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AN INVERSE EIGEN VALUE PROBLEM FOR OPTIMAL LINEAR QUADRATIC CONTROL

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ABSTRACT

This paper considers linear quadratic optimal control from the perspective of a matrix inverse eigenvalue problem. The approach employed uses a Newton's method for solving the Inverse Eigenvalue problem for a class of Hermitian/Hamiltonian matrices in the neighborhood of a related singular matrix of rank 1.A few numerical examples are presented to illustrate the result.

Key words: *Linear, quadratic, optimal control, eigenvalues, Hammiltonian, Riccati equations.*

INTRODUCTION

Recent theoretical results on the solvability of the inverse eigenvalue problem for Hermitian matrices together with numerical examples are systematically reviewed and discussed in respect of the inverse eigenvalue problems for certain singular and non-singular Hermitian matrices. See Oduro *et al* (2012) and Oduro (2012a, b) as well as Baah Gyamfi (2012).

This paper deals with methods of solving the inverse eigenvalue problem for certain matrices namely singular symmetric matrices of rank 1 via Newton's method for solving the inverse eigenvalue problem for non-singular symmetric matrices trying to determine how to drive a system from some initial state to a target final state by finding a set of parameters which gives a right solution. It is clear therefore that every control problem is an inverse problem.

Linear Quadratic Optimal Control Problem (LQOCP)

Here we consider a linear system of the form:

$$\dot{x} = Ax + Bu \quad x_0 = x_0 \tag{1}$$

Where: u is the admissible control unit and be of the form: $u = \phi(t)$

The control objective is to find a control strategy that minimizes the cost functional.

$$J(x,\phi) = \int_0^\infty \left[X^T(t)Qx(t) + \phi^T(t)R\phi(t)\right]dt \tag{2}$$

Where:

Q is a symmetric positive semi definite matrix.
R is a symmetric positive definite matrix.

Then, this type of control problem is called **Linear Quadratic Control Problem**

Since Q is positive semi definite, then, $x^T(t)Qx(t) \geq 0$ and R is positive definite i.e. $\phi^T(t)R\phi(t) > 0$ unless $\phi(t) = 0$.

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Theorem: Riccati Assume (A,B) is stabilizable and $(\sqrt{Q},A)$ is detectable. Then, there exists a unique solution P in the class of positive semi definite matrices and the closed loop system matrix $A-BR^{-1}B^TP$ is stable.

Proof: If (A,B) is stabilizable and $(\sqrt{Q},A)$ is detectable, then the equation will be an admissible as it is stabilizing, we then verify that it is optimal by completing the square.

For any admissible u.

$$\begin{aligned} J(x_0,u) &= \int_0^\infty \left[x^T(t)Qx(t)+u^T(t)Ru(t)\right]dt \\ &= \int_0^\infty \left[x^T(t)PBR^{-1}B^TPx(t)+u^T(t)Ru(t)-x^T(t)(A^TP+PA)x(t)\right]dt \\ &= \int_0^\infty \left[u(t)+R^{-1}B^TPx(t)\right]^T R\left[(u(t)+R^{-1}B^TPx(t)\right]dt \\ &= -\int_0^\infty \left[u^T(t)B^TPx(t)+x^T(t)PBu(t)+x^T(t)APx(t)+x^T(t)PAx(t)\right]dt \end{aligned} \tag{3}$$

$$\begin{aligned} &\int_0^\infty \left[(u(t)+R^{-1}B^TPx(t)\right]^T R\left[(u(t)+R^{-1}B^TPx(t)\right]-\dot{x}^T(t)Px(t)-x^T(t)P\dot{x}(t)]dt \\ &= x_0^TPx_0+\int_0^\infty \left[(u(t)+R^{-1}B^TPx(t)\right]^T R\left[(u(t)+R^{-1}B^TPx(t))\right]dt \end{aligned} \tag{4}$$

Since x_0Px_o is constant and $u=-R^{-1}B^TPx$ is admissible with $R>0$,

Then, the optimal control will be: $u(t)=-R^{-1}B^TPx(t)$ (5)

While the optimal cost is: $V(x)=x^TPx$ (6)

Linear System and the Riccati Equation

We let $A\in\Re^{n\times n}, B\in\Re^{n\times m}, Q\in\Re^{n\times n}: Q=Q^T\geq 0$ and $R\in\Re^{m\times m}: R=R^T>0$

