

# Optimum Multi-user Signal Detector in SSMA Communication Systems Based on Neural Networks

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**Abstracts:-** This paper is motivated by the fact that, in a multi-user CDMA system, the conventional receiver suffers severe performance degradation as the relative powers of the interfering signals becomes large (i.e., "near-far problem"). Furthermore, in many cases the optimum multiuser receiver, which alleviates the near-far problem, is too complex to be of practical use. And by viewing this optimum multi-user detector problem in CDMA channel as an optimum nonlinear classification decision problem, we apply a simple feedforward multilayered perceptrons refer to as Probabilistic Neural Network based Maximum Likelihood Rule that has the abilities of arbitrary nonlinear transformations, adaptive learning and tracking to implement this classification decision optimally and adaptively. The performance of this proposed neural detector is evaluated via computer simulations in terms of probability of detection and compared with other neural and conventional detector schemes in a multi-user environment.

**Keywords:-** Spread Spectrum Multiple Access (SSMA), multi-user detector (MUD), Probabilistic Neural Network (PNN), Maximum Likelihood (ML), MultiLayer Perceptron (MLP)

## 1: Introduction

Code divisional Multiple Access (CDMA / or SSMA) due to its high capacity potential and other desirable features such as anti-multipath, fading and antijamming capabilities is aim at providing a much higher capacity than the previous generations of mobile communication system. This technique permits all the users in the system to transmit simultaneously, operate at the same nominal frequency, and use the entire system bandwidth. It is achieved by spreading the spectrum of the transmitted signals using pre-assigned code waveforms. The traditional approach of demodulating the spread spectrum signals in multi-user system is that the spread spectrum signals is made to pass through a filter matched to the desired signal, then the decision is made directly [1]. This approach ignores the multiple-access interference (MAI), or equivalently, ignoring the cross-correlation between the modulating signals of different users. For this reasons, it has stimulated the research

interest in optimum receivers in multi-user communication system. And in [2], the scheme has been shown to have a complexity that is exponential in the number of users. Therefore, research efforts now focused on receivers that will; exhibits good near-far resistance properties, have low computational complexity to ensure their practical implementation, and achieve good bit error rate.

The main idea of optimum multiuser detectors is to find the supposed vector  $\hat{b}^* = (b_1^*, b_2^*, \dots, b_k^*)^T$  which makes the likelihood function maximum on the basis of the received signal's waveform 'r(t)' for the interval  $\{0, T_b\}$  [1,2]:

$$r(t) = \sum_{i=-M}^M \sum_{k=1}^K b_k^{(i)} s_k(t - iT - \tau_k) + n(t) \quad t \in [iT, (i+1)T] \quad (1)$$

In equ. (1),  $s_k(t)$  for  $k = 1, 2, \dots, K$ , is a signature waveform time limited in the interval  $[0, T]$  assigned to each transmitter.  $b_{ki} \in \{+1, -1\}$  is the  $i$ -th data bit of the  $k$ -th user.

$\tau_k \in [0, T]$  are the relative time delays associated with all users and  $2M+1$  is the packet size.  $n(t)$  is the AWGN with a spectral density with height  $\sigma^2$ . When all the possible sequences are independent and equally likely, the optimum receiver and ML decision formula can be written as  $b^*$

$$= \arg \max_{b \in \{-1, 1\}^k} \left\{ 2 \sum_k \int_0^{T_b} b_k s_k(t) r(t) dt - \int_0^{T_b} \left[ \sum_k b_k s_k(t) \right]^2 dt \right\} \quad (2)$$

$$\text{Or, } b^* = \arg \max_{b \in \{-1, 1\}^k} \left\{ 2 y^T b - b^T H b \right\} \quad (3)$$

where  $y = (y_1, y_2, \dots, y_{k-1})^T$  is the sampled vector of output  $y_k$  at time  $t = T_b$ , which is the sampled output of the normalized correlation receiver matched to the  $i$ -th signal passed by the received signal  $r(t)$ . The sampled vector  $y$  is called the observation vector and it's a sufficient statistic for demodulating  $b$ , is written as

$$y = Hb + n = RWb + n \quad (4)$$

Where  $H = WR$  is the equivalent transfer matrix.  $H \in \mathbb{R}^{k \times k}$  is the symmetric matrix of signal cross-correlations.

$$h_{ii} = \int_0^T s_k(t) s_i(t) dt = \int_{-\infty}^{\infty} s_k(t - \tau_k) s_i(t - jT - \tau_i) dt, \quad (5)$$

Where  $W$  is called the energy matrix, and is the diagonal matrix in which the diagonal elements  $w_{ii} > 0$ , denoting the signal energy of the  $i$ -th user in the received signal  $r(t)$ . When the transmitted power is constant,  $W_{ki}$  depends on the distance. When the  $k$ -th user changes his location,  $W_{ki}$  will be changed.  $R = [r_{ij}]_{k \times k}$  is the correlation matrix of each user's PN code, which is a diagonal matrix. Since each user's PN code is constant, matrix  $R$  is not changed. As there is no absolute orthogonality between PN codes, the non-diagonal elements of  $R$  is not zero. Therefore, there unavoidably exist MAI in the system, and  $n$  is a zero-mean

Gaussian  $K$ -vector with covariance matrix equal to  $\sigma^2 H$ , given as  $n = \int_0^T n(t) s_k(t) dt$ .

