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Article

Note on the Investigation of Nonlinear Optimal Stochastic Control Theory: Its Applications

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Abstract: This paper deals with investigation of nonlinear optimal stochastic control theory: its applications. During the last two decades, a large number of researchers have applied the theory of stochastic optimal control and stochastic approaches in different fields of engineering, economics, operations research, production planning, investment and medicine. Stochastic optimal control helps to study the problem of optimal control of a stochastic production, planning and investment model. In fact, the stochastic optimal control theory can be considered as a combination of optimal control, stochastic models and mathematical analysis. In this paper the investigation into the applications of nonlinear optimal stochastic control theory is highly emphasized.

Keywords: Nonlinear optimal stochastic, Control theory, Optimal Control, Production planning, Dynamical system, Consumption.

1. Introduction

Many questions of stochastic dynamic models and optimal controls arise in management, economics and engineering. For example, it may be required to calculate the conditions of operation of an industrial process that gives the maximum output or quality or that gives the minimum cost. Investment and production planning models have a relatively long tradition in dynamic optimization theory. Holt et al. (1960) proposed the first major contribution in this field by applied the calculus of variation principle to solve a production investment model. Bensoussan et al. (1974) provide a nice introduction in the applications of optimal control theory to investment and production models. A very good introduction about the dynamics of economics and management models, as well as their stochastic version, can be found in the book of Sethi and Thompson (1981, 2000). Pekelman (1974) was probably the first who introduced the price as an additional decision variable in a production-investment model. Assuming that the firm faces time-dependent linear demand duration, a convex production cost and a linear investment cost, he treated the problem of simultaneously determining the optimal price and production rate of a firm for a known time horizon.

Also, the investment level was assumed to be restricted by a non-negativity state constraint. Pekelman (1974) characterized the optimal inter-temporal price and production decisions depending on the sum of the adjoint variable of the inventory level and the Lagrange multiplier of the state constraint by using the optimal control theory. Production has attracted a growing attention due to its increasing importance in today's highly competitive environment, as propounded by Applequist et al. (1907), Balasubramanion and Grossmann, (2002), Bassett et al. (1997), Petkov and Mararas (1997)

A production problem that contains making decision is usually formulated as an optimal control problem. For optimization problems under randomness, two most important characteristics are not presented in its deterministic counterpart: they are the need of considering risks in the model formulation and the possibility of information gathering during the decision process. To solve this problem entailed the minimization or maximization of the expected value of a cost function. The stochastic production model evolves as a dynamics process and the optimal control law obtained is a feedback control policy. In stochastic control theory, the dynamical system can be formulated by a stochastic dynamic system to describe the functional relation between the state variables of the model and its inputs and disturbances. The surplus amount of the production system can be taken as the state variable that is affected by both production and demand rates. Production planners face substantial uncertainty in the demand for their products that comes from a variety of sources. For example, demand could be sensitive to varying economic conditions such as interest rates, and alternatively technical innovation may imply unexpected easily obsolescence.

The last three decades have seen a growing number of optimal control theory and applications in the field of investment. This part of the present paper is concerned with an important stochastic dynamical case of investment model by considering a variation of the Vidale-Wolfe investment model for which the maximum value of the objective function and the form of the optimal feedback investment control are, in both a deterministic and a stochastic environment, identical. In this model we assumed that the stochastic environment is due to a white noise disturbance in the deterministic sales investment dynamics. The term containing the stochastic element vanishes, because it involves the second derivative of the current value function, which is zero for the linear current value function case.

The Vidale-Wolfe investment model is one of the earliest management science applications in finance. It is well known that Vidale and Wolfe have developed a simple model of sales response to investment which was consistent with their experimental observations. Sethi, (1973) considered the optimal control of the Vidale-Wolfe investment model. In this study, the optimal control is the rate of investment expenditure to achieve a terminal market share within specified limits in a way which maximizes the current value of net profit streams over a finite time horizon. In his study the special polar cases of fixed and free end points are solved with and without an upper limit to investment rate. The resulting optimal investment rate is characterized by a combination of bang-bang, impulse, and singular control with the singular arc forming a turnpike.

