offers the lower bounds for the classification error is used here as a criterion for evaluating the proposed classifier employed in the classification problem for both the 6 and 10 active users. Indication from extensive simulation results shows that for an infinitely large number of training samples, the classification

TABLE 1. PNN Classification Errors

K=6 users	Classification Error for **Optimum Classifier**	Classification Error for neural nets **Classifier trained Without Noise**	Classification Error for neural nets **Classifier trained With Noise**
$E_s/\sigma_n^2 = 10$	**0.0370**	**0.0421**	**0.0820**
$E_s/\sigma_n^2 = 1$	**0.4190**	**0.4120**	**0.4910**

TABLE 2

K=10 users	Classification Error for **Optimum Classifier**	Classification Error for neural nets **Classifier trained Without Noise**	Classification Error for neural nets **Classifier trained With Noise**
$E_s/\sigma_n^2 = 10$	**0.0821**	**0.0950**	**0.2150**
$E_s/\sigma_n^2 = 1$	**0.4500**	**0.4721**	**0.7880**

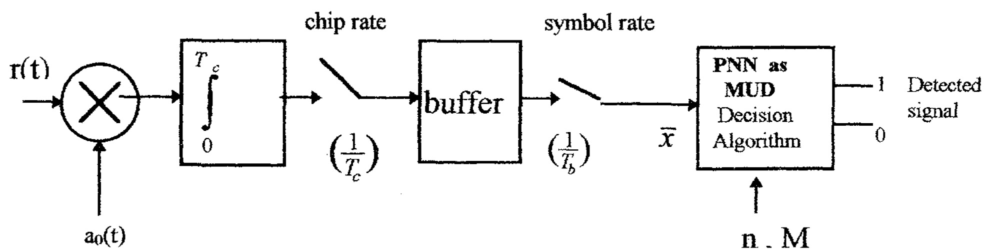

Figure 1. The Neural Network multi-user receiver and Its conversion into Discrete time received vector

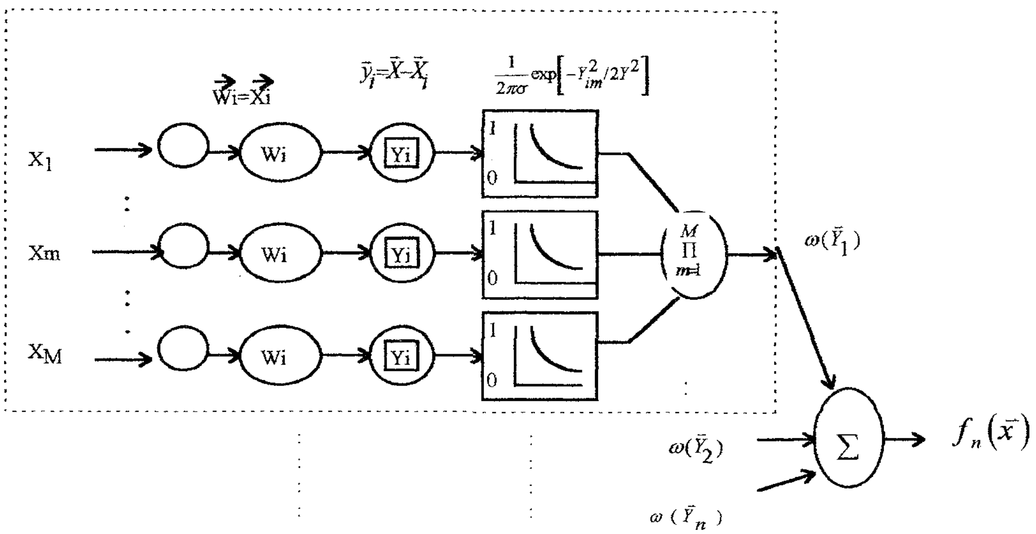

Figure 2. Detailed structure of the inputs and the 'n' pattern units of the networks

error of the PNN classifier is minimum compared to that of small number of training samples.

4. Conclusions

In this paper, we have proposed a Probabilistic Neural Network detector and classifier, in which extensive simulation results shows an improved performance over the conventional multi-user detector. And, when compared with other neural networks multi-user detector schemes in a multi-user environment, the proposed PNN detector exhibits some desirable properties and significantly outperforms them in terms of low computational complexity, high processing speed and low classification error. Also, the PNN algorithm shows good adaptivity to any change in system parameters during the training and detection periods. As a result, displays a great resistance to the near-far problem, and thereby eliminating the need for strict power control. It also shows good promise as a single user detector. Because memory is dense and inexpensive with the availability of ASICs, the need for acceptable storage for the training samples may not pose a problem to the neural detector hardware implementation. More research works into the PNN detector's capability to perform in unsymmetric noise case, and in multipath or fading communications environment is still going on.

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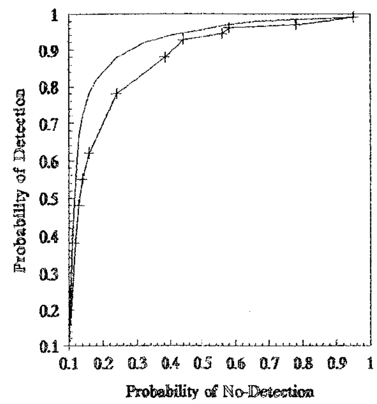
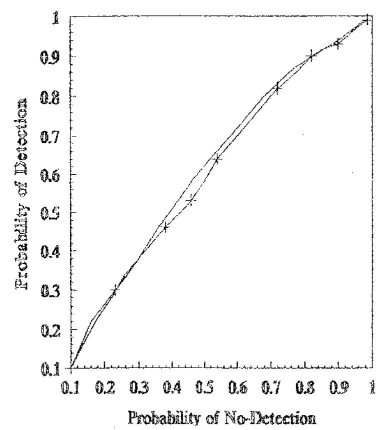


(a)



(b)

Figure 4. The ROC curves showing the performance of PNN detector trained without noise (plain line) and with noise (crossed line) for K = 6 asynchronous CDMA users for a signal-to-noise variance ratio of (a) 10 and (b) 1.

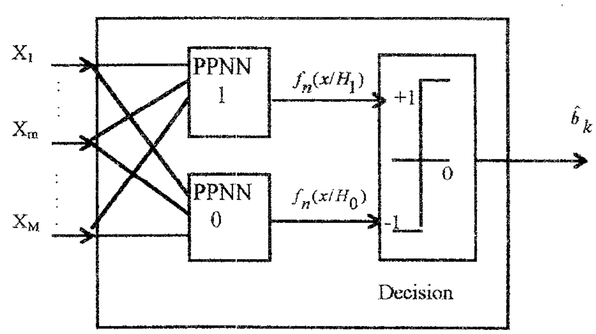


Figure 3. The Probabilistic Neural Network Detector.