## 2: The PNN Implementation and Architecture

The optimum MUD task is to decide and estimate each user's transmitted signal  $b = (b_1, b_2, \dots, b_k)^T$  from the observation vector  $y = (y_1, y_2, \dots, y_{k-1})^T$  at time  $jT_b$  according to equ. (3) and figure 1. How to do this is the problem to be solved by the MUD. So, the MUD problem is a problem of decision and classification. And because neural networks (NN) with MLP structure could implement arbitrary complex nonlinear functions by the use of proper training algorithm, provided that the number of neurons in the hidden layers is adequate. So, we consider the use of PNN algorithm based on Parzen probability density function estimation theory to implement the nonlinear classification decision function problem and to approximate a solution to the maximization problem in equ. (3) that is known to be NP-hard complete[1]. The training process adapts to the asynchronous nature of the system if the relative delays and phases of the active users' signals are assumed to be unknowns constant during the training and data transmission periods. And since MLPs allow only for discrete time inputs, the continuous time received CDMA signal is converted to its corresponding discrete time signal version to form the input to the neural network. Essentially there exist two relevant possibilities: our active  $k$ -user's transmit +1 or a -1. This user's bits are embedded in noise that consists of MAI and Gaussian noise. And for the proposed NN to implement a ML detector, we viewed the input vectors (i.e., active user's bits embedded in MAI and Gaussian noise) to the network for detection as the

following classification problem, or as a binary hypothesis-testing problem  $H_1$  and  $H_0$  [5]:

$$H_1 : y = +A_k S_k + I + \mu \quad (6a)$$

$$H_0 : y = -A_k S_k + I + \mu \quad (6b)$$

where  $y$  is the observation of the active user's signal in noise, the desired signals vector is  $\pm A_k S_k$ . The received signal amplitude of desired users is  $A_k$ , and its spreading code denoted by sequence  $S_k$ . The sum of the interfering user's signals is ' $I$ ', and ' $\mu$ ' is the length- $N$  vector of additive white Gaussian noise (AWGN) noise samples. A ML detector calculates a likelihood ratio ' $\lambda$ ' for a binary decision problem and compare it to a threshold ' $\beta$ ' [5]:

$$\lambda(y) = \frac{f(y/H_1)}{f(y/H_0)} \underset{H_0}{\overset{H_1}{>}} \beta \quad (7)$$

$f(y/H_1)$  is the conditional density function of the observation of desired users' signals in noise given that ( $H_1$ ) is true

$$f(y/H_1) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} \frac{1}{(2\pi)^{\frac{N}{2}} \sigma^N} e^{-\frac{1}{2\sigma^2} (y_m - y_{+im})^T R^{-1} (y_m - y_{+im})}$$

and similarly,  $f(y/H_0)$  is the conditional density function of the observation given that hypothesis ( $H_0$ ) is true:

$$f(y/H_0) = \frac{1}{2^{K-1}} \sum_{i=1}^{2^{K-1}} \frac{1}{(2\pi)^{\frac{N}{2}} \sigma^N} e^{-\frac{1}{2\sigma^2} (y_m - y_{-im})^T R^{-1} (y_m - y_{-im})}$$

For this work, the threshold  $\beta$ , is taken to be equal to one. Thus, the following decision rule is applied:

$$H_1 \text{ is true : } f(y/H_1) > f(y/H_0) \quad (10a)$$

$$H_0 \text{ is true : } f(y/H_1) < f(y/H_0) \quad (10b)$$

where  $y_{im}$  is the  $i$ -th samples (+1or-1) of the  $m$ -th signal.  $y_{+im} \in \{y = Hb = RWb / b_k = +1\}$  and  $y_{-im} \in \{y = Hb = RWb / b_k = -1\}$ , for  $i = 1, 2, \dots, 2^{k-1}$ , are the non-noise

observing vectors which has  $2^{k-1}$  vectors when the  $k$ -th users transmits signals  $b_k = +1$  and  $b_k = -1$ , respectively. All the vectors and the nonlinear decision function  $f_k(y)$  are only been determined by matrix  $H = R W$  and the noise power,  $\sigma^2$ . To make a decision one has to calculate  $f(y/H_1)$  and  $f(y/H_0)$  using equations (8) and (9). The key to using the likelihood ratio equation in (7) in the network is the ability to estimate accurately the PDF based on the training samples, and it is now used for the construction of the Receiver Operating Characteristics (ROC) curves for this detection and classification problem, and it is a random variable distributed according to  $f(y/H_0)$  and  $f(y/H_1)$  for hypotheses  $H_0$  and  $H_1$ . An ROC curve for a binary hypotheses problem is a plot of the probability of detected signals ' $P_D$ ' versus the probability of undetected signals ' $P_{NoD}$ ' calculated from equ.(11) below by simulation for different values of varying threshold,  $\beta = \{1, -1\}$  to forms the two performance measures for the detection problem.