Sethi and Lee (1981) considered the problem of optimal investment for the Nerlove-Arrow model under a replenish-able budget. In this study, Sethi and Lee (1981) have considered an optimal control problem with two state variables for the dynamics of this model and the optimal control is the rate of investment expenditure that is required to maximize the present value of net streams over an infinite time horizon subject to a replenish-able budget. Merton (1969) examined the continuous-time consumption-portfolio problem for an individual whose income is generated by capital gains and investments in assets with prices assumed to satisfy the geometric Brownion motion. In Merton (1971, 1973), he extend these results for more general utility function, price behavior assumptions, and for income generated also from non-capital gains sources.

Derzko and Sethi (1981) have studied the problem of optimal exploration and consumption of a natural resource in the stochastic case. Docker and Feichtinger (1993) also studied the cyclical consumption patterns and rational addition. Feichtinger and Novak (1992) studied the optimal consumption training, working time and leisure over the life cycle. Sethi (1997) investigated the optimal consumption and investment with bankruptcy. In Sethi and Thompson (2000) the stochastic optimal control problem of consumption model which is subject to It'o differential equation was discussed. In their study they assumed that the utility function is an increasing function of the consumption rates. The work done by the researchers like, Dockner, E. J. and Feichtingor, G. (1993), Stroock and Varadhan (1979) and Wim Wiegerinck Bert Kappen (2007) are also of great importance in the area of our subject matter.

Investment contains making decisions for a manufacturing system, in which different types of events such as operations, failures, preventive maintenance and raw material supply as well as customer demand fluctuation occur at the same time. To better understand and more effectively deal with randomness from various sources, mathematical models that can characterize the unique feature of every major event are needed.

2. Applications of Stochastic Optimal Control

During the last two decades, a large number of researchers have applied the theory of stochastic optimal control and stochastic approaches in different fields of engineering, economics, operations research, production planning, investment and medicine. There are many important examples in different areas; for example, an interesting example in one dimension arises in Merton's model of optimal portfolio. Also the stochastic approach is used to study the stability and control of tumor and cancer modelled by El-Gohary (2006) where he used the optimal control approach to stabilize the tumor and cancer unstable equalized states.

El-Gohary, Tadj and Al-Rahmah (2006) applied the stochastic optimal control to study the problem of optimal control of a stochastic production planning and investment model. An important example of the stochastic models is the cash balance model which can be represented by Itˆo stochastic differential equation. In fact, the stochastic optimal control theory can be considered as a combination of optimal control, stochastic models and mathematical analysis. The stochastic models have a variety of stochastic counterparts corresponding to different assumptions about the nature of the uncertainty. The process of estimating the values of the state of the stochastic models is known as the optimal filtering. Really, the state of the controlled stochastic system can be represented by a controlled stochastic process. In the stochastic models there two types of random disturbances are presented. The first disturbances are due to the noises occurred in the measurements of the state and the second are due to the noises which affect the system state.

Viktoria Blüschke-Nikolaeva (2010) provides an algorithm for the optimal control of nonlinear stochastic models where the new version of the algorithm OPTCON2, named OPTCON2, is presented. This algorithm has been developed to obtain approximate solution of control optimum problems where the objective function is quadratic and the dynamic multivariable system is nonlinear. The additive and multiplicative uncertainties are present in dynamic models. OPTCON2 differs from the basic algorithm OPTCON in the dealing with stochastic parameters during the computation of optimal control variables. OPTCON2 uses passive learning, i.e. the stochastic parameters are updated in each time period. This fact can make the results of optimum stochastic control problems more accurate and more reliable than computation without update.

Kushner, H. and Kleinman, A. (1968) give the numerical methods for the solution of the degenerate nonlinear elliptic equations arising in optimal stochastic control theory where three distinct but related results are obtained. First, an iterative method is derived for obtaining the solution of optimal control problems for Markov chains. The method usually converges much faster, and requires less computer storage space, than the methods of Howard or Eaton and Zadeh.

Secondly, nonlinear finite difference equations, which "approximate" the nonlinear degenerate elliptic equation arising out of the stochastic optimization problem, are found. The difference equations, and their solution, may have a meaning for the control problem even when it cannot be proved that the equation has a solution. The iterative methods for the iterative solution of these nonlinear systems are discussed and compared, and discovered that both converge to the solution.

