Abstract—A new modular relative Jacobian formulation for single end-effector control of combined 3-arm cooperating parallel manipulators is derived. It is based on a previous method of derivation for dual-arm robots, with the same approach of modularity and single end-effector control for combined manipulators. This paper will present this new formulation, as well as investigate task prioritization scheme to verify the claim that a single end-effector controller of combined manipulators will indeed implement a strict task prioritization, by intentionally adding more tasks. In addition, this paper will investigate a claim that the holistic approach to control of combined manipulators affords easier control coordination of each of the stand-alone components. Switching control from an individual manipulator control in the null space to relative control in the tasks space is shown to investigate the smoothness of task execution during switching. Simulation results using Gazebo 2.2.5 running in Ubuntu 14.04 is shown.

Index Terms—Task prioritization, holistic control coordination, 3-arm cooperating parallel manipulators, single end-effector control, relative Jacobian, modular kinematics

I. INTRODUCTION

This work is geared towards a future goal of achieving a holistic control of combined manipulators, particularly, a humanoid that can perform more complicated motion such as performing a dive with somersault, jump and kick in the air, doing a cartwheel, etc. which are not possible at the current state of the art control for humanoids or for other combined manipulators such as quadrupeds and hexapods. Part of the challenge is on the complexity of the combined physical structures such that a holistic approach in its kinematics model and an accurate cancellation of its resulting dynamics require considerable effort.

This work is part of a series of studies to utilize modularity in the kinematics and dynamics expressions of the combined manipulators, expressed as a single manipulator (with single end-effector). In particular, this work considers 3-arm cooperating parallel manipulators controlled as a single manipulator. A modular kinematics expression is derived that is expressed in terms of the kinematics of each of the stand-alone manipulators. Of the single end-effector control of combined manipulators, its claims include: (1) strict implementation of task prioritization, and (2) a holistic approach to coordinated control. These two claims may prove to be crucial towards more complicated combined manipulators motions.

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In this work, a modular relative Jacobian for the 3-arm cooperating parallel manipulator is derived. The concept of a relative Jacobian was first introduced in [1], [2]. In a new derivation of a modular relative Jacobian for dual-arms [3], a wrench transformation matrix was revealed that was not present or was not explicitly expressed in the previous relative Jacobians. Further studies shown in [4] highlighted the effects of the omission of the wrench transformation matrix on the exerted forces and moments at the dual-arm end-effectors, such that at certain configurations, its omission lead to non-contact for task that requires contact all the time. Asymmetric bimanual task was shown [5] for dual-arm performing at writing task using a relative Jacobian.

This work proposes to investigate more closely the task prioritization [6] and holistic coordinated control of combined 3-arm cooperating parallel manipulators (shown in Fig 1) as a one single robot with a single end-effector. The two main reason behind this type of control includes: (1) a drastic increase the null-space dimension of the resulting combined manipulators and (2) the principles of single manipulator control can now be applied to the combined manipulators. In terms of the drastic control of the null-space dimension, consider a dual-arm robot with each arm having seven degrees-of-freedom (7-DOFs). When each of the two arms is independently controlled in the full space, the resulting dual-arm robot has two degrees of redundancy (2-DORS). But when the arms are controlled in the relation full space, the resulting dual-arm robot has 8-DORS.

Modularity on the proposed approach enhances ease of
A Jacobian of the standalone manipulators. The Jacobian for a robot $i$ can be expressed with respect to those frames. Consider reference frames $\{i\}$ and $\{j\}$, such that $^i p_j$ is the position of frame $\{j\}$ with respect to frame $\{i\}$, and $^i R_j$ is the rotation of frame $\{j\}$ with respect to frame $\{i\}$. In addition, a Jacobian $^i J_j$ can be expressed with respect to those frames. From the figure, we state the following conventions for the Jacobians of the stand-alone manipulators. The Jacobian for robot $A$ is $^2 J_2$, for robot $B$ is $^3 J_4$ and for robot $C$ is $^5 J_6$, each is expressed with respect to the indicated reference frame indices.

**II. Naming Convention for Symbols**

The naming convention for most symbols used in this work are shown in Table I. Based on the schematic diagram of the 3-arm cooperating parallel manipulators in Fig. 2, the reference frames are assigned. The base reference frames are odd-numbered, while the end-effector reference frames are even-numbered. Relative position vectors connect the end-effectors.