$$P_D = \int_{\beta}^{\alpha} f(\lambda/H_1) d\lambda \quad \text{and} \quad P_{NoD} = \int_{\beta}^{\alpha} f(\lambda/H_0) d\lambda \quad (11)$$

The network architecture realized for implementation of a ML detector and classifier is as shown in figure 2, and described by equ.(10) above. It is a MLP networks trained with the PPNN algorithm that uses a sum of Gaussian distributions to estimate the PDF for a training data samples. The trained network is then used to classify new data samples based on the learned pdfs, and further, to provide a probability factor associated with each class. The developed network though similar in structure to the Back Propagation NN with three layers structure of: input, pattern (or hidden) and summation (or output) layers; but differs primarily by employing exponential function instead of sigmoid function as activation function in the pattern

layer[4,8]. The input layer of the network consist of  $M$  nodes and distributes the sampled signals to the pattern units which has  $2^K$  nodes, while the output or decision device which is a two-input neurons compares the likelihood ratio functions outputted by the two networks and produces a binary signal output (-1 or 1) to declare that an output containing positive value samples which is larger than the threshold is detection, and no-detection for a negative value samples exceeding the threshold at the network output. The number of detection and no-detection for every threshold value over the total number of propagated positive and negative input value samples, respectively, yield the estimates of the PDF of detection and no-detection, which forms the PPNN ROC curves. In the pattern layer, the pattern units number corresponds with the training samples number. The weight coefficients of network 1 and 0 are determined by training samples number 'n' from both hypotheses  $H_0$  and  $H_1$ . The output of the two networks now produces the conditional density functions  $f(y/H_1)$  and  $f(y/H_0)$ .

### 3: Discussion of Simulation Results

Here, we consider the performance of a feedforward MLPs referred to as PPNN based ML detector for signal detection and classification in an asynchronous multi-user communications Gaussian channel with near-far problem. Shown in figure 3 is the ROC curves obtained for the neural network CDMA multi-user detector for  $K = 5$  active users with AWGN and employing a spreading code of length  $N = 63$ . The training samples number generated vary from 100 to 1000 for a ratio of signal energy over noise variance of 10 and 1. The effectiveness of the proposed PNN detector in a near-far situation is fully shown in these graphs. The probability of detection versus the probability of no-detection when the network is trained with and without

noise (i.e., active users and AWGN) added for different signal-energy-to-noise variance ratios are also shown. As can be seen, the distances between the curves obtained from simulations are not too far away from each other, which indicate no much performance degradation with addition of the noise interference. High detection probability were attained for even a quiet large number of active users in the system, but this tend to fall as the signal-energy-to-noise variance ratios becomes smaller, i.e.,  $E_s/\sigma_n^2=10$ .

### 4: Conclusions

In this paper , we have proposed a PNN detector and classifier, in which simulation results shows an improved performance over the conventional CDMA detector. And compared with other neural network multi-user detector schemes in a multi-user environment[6,7], the PNN detector exhibits a number of attractive properties and significantly outperforms them in terms of low computational complexity, high processing speed, low classification error and near-far resistance. Our proposed neural detector shows adaptability to some of the unknown parameter in the system, especially, the energy matrix (power) which vary with the distance between users. These change of the energy matrix which is caused by the users movement are adaptively traced by the neural network. As a result, the detector displays a great resistance to the near-far problem, thereby eliminating the need for strict power control. Also, because memory is dense and inexpensive with the development and availability of ASICs and programmable general-purpose digital signal processing techniques, the need for acceptable buffer size for the training samples will not pose a problem to the neural detector hardware implementation. More research work into its ability to be applied in unsymmetric noise case, and in multipath fading environment is still going on.

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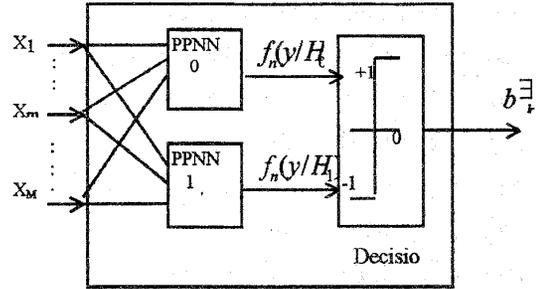


Figure 2. The Probabilistic Neural Network Detector.

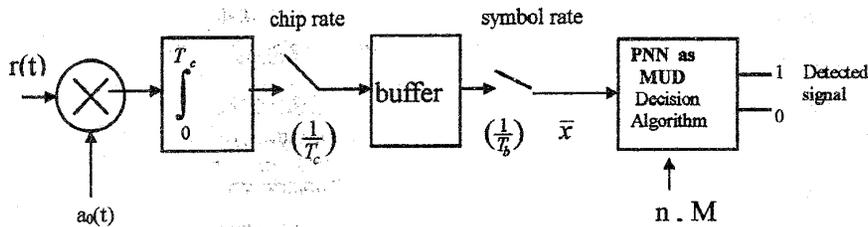


Fig. 1. The Neural Network Multi-user Receiver and Its Conversion into Discrete Time Received Vector