V. Blueschke-Nikolaevaa, D. Blueschkea, R. Neck (2010) propounded an optimal control of nonlinear dynamic econometric models: An Algorithm and an Application. He proposed that OPTCON is an algorithm for the optimal control of nonlinear stochastic systems which is particularly applicable to econometric models. It delivers approximate numerical solutions to optimum control problems with a quadratic objective function for nonlinear econometric models with additive and multiplicative (parameter) uncertainties. The algorithm was programmed in C+ and allows for deterministic and stochastic control, the latter with open-loop and passive learning (open-loop feedback) information patterns. The applicability of the algorithm is demonstrated by experiments with a small quarterly macro econometric model for Slovenia. This illustrates the convergence and the practical usefulness of the algorithm and (in most cases) the superiority of open-loop feedback over open-loop controls.

Erhan Bayraktar (2010) considered the valuation equations for stochastic volatility models where he study the valuation partial differential equation for European contingent claims in a general framework of stochastic volatility models and discovered that the standard Feynman-Kac theorem cannot be directly applied because the diffusion coefficients may degenerate on the boundaries of the state space and grow faster than linearly. He allow for various types of model behavior; for example, the volatility process in the model to potentially reach zero and either stay there or instantaneously reflect, and asset-price processes may be strict local martingales under a given risk-neutral measure.

Andrea Pascucci (2010) considers the Kolmogorov equations and Asian options. He presented a survey of the theory of partial differential equations of Kolmogorov type arising in physics and in mathematical finance. These evolutionary equations, which are generally non uniformly parabolic, are naturally associated to stochastic models with memory. Financial derivatives with dependence on the past provide some typical examples in particular, Asian options of European and American style was discussed.

Douglas Borden (2010) studied stochastic control and automated market-making trading decisions in Automated Market-Making involve a subtle interplay between expected price movement, transaction costs, market impact and risk. The decisions were made in milliseconds, millions of times a day. Standard practice is to derive a set of trading rules, with the rule parameters optimized through simulation back testing. He presented a different approach to the decision process in Automated Market-Making, making use of techniques from Stochastic Control Theory.

Igor Halperin (2010) presented an option pricing and hedging by risk minimization where he discusses the risk minimization approach to the pricing and hedging of options under incomplete market scenarios in the presence of transaction costs and presents a practical framework which assumes a multifactor stochastic model for the underlying assets.

Thaleia Zariphopoulou (2010) considered an approximation scheme for investment performance processes in incomplete markets where an approximation scheme for the maximal expected utility in an incomplete market was presented. He opined that the market incompleteness comes from a stochastic factor affecting the dynamics of the stock price.

The scheme yields an intuitively pleasing decomposition of the value function process at each splitting step. Specifically, in the first sub-step, the current utility was adjusted while the component of the investment opportunity set is perfectly correlated with the stock remains unchanged. In the second sub-step the reverse happens. The "orthogonal" decomposition highlights how dynamic preferences and investment decisions behave in terms of the imperfect correlation and, moreover, explains some effects of the stochastic factors on the risk preferences and the investment decisions.

Lee, M.H.Kolodziej, W.J.Mohler, R. R. (2007) studied the dynamic system of suboptimal control with uncertain parameters where the control of a linear system with random coefficients where the cost function is of a quadratic form and the random coefficients are assumed to be partially observable by the controller. He discovered that by means of the stochastic Bellman equation, the optimal control of stochastic dynamic models with partially observable coefficients is derived. The optimal control was shown to be a linear function of the observable states and a nonlinear function of random parameters. The theory is applied to an optimal control design of an aircraft landing in wind gust.

H. J. Kappen (2005) studied the path integrals and symmetry breaking for optimal control theory where he considers linear-quadratic control of a non-linear dynamical system subject to arbitrary cost. He shows that in some class of stochastic control problems the non-linear Hamilton–Jacobi–Bellman equation can be transformed into a linear equation.

The transformation is similar to the transformation used to relate the classical Hamilton–Jacobi equation to the Schrodinger equation. As a result of the linearity, the usual backward computation can be replaced by a forward diffusion process that can be computed by stochastic integration or by the evaluation of a path integral. It is shown how in the deterministic limit the Pontryagin minimum principle formalism is recovered.

The significance of the path integral approach is that it forms the basis for a number of efficient computational methods, such as Monte Carlo sampling, the Laplace approximation and the variational approximation.

Diogo Aguiar Gomes (2007) considered the duality principles for fully nonlinear elliptic equations where he applied duality theory to associate certain measures to fully-nonlinear elliptic equations. These measures are the natural extension of the Mather measures to controlled stochastic processes and associated second-order elliptic equations. He apply these ideas to prove new a-priori estimates for smooth solutions of fully nonlinear elliptic equations.