Finding the linear quadratic optimal control for the functional;

$$Ix_i(u)=\int_{t_i}^{t_f}\frac{1}{2}\left[x(t)^TQx(t)+u(t)^TRu(t)\right]dt \tag{7}$$

Subject to differential equation

$$\dot{X}(t)=Ax(t)+Bu(t), t\in\left[t_i,t_f\right], x(t_i)=x_i \tag{8}$$

Then the Hamiltonian functional is given by:

$$H(p,x,u,t)=\frac{1}{2}\left[x^TQx+u^TRu\right]+p^T\left[Ax+Bu\right] \tag{9}$$

From the above theorem, it then follows that any optimal input $u_\bullet$ and the corresponding State $x_\bullet$ Satisfies:

$$\begin{aligned} &\frac{\partial H}{\partial u}(p_\bullet(t),x_\bullet(t),u_\bullet(t),t)=0 \\ &\Rightarrow u_\bullet(t)^TR+p_\bullet(t)^TB=0 \end{aligned} \tag{10}$$

Thus, $u_\bullet(t)=-R^{-1}B^Tp_\bullet(t)$ and the adjoint equation is given as:

$$\left[\frac{\partial H}{\partial x}(p_\bullet(t),k_\bullet(t),u_\bullet(t),t)\right]^T=-\dot{p}_\bullet(t), t\in\left[t_i,t_f\right], p_\bullet(t_f)=0$$

$$\Rightarrow \left(x_{\bullet}(t)^T Q + p_{\bullet}(t)^T A\right)^T = -\dot{p}_{\bullet}(t), t \in [t_i, t_f], p_{\bullet}(t_f) = 0$$

Then,

$$\dot{p}_{\bullet}(t) = A^T p_{\bullet}(t) - Qx_{\bullet}(t), t \in [t_i, t_f], p_{\bullet}(t_f) = 0 \tag{11}$$

Consequently;

$$\frac{d}{dt}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t) \end{bmatrix}, t \in [t_i, t_f], x_{\bullet}(t_i) = x_i, p_{\bullet}(t_f) = 0 \tag{12}$$

Equation (12) is a linear, time variant differential equation in $(x_{\bullet}, p_{\bullet})$ which is called Hamilton's Equation.

From equation (12), we consider the case where the Hamiltonian matrix

$H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$ is $2n \times 2n$ so that *A, Q, $BR^{-1}B^T$* are all is 2×2 sub-matrices of *H*.

Using appropriate row dependence relations, a 4×4 singular Hermitian matrix representing H above can be constructed as follows

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = a_{11}\begin{bmatrix} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & \bar{k}_1 k_2 & k_1 \bar{k}_3 \\ k_2 & k_2 \bar{k}_1 & |k_2|^2 & k_2 \bar{k}_3 \\ k_3 & k_3 \bar{k}_1 & k_3 \bar{k}_2 & |k_3|^2 \end{bmatrix}$$

Here we assume that the singularity of the matrix is due to the row dependence relations specified below:

$$R_{i+1} = k_i R_1$$

$$\Rightarrow a_{21} = k_1 a_{11} = -a_{12} = \bar{k}_1; a_{31} = k_1 a_{11} = -a_{13} = \bar{k}_2; a_{41} = k_1 a_{11} = -a_{14} = \bar{k}_3$$

$$a_{32} = k_2 a_{11}(-k_1 a_{12}) = k_2 \bar{k}_1; a_{42} = k_3 a_{11}(-k_1 a_{12}) = k_3 \bar{k}_1; a_{43} = k_3 a_{11}(-k_2 a_{12}) = k_3 \bar{k}_2$$

$$\Rightarrow a_{22} = k_1(a_{12}) = k_1(a_{21}) = k_1^{\ 2} a_{11} = |k_1|^2 \qquad a_{23} = k_1(a_{13}) = k_1(a_{31}) = k_1 k_2 a_{11} = \bar{k}_1 k_2$$

$$a_{24} = k_1(a_{14}) = k_1(a_{41}) = k_1 k_3 a_{11} = k_1 k_3 \qquad a_{33} = k_2(a_{13}) = k_2(a_{31}) = k_2^{\ 2} a_{11} = |k_2|^2$$

$$a_{34} = k_2(a_{14}) = k_2(a_{41}) = k_2 k_3 a_{11} = k_2 \bar{k}_3 \qquad a_{44} = k_3(a_{14}) = k_3(a_{41}) = k_3^{\ 2} a_{11} = |k_3|^2$$