**III. The Modular Relative Jacobian of 3-Arm Cooperating Parallel Manipulators**

Based on the frame assignment shown in Fig. 2, we present here the modular relative Jacobians for dual-arms as derived in [3]. The relative Jacobian for a dual-arm consisting of robots $A$ and $B$ is

$$ ^2 J_4 = \begin{bmatrix} -2 \psi_4 & 4 \Omega_1 & 2 \Omega_3 & 3 J_4 \end{bmatrix}, $$

and the relative Jacobian for a dual-arm consisting of robots $B$ and $C$ is

$$ ^4 J_6 = \begin{bmatrix} -4 \psi_6 & 4 \Omega_4 & 4 \Omega_5 & 5 J_6 \end{bmatrix}, $$

and lastly, the relative Jacobian for dual-arm robots $A$ and $C$ is

$$ ^2 J_6 = \begin{bmatrix} -2 \psi_6 & 2 \Omega_1 & 2 \Omega_5 & 3 J_6 \end{bmatrix}. $$

Such that the wrench transformation matrix $^i \Psi_j$ is defined as

$$ ^i \Psi_j = \begin{bmatrix} I & -S(^i p_j) \end{bmatrix} $$

![Fig. 2. An schematic diagram of a 3-arm cooperating parallel manipulator, with the corresponding reference frames and the relative position vectors.](image-url)
and the rotation matrix \( ^i\Omega_j \) is expressed as

\[
^i\Omega_j = \begin{bmatrix}
^iR_j & 0 \\
0 & ^iR_j
\end{bmatrix}.
\] (5)

Given \( \omega = [\omega_x, \omega_y, \omega_z]^T \), the operator \( S(\omega) \) is the skew symmetric operator used to replace the cross-product operator and is expressed as

\[
S(\omega) = \begin{bmatrix}
0 & -\omega_z & \omega_y \\
\omega_z & 0 & -\omega_x \\
-\omega_y & \omega_x & 0
\end{bmatrix}.
\] (6)

To complete the definition of the modular dual-arm manipulators, we express them here as paired robots. We express them here as

\[
\begin{align*}
^2p_4 &= 2R_1(1p_3 + 1R_3^3p_4 - 1p_2) \\
^4p_6 &= 4R_3^3(3p_5 + 3R_5^5p_6 - 3p_4) \\
^2p_6 &= 2R_1(1p_5 + 1R_5^5p_6 - 1p_2).
\end{align*}
\] (7)

To derive the modular relative Jacobian for the 3-arm cooperating parallel manipulators, we invoke the approach used in [3], that is, we express translational and rotational velocities of the end-effectors with respect to each other. Thus the relative position of frame \{6\} with respect to frame \{2\} is expressed as

\[
\frac{2}{3}p_6 = 2p_4 + 2R_4^4p_6.
\] (8)

We take the derivative of the above equation to get

\[
\frac{2}{3}\dot{p}_6 = \frac{2}{3}\dot{p}_4 + \frac{2}{3}R_4^4\dot{p}_6 + \frac{2}{3}R_4^4\dot{p}_6
\] (9)

\[
= \frac{2}{3}\dot{p}_4 + S(2\omega_4)2R_4^4\dot{p}_6 + \frac{2}{3}R_4^4\dot{p}_6
\]

\[
= \frac{2}{3}\dot{p}_4 - S(2R_4^4\dot{p}_6)2\omega_4 + \frac{2}{3}R_4^4\dot{p}_6.
\]

The linearity of angular velocities allows us to express the relative angular velocity of frame \{6\} with respect to frame \{2\} as

\[
\frac{2}{3}\dot{\omega}_6 = 2\omega_4 + \frac{2}{3}R_4^4\dot{\omega}_6.
\] (10)

By combining (9) and (10), we get

\[
\begin{bmatrix}
\frac{2}{3}\dot{p}_6 \\
\frac{2}{3}\dot{\omega}_6
\end{bmatrix} = \begin{bmatrix}
\frac{2}{3}\dot{p}_4 - S(2R_4^4\dot{p}_6)2\omega_4 + \frac{2}{3}R_4^4\dot{p}_6 \\
\frac{2}{3}\dot{p}_4 + S(2\omega_4)2R_4^4\dot{p}_6 + \frac{2}{3}R_4^4\dot{p}_6
\end{bmatrix}.
\] (11)