Fabio Camilli, Lars Grüne, and Fabian Wirth (2010) considered the control Lyapunov functions and Zubov's method for finite-dimensional nonlinear control systems where the relation between asymptotic null-controllability and control Lyapunov functions are studied.

It was shown that control Lyapunov functions (CLFs) may be constructed on the domain of asymptotic null-controllability as viscosity solutions of a first order PDE that generalizes Zubov's equation. The solution is also given as the value function of an optimal control problem from which several regularity results may be obtained.

Ioannis Karatzas, John P.e'tal (2010) studied martingale and duality methods for utility maximization in an incomplete market where by the problem of maximizing the expected utility from terminal wealth is well understood in the context of a complete financial market. He also considered the same problem in an incomplete market containing a bond and a finite number of stocks whose prices are driven by a multidimensional Brownian motion process It was discovered that the coefficients of the bond and stock processes are adapted to the filtration and incompleteness arises when the number of stocks is strictly smaller than the dimension of the price. It was shown that there is a way to complete the market by introducing additional "fictitious" stocks so that the optimal portfolio for the thus completed market coincides with the optimal portfolio for the original incomplete market. The notion of a "least favorable" completion was introduced and is shown to be closely related to the existence question for an optimal portfolio in the incomplete market. This notion was expounded upon using martingale techniques; several equivalent characterizations are provided for it, examples are studied in detail, and a fairly general existence result for an optimal portfolio was established based on convex duality theory.

Arnab Basu and Vivek S. Borkar (2010) studied the stochastic control with imperfect models. They considered the problem of worst case performance estimation for a stochastic dynamic model in the presence of model uncertainty. It was cast as a nonclassical controlled diffusion problem. An infinite dimensional linear programming formulation is given and its dual is derived. The dual is successively approximated on a bounded domain by a semi-infinite and a finite linear program. This uses function approximation based on a reproducing kernel Hilbert space. Error analysis for the approximation was provided along with an estimate of the sample complexity.

Hua Deng, Miroslav Krstic, and Ruth J. Williams (2001) studied the stabilization of stochastic nonlinear systems driven by noise of unknown covariance. This paper poses and solves a new problem of stochastic (nonlinear) disturbance attenuation where the task is to make the system solution bounded (in expectation, with appropriate nonlinear weighting) by a monotone function of the supremum of the covariance of the noise. This was a natural stochastic counterpart of the problem of input-to-state stabilization in the sense of Sontag. The development started with a set of new global stochastic Lyapunov theorems. For an exemplary class of stochastic strict-feedback systems with vanishing nonlinearities, where the equilibrium was preserved in the presence of noise, he developed an adaptive stabilization scheme (based on tuning functions) that requires no a priori knowledge of a bound on the covariance.by introducing a control Lyapunov function formula for stochastic disturbance attenuation and address optimality and solve a differential game problem with the control and the noise covariance as opposing players for strict-feedback systems the resulting Isaacs equation has a closed-form solution.

Daniel Andersson, (2008) considered the necessary optimality conditions for two stochastic control problems. He studied the problem of controlling a linear stochastic differential equation (SDE) where the coefficients are random and not necessarily bounded. He also consider relaxed control processes and define the control as a process taking values in the space of probability measures on the control set. The main motivation is a bond portfolio optimization problem. The relaxed control processes are then interpreted as the portfolio weights corresponding to different maturity times of the bonds. He also establishes existence of an optimal control and necessary conditions for optimality in the form of a maximum principle, extended to include the family of relaxed controls.

Kurt Helmes and Richard H. Stockbridge (2010) considered the construction of the value function and optimal rules in optimal stopping of one-dimensional diffusions .A new approach to the solution of optimal stopping problems for one-dimensional diffusions was developed. It arises by imbedding the stochastic problem in a linear programming problem over a space of measures. Optimizing over a smaller class of stopping rules provides a lower bound on the value of the original problem. Then the weak duality of a restricted form of the dual linear program provides an upper bound on the value.

An explicit formula for the reward earned using a two-point hitting time stopping rule allows us to prove strong duality between these problems and, therefore, allows to either optimize over these simpler stopping rules or to solve the restricted dual program. Each optimization problem was parameterized by the initial value of the diffusion and, enables to construct the value function by solving the family of optimization problems.