To solve the inverse eigenvalue problem (IEP) for the singular matrix of rank 1 we use the given nonzero eigenvalue as follows

$$tr(A) = \lambda = a_{11}(1 + |k_1|^2 + |k_2|^2 + |k_3|^2)$$

So that

$$H = \frac{\lambda}{1 + |k_1|^2 + |k_2|^2 + |k_3|^2}\begin{bmatrix} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & \bar{k}_1 k_2 & k_1 \bar{k}_3 \\ k_2 & k_2 \bar{k}_1 & |k_2|^2 & k_2 \bar{k}_3 \\ k_3 & k_3 \bar{k}_1 & k_3 \bar{k}_2 & |k_3|^2 \end{bmatrix}$$

Since we have assumed that the above is also a Hamiltonian matrix of the linear quadratic optimal control problem, we may partition it as follows:

$$\begin{bmatrix} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & \bar{k}_1 k_2 & k_1 \bar{k}_3 \\ k_2 & k_2 \bar{k}_1 & |k_2|^2 & k_2 \bar{k}_3 \\ k_3 & k_3 \bar{k}_1 & k_3 \bar{k}_2 & |k_3|^2 \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$$

Where

R and Q are Hermitian symmetric matrices and $A = -A^T$

Thus we have

$$\bar{k}_1 = k_3 \bar{k}_2$$

$$k_1 = -k_3 \bar{k}_2 \,......................(i)$$

$$\bar{k}_3 = k_2 k_1 \Rightarrow k_3 = \bar{k}_2 k_1$$

Substituting for k_3 in equation (i)

$$k_1 = -\left(\bar{k}_2 k_1\right) k_2 ....(ii)$$

$$k_1 = \left(\bar{k}_2\right)^2 k_1 \Rightarrow \left(\bar{k}_2\right)^2 = -1$$

$$\bar{k}_2 = \sqrt{-1} \Rightarrow k_2 = -i ....(iii)$$

But,

$$k_3 = \bar{k}_2 k_1 \Rightarrow k_3 = -(i) k_1$$

$$k_3 = ik_1 ....(iv)$$

Substituting k_1, k_2, k_3 into the Hamiltonian matrix gives;

$$a_{11} \begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix}$$

Thus; $tr(A) = \lambda = 2a_{11}\left(1 + |k_1|^2\right)$

Thus the solution of the IEP is given by

$$H = \frac{\lambda}{2\left(1 + |k_1|^2\right)} \begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix}$$

ILLUSTRATION

Given that $a_{11} = 1, k_1 = 2 \Rightarrow \lambda = 10$

Hence;

$$\begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & i & 2i \\ 2 & 4 & 2i & -4i \\ i & -2i & 1 & 2 \\ 2i & 4i & -2 & 4 \end{bmatrix}$$

SOLUTION OF THE IEP FOR NONSINGULAR HAMILTONIAN MATRIX

Since there are repeating diagonal elements we solve the IEP by Newton's method for two distinct target eigenvalues λ_1, λ_2 which therefore give rise to two (2) functions with independent variables being the diagonal elements of matrix A which is a sub-matrix of H:

$$f_1(a_{11}, a_{22}) = \lambda_1^2 - 2(trA)\lambda_1 + \det H$$
$$f_2(a_{11}, a_{22}) = \lambda_2^2 - 2(trA)\lambda_2 + \det H$$

Thus;

$$f_1(a_{11}, a_{22}) = \lambda_1^2 - 2(a_{11} + a_{22})\lambda_1 + \det H$$

$$f_2(a_{11}, a_{22}) = \lambda_2^2 - 2(a_{11} + a_{22})\lambda_2 + \det H$$

Then the Jacobian of the above functions is given as:

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial a_{11}} & \dfrac{\partial f_1}{\partial a_{22}} \\ \dfrac{\partial f_2}{\partial a_{11}} & \dfrac{\partial f_2}{\partial a_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda_1 + 2a_{22} & -\lambda_1 + 2a_{11} \\ -\lambda_2 + 2a_{22} & -\lambda_2 + 2a_{11} \end{bmatrix}$$

While the general Newton's method is given by the following iteration;