We then simplify the above expression by combining linear and rotational terms together and express the result in terms of the dual-arm relative Jacobians shown from (1) to (3) to get

\[
\begin{bmatrix}
\frac{2}{3}\dot{p}_6 \\
\frac{2}{3}\dot{\omega}_6
\end{bmatrix} = \begin{bmatrix}
\frac{2}{3}J_{p_6} \\
\frac{2}{3}J_{\omega_6}
\end{bmatrix} = \begin{bmatrix}
2J_{p_4}\dot{q}_{24} - S(2R_4^4\dot{p}_6)2J_{p_4}\dot{q}_{24} + 2R_4^4\dot{p}_6\dot{q}_{46} \\
\frac{2}{3}J_{p_4} \dot{q}_{24} + 2R_4^4\dot{J}_{p_4}\dot{q}_{46}
\end{bmatrix}
\] (12)

\[
= \begin{bmatrix}
I - S(2R_4^4\dot{p}_6) \frac{2}{3}J_{p_4} \dot{q}_{24} + 2R_4^4\dot{J}_{p_4}\dot{q}_{46}
\end{bmatrix}
\] (13)

\[
= \begin{bmatrix}
2A\Psi_6^2\dot{\Psi}_4^2\dot{\Omega}_1^1J_2 & \ldots & 2A\Psi_6^2\dot{\Psi}_4^2\dot{\Omega}_5^5J_6
\end{bmatrix}.
\]

We then need to simplify (13) column by column. We invoke Matlab matrix notation to do this. Thus the first column of \( \frac{2}{3}J_6 \) is

\[
\frac{2}{3}J_6(:, 1) = -2A\Psi_6^2\dot{\Psi}_4^2\dot{\Omega}_1^1J_2
\] (14)
The null space projection of \( \frac{2}{3} J_6 \) is

\[
\begin{aligned}
\frac{2}{3} J_6 &= 2 \Psi_6^2 \Omega_3 - 2 \Omega_4 \Psi_6^4 \Omega_3 \frac{J_4}{3} \\
&= \left( 2 \Psi_6^2 \Omega_3 - 2 \Omega_4 \Psi_6^4 \Omega_3 \right) \frac{J_4}{3} \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right) \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right) \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right) \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right) \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right)
\end{aligned}
\]

This makes the relative Jacobian of the 3-arm cooperating parallel manipulator to be

\[
\frac{2}{3} J_6 = \left[ -2 \Psi_6^2 \Omega_3 \frac{1}{3} J_6 \right] 2 \Omega_5 \frac{J_6}{5}
\]

which is identical to (3), except for the middle zero column. Comparing (16) to (3) it would seem that we have not gained enough in terms of expressing the relative Jacobian of the 3-arm cooperating parallel manipulators. However, this new formulation is in fact a consequence of the method of formulation based on paired-arm manipulation. This approach is commonly found in nature [18], [19]. Thus, the third arm will always move in the null-space of the dual arm. A holistic modular kinematic expression for the 3-arm cooperating parallel manipulator can be expressed as

\[
\begin{aligned}
\dot{q}_{246} &= \frac{2}{3} J_6^T \dot{x}_6 + (I - \frac{2}{3} J_6^T \frac{J_5}{3} J_6) \frac{2}{3} J_4^T \dot{x}_4 \\
&= (I - \frac{2}{3} J_6^T \frac{J_5}{3} J_6) \frac{2}{3} J_4^T \dot{x}_4 \\
&= \left( \begin{bmatrix}
2R_4 & 0 & I \\
0 & 2R_4 & 0 \\
I & 0 & I
\end{bmatrix} \right) \left( \begin{bmatrix}
\frac{1}{2} J_4 \\
0 \\
0
\end{bmatrix} \right)
\end{aligned}
\]

where \( \dot{q}_{246} = [\dot{q}_2, \dot{q}_4, \dot{q}_6]^T \), \( \frac{2}{3} J_4 = [\frac{1}{3} J_4 0] \), \( \dot{x}_6 = [\dot{x}_6 0] \), \( \frac{2}{3} \dot{x}_6 = [\dot{x}_6 0] \), \( \dot{x}_4 = [\dot{x}_4 0] \), \( \frac{2}{3} \dot{x}_4 = [\dot{x}_4 0] \), and \( \nabla z \) is the null-space posture.

The expression in (17) shows modularity in expressing the complete kinematics of the 3-arm cooperating parallel manipulators in both task space and null-space velocities. The null space projection of \( \nabla z \) can be computed as shown in [6], where maximum number of tasks was utilized and prioritized despite singularities.