Jussi Keppo (1998) in his study on arbitrage, optimal portfolio, and equilibrium under incomplete market and transaction costs considers mathematical finance in an incomplete market with transaction costs. The practical aspect can be seen as an application of optimal portfolio selection and the theoretical aspect study the market equilibrium conditions in the presence of incompleteness. The results indicated that the optimal hedging strategy differs significantly from the corresponding strategy in a frictionless market even at one or two-day trading intervals and that under stochastic real foreign exchange rates international equilibrium is curved. In addition, it was shown that the market conditions hold in an incomplete market under frictions if the conditions hold in the projected markets that exist inside the initial market.

J. J. Westman and F. B. Hanson, (1999) considered the computational method for nonlinear stochastic optimal control discovered that nonlinear stochastic optimal control problems are treated that are nonlinear in the state dynamics, but are linear in the control. The cost functional is a general function of the state, but the costs are quadratic in the control. The system is subject random fluctuations due to discontinuous Poisson noise that depends on both the state and control, as well as due to discontinuous Gaussian noise. This general framework provides a comprehensive model for numerous applications that are subject to random environments. A stochastic dynamic programming approach was used and the theory for an iterative algorithm was formulated utilizing a least squares equivalent of a genuine LQGP problem to approximate the nonlinear state space dependence of the LQGP problem in control only in order to accelerate the convergence of the nonlinear control problem. A particular contribution was the treatment of a Poisson jump process that is linear in the control vector within the context of a nonlinear system.

S. S. Sritharan (2000) considered the deterministic and stochastic control of Navier–Stokes equation with Linear, Monotone, and Hyperviscosities. Where he deals with the optimal control of space–time statistical behavior of turbulent fields. He provides a unified treatment of optimal control problems for the deterministic and stochastic Navier–Stokes equation with linear and nonlinear constitutive relations. Tonelli type ordinary controls as well as Young type chattering controls are analyzed.

L.A. Socha (2000) considered the application of true linearization in stochastic quasi-optimal control problems where the problem of quasi-optimal linear feedback control laws for nonlinear systems with quadratic performance criteria under Gaussian white noise excitations was considered. To determine the quasi-optimal control a modified version of standard iterative procedure was adopted, where three versions of statistical linearization methods and true linearization method were combined with the optimal control method for linear systems with the mean-square criterion.

David Siska (2007) considered numerical approximations of stochastic optimal stopping and control problems where he studied the numerical approximations for the payoff function of the stochastic optimal stopping and control problem. It was discovered that the payoff function of the optimal stopping and control problem corresponds to the solution of a normalized Bellman PDE.

The principal aim was to study the rate at which finite difference approximations, derived from the normalized Bellman PDE, converge to the payoff function of the optimal stopping and control problem. For the deterministic case with monotone viscosity he uses the Minty–Browder technique to prove the existence of optimal controls for the stochastic case with monotone viscosity, he combines the Minty–Browder technique with the martingale problem formulation of Stroock and Varadhan to establish existence of optimal controls.

Z.G. Ying (2008) studied the stochastic optimal control of structural systems he opined that the stochastic optimal control is an important research subject in structural engineering. Recently, a stochastic optimal nonlinear control method has been proposed based on the stochastic dynamical programming principle and stochastic averaging method. The active and semi-active stochastic optimal control methods have been further developed for structural systems. The control saturation or bound, partial state observation, etc. was taken into account by the stochastic optimal control.

Kumar, S. and Sethi, S.P. (2009) studied the dynamic pricing and advertising for web content providers where he discovered that the accumulated evidence indicates that pure revenue models, such as free-access models and pure subscription fee based models, are not sufficient to support the survival of online information sellers. Hence, hybrid models based on a combination of subscription fees and advertising revenues are replacing the pure revenue models. In response to increasing interest in hybrid models, the problem of dynamic pricing on web content on a site where revenue is generated from subscription fee as well as advertisements. He adopted the optimal control theory to solve the problem and obtain the subscription fee and the advertisement level over time by considering the case when the subscription fee varies over time, but the advertisement level stays the same. He then extends it by optimizing both the subscription fee and the advertisement level dynamically and presents several analytical and numerical results which provide some important managerial insights.

E. Fern´andez-Cara and E. Zuazua (2007) considered the control theory: history, mathematical achievements and perspectives where he presented some of the mathematical milestones of control theory by overviewing its origins and some of the main mathematical achievements and discuss some of the main areas of sciences and technologies where Control theory arises and applies as well as address modelling issues and distinguish between the two main control theoretical approaches, controllability and optimal control, discussing the advantages and drawbacks of each of them.