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(0)})\underline{f}(X^{(n)})$$

$$X^{(0)} = \begin{bmatrix} a_{11}^{(0)} \\ a_{22}^{(0)} \end{bmatrix}$$

While the Determinant $Det = 2(\lambda_1 - \lambda_2)(a_{22} - a_{11})$

Then the formula for finding inverse of 2×2 Jacobian matrix is given as;

$$J^{-1} = \frac{1}{2(\lambda_1 - \lambda_2)(a_{22} - a_{11})} \begin{bmatrix} 2a_{22} - \lambda_1 & 2a_{11} - \lambda_1 \\ 2a_{22} - \lambda_2 & 2a_{11} - \lambda_2 \end{bmatrix}$$

ALGORITHM

To solve the inverse eigenvalue problem (IEP) for the Hamiltonian equation associated with the LQOC problem:

$$Ix_i(u) = \int_{t_i}^{t_f} \frac{1}{2}\left[x(t)^T Qx(t) + u(t)^T Ru(t)\right]dt$$

Subject to differential equation

$$\dot{X}(t) = Ax(t) + Bu(t), t \in [t_i, t_f], x(t_i) = x_i$$

Given the Hamiltonian equation of the form:

$$\frac{d}{dt}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix}, t \in [t_i, t_f], x_\bullet(t_i) = x_i, p_\bullet(t_f) = 0$$

Solving the IEP for the matrix; $H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}$

For the case when it is Non-Singular matrix:

Given two distinct target eigenvalues λ_1, λ_2 (repeated for each)

Step 1: Determine the characteristic functions i.e.

$$f_1(a_{11}, a_{22}) = \lambda_1^2 - 2(trA)\lambda_1 + \det H$$
$$f_2(a_{11}, a_{22}) = \lambda_2^2 - 2(trA)\lambda_2 + \det H$$

Step 2: Find the Jacobian from the function where:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial a_{11}} & \frac{\partial f_1}{\partial a_{22}} \\ \frac{\partial f_2}{\partial a_{11}} & \frac{\partial f_2}{\partial a_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda_1 + 2a_{22} & -\lambda_1 + 2a_{11} \\ -\lambda_2 + 2a_{22} & -\lambda_2 + 2a_{11} \end{bmatrix}$$

Step 3: Apply the Newton's method in H .i.e.

$$X^{(1)} = X^{(0)} - J^{-1}(X^{(0)})\underline{f}(X^{(0)})$$

Step 4: Substitute $X^{(0)} = \begin{bmatrix} a_{11}^{(0)} \\ a_{22}^{(0)} \end{bmatrix}$ into H replacing the original diagonal element.

NUMERICAL EXAMPLES

I. Positive Definite Case

Given the target eigenvalues

$\lambda_1 = 1, \lambda_2 = 2$

And an initial rank 1 singular matrix with

$a_{11} = 1, k_1 = 2 \Rightarrow \lambda = 10$

i.e.

$$\Rightarrow \begin{bmatrix} 1 & 2 & i & 2i \\ -2 & 4 & -2i & 4i \\ i & -2i & 1 & 2 \\ 2i & 4i & -2 & 4 \end{bmatrix}$$

To solve the IEP by Newton's method using the above as initial matrix, we proceed as follows

$$f_1(a_{11}, a_{22}) = \lambda_1^2 - 2(a_{11} + a_{22})\lambda_1 + 0$$

$$\Rightarrow 1 - 10(1) + 0 = -9$$

$$f_2(a_{11}, a_{22}) = \lambda_2^2 - 2(a_{11} + a_{22})\lambda_2 + 0$$

$$\Rightarrow 4 - 10(-2) + 0 = 24$$

$$\underline{f}(X^{(0)}) = \begin{bmatrix} -9 \\ 24 \end{bmatrix}$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial a_{11}} & \frac{\partial f_1}{\partial a_{22}} \\ \frac{\partial f_2}{\partial a_{11}} & \frac{\partial f_2}{\partial a_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda_1 + 2a_{22} & -\lambda_1 + 2a_{11} \\ -\lambda_2 + 2a_{22} & -\lambda_2 + 2a_{11} \end{bmatrix} = \begin{bmatrix} -1+8 & -1+2 \\ 2+8 & 2+2 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 10 & 4 \end{bmatrix}$$

$$J^{-1} = \frac{1}{2(\lambda_1 - \lambda_2)(a_{22} - a_{11})} \begin{bmatrix} 2a_{22} - \lambda_1 & 2a_{11} - \lambda_1 \\ 2a_{22} - \lambda_2 & 2a_{11} - \lambda_2 \end{bmatrix} = \frac{1}{2(3)(3)} \begin{bmatrix} 8-1 & 2-1 \\ 8+2 & 2+2 \end{bmatrix}$$

$$J^{-1} = \frac{1}{18} \begin{bmatrix} 7 & 1 \\ 10 & 4 \end{bmatrix}$$

Substituting into the Newton's equation

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(0)})\underline{f}(X^{(n)})$$

$$X^{(1)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \frac{1}{18} \begin{bmatrix} 7 & 1 \\ 10 & 4 \end{bmatrix} \begin{bmatrix} -9 \\ 24 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} -2.166 \\ 0.333 \end{bmatrix}$$