IV. SIMULATION USING GAZEBO

The section shows the results using Gazebo simulator. The controller in the simulation is a controller with purely kinematic information, without any dynamics information included. This can be a limitation in the simulation. The velocity controller is expressed as

\[
\begin{align}
\dot{q}_{246} &= J_K^T \Delta(x_d) + (I - J_K^T J_K) \frac{2}{3} J_4^T \Delta(\dot{x}_2) \\
&= (I - J_K^T J_K)(I - \frac{2}{3} J_4^T J_4) \frac{2}{3} J_4^T \Delta(\dot{x}_2) \\
&= (I - J_K^T J_K)(I - \frac{2}{3} J_4^T J_4) \frac{2}{3} J_4^T \Delta(\dot{x}_2) \\
\end{align}
\]

where \( J_K = \frac{2}{3} J_6 \) and \( \dot{x}_2 = \frac{2}{3} \dot{x}_6 \) is the relative position and orientation vector. For the delta function, given \( x \) as the input,

\[
\Delta(x) = k_p(x_d - x) + k_v(\dot{x}_d - \dot{x}) + k_i \sum_{t=0}^\infty (x_d - x)
\]

where \( x_d \) is the desired \( x \), \( \dot{x}_d \) is the desired velocity of \( x_d \), \( \dot{x} \) is the velocity of \( x \), \( t \) is the time, and \( k_p \), \( k_v \), and \( k_i \) are the proportional, derivative, and integral gains. The 3-arm null-space Jacobians are \( \frac{2}{3} J_4 = [\frac{2}{3} J_4 0] \) and \( \frac{1}{3} J_2 = [\frac{1}{3} J_2 0] \).

The \( \nabla_z \) is the null-space gradient that controls the posture of the arms, such that \( \nabla_z = (\Delta(q_2), \Delta(q_4), \Delta(q_6))^T \).

The desired values are the following (with lengths in meters and angles in degrees): \( x_{d\text{sd}} = [0, 0, 0, 3, 0, 180, 0]^T \), \( x_{d\text{sd}} = [0, 0, 0, 3, 0, 180, 0]^T \), \( q_{d\text{sd}} = [0, 0, 0, 3, 0, 180, 0]^T \), \( q_{d\text{sd}} = [0, 0, 0, 3, 0, 180, 0]^T \), \( q_{d\text{sd}} = [0, 0, 0, 3, 0, 180, 0]^T \). All desired velocities are zero. The desired values \( x_{d\text{sd}} \) changes according in a point-to-point motion to the time increment of \( 1s \) as follows: \( x_{d\text{sd}} = [0.5, 0, 0, 5, 0, 90, 0, 0]^T \), \( x_{d\text{sd}} = [0.5, 0, 0, 5, 0, 90, 0, 0]^T \), \( x_{d\text{sd}} = [0.5, 0, 0, 5, 0, 90, 0, 0]^T \), \( x_{d\text{sd}} = [0.5, 0, 0, 5, 0, 90, 0, 0]^T \), and \( x_{d\text{sd}} = [0.5, 0, 0, 5, 0, 90, 0, 0]^T \). Then \( x_{d\text{sd}} \) loops back in a 4s cycle of desired values.

Note that Gazebo simulator does not run in real-time. The gains are set at \( k_p = 3000 \), \( k_v = 200 \), and \( k_i = 0.1 \). Note that the \( \Delta(q) \) function in the null-space used \( k_p = 200 \), and \( k_v = k_i = 0 \).

Three sets of simulation experiments are shown here: (1) when robot \( B \) is stationary such that \{6\} moves w.r.t. to \{2\} while \{4\} is not moving (as shown in Fig. 3 with performance errors shown in Fig. 6), (2) all three robots end-effectors are moving in coordinated motion (as shown in Fig. 4 with performance errors shown in Fig. 7), and (3)
Position Error (m)  
Orientation Error (deg)  
Relative Orientation Error

robot B goes in and out of coordinated motion while \{6\} uninterrupted moves w.r.t. \{2\} (as shown in Fig. 5 with numerical errors shown in Fig. 8).

V. CONCLUSION

Performance errors shown from Figs. 6 to 8 showed consistent results such that the motion of robot B, being stationary or moving in a holistic coordinated motion, does not affect the relative motion between robots A and C. Thus the shown identical errors in columns one and two of Figs. 6, 7, and 8, while the errors in column three of the same figures vary. This kind of strict task prioritization results is consistent with a single end-effector controller.

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Fig. 7. The position and orientation errors as in Fig. 6, but with all the robots moving.

Fig. 8. Position and orientation errors as in Figs. 6 and 7 when robot B alternates from moving with the rest of the robots to being stationary.