The basic concepts related to the dynamical systems approach to (linear and nonlinear) mathematical programming and calculus of variations were discussed and present a simplified version of the outstanding results by Kalman on the controllability of linear finite dimensional dynamical systems, Pontryaguin's maximum principle and the principle of dynamical programming.

Wim Wiegerinck Bert Kappen (2007) considered a Gaussian approximation for stochastic non-linear dynamical processes with annihilation where a stochastic nonlinear dynamical process with annihilation of particles were stuied. The process was viewed as the continuous time version of the extended Kalman filter/smoother.

It also plays an important role in stochastic optimal control theory by derive a Gaussian approximation for this process. With the use of the path integral formalism he derives Euler-Lagrange equations for the mode and derives a linear noise approximation to estimate the size of the fluctuations around the mode, and estimates of the partition function, based on the mode and Gaussian corrections. He applied numerical experiments to confirm the validity of the approximation method and show that the Gaussian correction provides a significant improvement of the estimate of the partition function.

Artur Nowoświat (2002) studies the optimal control of a duffing oscillator under parametric and external excitations which deals with the investigations of a stochastic model of the Duffing oscillator operating continuously. The standard theory of the optimal stochastic control described in Fleming and Richel (1975) was presented, as well as the application of these concepts to the perturbation control techniques suggested by Suhardio et al. (1992).

The method presented was a generalization of the method of the development of nonlinear control units into a series, as described for Duffing's deterministic oscillator involving stochastic systems derived by the author and applied in his master thesis (Nowoświat, 1999).Detailed analysis and numerical calculations was done using the Runge-Kutta procedures.

Mohamed Ben Alaya and Benjamin Jourdain (2006) considered asymptotic behavior of the maximum likelihood estimator for ergodic and nonergodic square-root diffusions where they discussed the problem of parameter estimation in the Cox-Ingersoll-Ross (CIR) model (Xt)t¸0. This model is frequently used in finance for example to model the evolution of short-term interest rates or as a dynamic of the volatility in the Heston model. He established asymptotic results on the maximum likelihood estimator (MLE) associated to the drift parameters of (Xt)t¸0.

Nan Chen and S.G. Kou (2009) studied a non-zero-sum game approach for convertible bonds: Tax benefits, bankrupt cost and early/late call and discovered that convertible bonds are hybrid securities that embody the characteristics of both straight bonds and equities. The conflict of interests between bondholders and shareholders affects the security prices significantly. He investigates how to use a non-zero-sum game framework to model the interaction between bondholders and shareholders and to evaluate the bond accordingly.

Mathematically, the problem was reduced to a system of variational inequalities and explicitly derives the Nash equilibrium to the game. The model shows that credit risk and tax benefit have considerable impacts on the optimal strategies of both parties. The shareholder may issue a call when the debt is in-the-money or out-of-the-money. This is consistent with the empirical findings of "late and early calls" (Ingersoll (1977 a,b), Mikkelson (1981), Cowan et al. (1993) and Asquith (1995)). In addition, the optimal call policy under the model offers an explanation for certain stylized patterns related to the returns of company assets and stock on calls.

L. Brenig (1980) studied the stochastic hydrodynamic theory for one-component systems A nonlinear diffusion approximation for a previously derived master equation describing an inhomogeneous Boltzmann gas in a lumped phase space was proposed where by a fluctuating kinetic equation was obtained which differs from the usual Langevin equations in three essential properties: the drift and random force are nonlinear, the random noise obeys a generalized fluctuation-dissipation theorem, and there was no reference to equilibrium relations with other approaches to hydrodynamic fluctuations are discussed.

Areski Cousin (2008) Model dynamic portfolio credit risk with common shocks by considering a bottom-up Markovian portfolio credit risk model where dependence among default times stems from the possibility of simultaneous defaults. A common shocks interpretation of the model is possible so that efficient convolution recursion procedures are available for pricing and hedging CDO tranches, conditionally on any given state of the Markov model. Calibration of marginals and dependence parameters can be performed separately using a two-steps procedure, much like in a standard static copula set-up. As a result this model allows us to hedge CDO tranches using single-name CDS-s in a theoretically sound and practically convenient way.