Hence,

$$(X^{(1)}) = \begin{bmatrix} 3.167 \\ 3.667 \end{bmatrix}$$

Thus the solution of IEP is given by

$$H = \begin{bmatrix} 3.167 & 2 & i & 2i \\ -2 & 3.667 & -2i & 4i \\ i & -2i & 3.167 & 2 \\ 2i & 4i & -2 & 3.667 \end{bmatrix}$$

Hence, the matrices is positive definite.

Ii. Negative Definite Case

Given the target eigenvalues

$$\lambda_1 = -1, \lambda_2 = -2$$

And an initial rank 1 singular matrix with

$$a_{11} = 1, k_1 = 2 \Rightarrow \lambda = 10$$

i.e.

$$\Rightarrow \begin{bmatrix} 1 & 2 & i & 2i \\ -2 & 4 & -2i & 4i \\ i & -2i & 1 & 2 \\ 2i & 4i & -2 & 4 \end{bmatrix}$$

To solve the IEP by Newton's method using the above as initial matrix, we proceed as follows

$$f_1(a_{11}, a_{22}) = \lambda_1^{\ 2} - 2(a_{11} + a_{22})\lambda_1 + 0$$

$$\Rightarrow 1 - 10(-1) + 0 = 11$$

$$f_2(a_{11}, a_{22}) = \lambda_2^{\ 2} - 2(a_{11} + a_{22})\lambda_2 + 0$$

$$\Rightarrow 4 - 10(-2) + 0 = 24$$

$$\underline{f}(X^{(0)}) = \begin{bmatrix} 11 \\ 24 \end{bmatrix}$$

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial a_{11}} & \dfrac{\partial f_1}{\partial a_{22}} \\ \dfrac{\partial f_2}{\partial a_{11}} & \dfrac{\partial f_2}{\partial a_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda_1 + 2a_{22} & -\lambda_1 + 2a_{11} \\ -\lambda_2 + 2a_{22} & -\lambda_2 + 2a_{11} \end{bmatrix} = \begin{bmatrix} 1+8 & 1+2 \\ 2+8 & 2+2 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ 10 & 4 \end{bmatrix}$$

$$J^{-1} = \frac{1}{2(\lambda_1 - \lambda_2)(a_{22} - a_{11})} \begin{bmatrix} 2a_{22} - \lambda_1 & 2a_{11} - \lambda_1 \\ 2a_{22} - \lambda_2 & 2a_{11} - \lambda_2 \end{bmatrix} = \frac{1}{2(3)(3)} \begin{bmatrix} 8+1 & 2+1 \\ 8+2 & 2+2 \end{bmatrix}$$

$$J^{-1} = \frac{1}{18} \begin{bmatrix} 9 & 3 \\ 10 & 4 \end{bmatrix}$$

Substituting into the Newton's equation

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(0)})\underline{f}(X^{(n)})$$

$$X^{(1)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \frac{1}{18} \begin{bmatrix} 9 & 3 \\ 10 & 4 \end{bmatrix} \begin{bmatrix} 11 \\ 24 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 1.5 \\ 0.777 \end{bmatrix}$$

Hence,

$$(X^{(1)}) = \begin{bmatrix} -0.5 \\ -3.223 \end{bmatrix}$$

Thus the solution of IEP is given by

$$H = \begin{bmatrix} -0.5 & 2 & i & 2i \\ -2 & -3.223 & -2i & 4i \\ i & -2i & -0.5 & 2 \\ 2i & 4i & -2 & -3.223 \end{bmatrix}$$

Hence, the matrix is Negative definite.

CONCLUSION

The usual approach to the LQOC problem has been reviewed. Recent theoretical results have also been systematically reviewed and discussed in respect of the inverse eigenvalue problem (IEP) for certain singular and non-singular Hermitian matrices. Based on these results, we have successfully developed a general form of the LQOC problem as an inverse eigenvalue problem involving a Hermitian Hamiltonian matrix and justified the claims by numerical examples for real distinct target eigenvalues.

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