To illustrate this we calibrate the model against market data on CDO tranches and the underlying single-name CDS-s. After the calibration, which renders good fits, we study the implied loss distributions as well as the implied min-variance hedging strategies in the calibrated portfolios.
Stephane Crepey (2003, 2012 a, b, c) studied counterparty risk on interest rate derivatives in a multiple curve setup whereby he considered the valuation and hedging of CSA interest rate derivatives. He considered the clean valuation of the portfolio, namely the valuation in a hypothetical situation where the parties would be risk-free, yet accounting for the post-crisis discrepancy between the risk-free discount curve, and the LIBOR fixing curve.

Secondly, the counterparty clean value process of the portfolio was used as an underlie to an option called Contingent Credit Default Swap (CCDS), which prices the correction in value known as the Credit Valuation Adjustment (CVA) to the portfolio due to the counterparty risk. The post-crisis multiple curve issue (including the above discrepancy) implies that the CVA was accounted for specific costs of funding a position in the portfolio and in its collateral, and of setting-up a related hedge. He model funding costs in the form of credit and liquidity bases by developed a reduced-form backward stochastic differential equations (BSDE) approach to the problem of pricing and hedging interest rate counterparty risk. In the simplest cases the problem was reduced to low-dimensional Markovian pre-default CVA BSDEs, or equivalent semi-linear PDEs.

Min Dai (2007) considered an optimal consumption and investment with differential long-term and short-term tax rates where he proposed a novel optimal consumption and optimal investment model with differential long-term/short-term tax rates formulated as a stochastic control problem and the associated value function satisfies a quasi-variational inequality equation. The optimal trading strategy is characterized by a time-varying no-transaction region outside which it is optimal to sell the stock and repurchase it to achieve an optimal fraction of after-tax wealth invested in stock. In contrast to the standard literature, it was showed that it can be optimal to defer capital loss even in the absence of transaction costs. In addition, a small investor subject to capital gain tax may be better off than a tax-exempt small investor, and a small investor may prefer a taxable security to a tax-exempt security. Moreover, raising the short-term tax rate can increase both consumption and stock investment.

Zhongxing Ye (2006) also considered some new results on pricing credit derivatives based on intensity model with interest rate risk and counterparty risk where he studied the pricing formulas of credit default swap (CDS) in intensity-based models with counterparty risk.

He assumed that the default intensity of firm depends on the stochastic interest rate driven by the jump-diffusion process and the default states of counterparty firms by make use of the hyperbolic function to illustrate the attenuation effect of correlated defaults between counterparties. This was an extensions of the models in Jarrow and Yu (2001) and Bai, Hu and Ye (2007); likewise, Michael Kohlmann, Dewen Xiong, Zhongxing Ye (2007), that was made use of in the techniques in Park (2008) to compute the conditional distribution of default times and present the explicit prices of bond and CDS in the primary-secondary and looping default frameworks.

J. Gough, V.P. Belavkin, and O.G. Smolyanov (2005) considered the Hamilton-Jacobi-Bellman equations for quantum optimal feedback control where he exploits the separation of the filtering and control aspects of quantum feedback control to consider the optimal control as a classical stochastic problem on the space of quantum states. He derive the corresponding Hamilton-Jacobi-Bellman equations using the elementary arguments of classical control theory and show that this is equivalent, in the Stratonovich calculus, to a stochastic Hamilton-Pontryagin setup and shows for cost functionals that are linear in the state, the theory yields the traditional Bellman equations treated so far in quantum feedback.

3. Conclusion

From the literature considered so far, evidences have shown that stochastic optimal control helps to study the problem of optimal control of a stochastic production, planning and investment model. In fact, the stochastic optimal control theory can be considered as a combination of optimal control, stochastic models and mathematical analysis.

References

Holt, C. C., Modigliani, F., Muth, J. F., and Simon, H., (1960), *Planning Production, Inventories, and the Work Force*, Prentice Hall, Englewood Cliffs.

Bensoussan, A., Jr., E.G.H. and Naslund, B. (1974), *Management Applications of Modern Control Theory*, North-Holland Publishing Company

A. Bensoussan, (1974), Points de Nash dans le cas de fontionnelles quadratiques et jeu differentiels linaires a N personnes. *SIAM Journal on Control*, 12: 460-499.

Sethi, S.P. and Thompson, G.L., (1981), *Optimal Control Theory: Applications to Management Science*, Martinus Nijhoff, Boston, 1981 (481 pages).

Sethi, S.P. and Thompson, G.L., (2000), *Optimal Control Theory: Applications to Management Science and Economics*, Second Edition, Springer.

Pekelman, D. (1974), Simultaneous Price-Production Decisions, *Oper. Res*., 22: 788-794.

Apllequist, G., Samikoglu, O., Pekny, J. and Reklaities, G., (1907). Issues in the use, design and evolution of process scheduling and planning system. *ISA Transactions*, 36 (2): 81-121.

Balasubramanion, J and Grossmann, I. E. (2002), Scheduling to minimize ex-pected completion time in flow shop with uncertain processing times. *Computers and Chemical Engineering*, 26: 41-57

Bassett, M. H., Perkny, J. F., and Reklaitis, G. V. (1997), Using detailed scheduling to obtain realistic operating policies for a batch processing facility, *Industrial Engineering and Chemical Research*, 36 (5): 1717-1726

Petkov, S. B. and Mararas, C. D. (1997), Multiparied planning and scheduling of multiproduct batch plants under demand uncertainty, *Industrial Engineering and Chemistry Research*, 36(1): 4864-4881

Sethi, S. P. (1973), Optimal control of Vidale-Wolfe advertising models, *Operations Research*, 21: 998-1013.

Sethi, S. P. and Lee, S. C. (1981), Optimal advertising for the Nerlove-Arrow model under a replenishable budget, *Optimal Control Applications and Methods*, 2: 165-173

Merton, R. C. (1971), Lifetime portfolio selection under uncertainty: the continuous-time case, *Rev. Eco. Statist*, 11: 247-257

Merton, R. (1973) optimal consumption and portfolio rules in continuous time, *Journal of Economic Theory*, 3: 373-413.

Derzko, N. A. and Sethi, S. P. (1981), Optimal exploration and consumption of a natural resource: stochastic case, *International Journal of Policy Analysis*, 5(3): 185-200

Dockner, E. J. and Feichtingor, G. (1993), Cyclical consumption patterns and rational addiction, *American Economic Review*, 83: 256-263.

Sethi, S. P. (1977a). Dynamic optimal control models in advertising: A survey, *SIAM Review*, 19(4): 685-725.

Sethi, S. P. (1997b). *Optimal consumption an Investment with Bankruptcy*, Kluwer Academic Publishers, Boston.

Sethi, S.P. and Thompson, G.L., (2000), *Optimal Control Theory: Applications to Management Science and Economics*, 2nd ed., Kluwer Academic Publishers, Dordrecht.

El-Gohary, A. (2008). Chaos and optimal control of cancer self-remission and tumer system steady-states, A stochastic approach, *Chaos, Solutions and Fractals*, 37(5): 1305–1316

El-Gohary, A., Tadj, L. and Al-Rahmah, A. (2006). Optimal control of a stochastic production planning model with different demand rates, *International Journal of Applied Mathematics* 29(5): 1085–1094.

Kushner, H. and Kleinman, A. (1968), Accelerated Procedures for the Solution of Discrete Markov Control Problems, *IEEE Trans. on Automatic Control*, *AC-13* (1968): 344-353.

Blueschke-Nikolaevaa, V. Blueschkea, D., Neck, R. (2010), *OPTCON2: An algorithm for the optimal control of nonlinear stochastic dynamic systems with passive learning*. Tech. rep., Department of Economics, Klagenfurt

Erhan Bayraktar and H. Sayit (2010), No Arbitrage Conditions For Simple Trading Strategies, *Annals of Finance*, 6 (1): 147-156.

Andrea Pascucci, (2010), *PDE and martingale methods in option pricing*, Bocconi & Springer Series, Springer-Verlag, New York.

Douglas Borden (2010): *Stochastic Control Theory and High Frequency Trading*, Purumi 77 Publisher.

Igor Halperin (2010), An-implied-multi-factor-model-bespoke-collateralized-debt-obligation-tranches-portfolio-credit-derivatives, *Journal of Credit Risk*, 6(3): 343

Thaleia Zariphopoulou and M. Musiela (2010), Portfolio choice under space-time monotone performance criteria, *SIAM Journal on Financial Mathematics*, 2010: 1-43

Lee, M.H.Kolodziej, W.J.Mohler, R. R. (2007), Stochastic Dynamic System Suboptimal Control with Uncertain Parameters, Aerospace and Electronic Systems, *IEEE Transactions on (Volume: AES-21, Issue: 5)*

H J Kappen (2005), Path integrals and symmetry breaking for optimal control theory, *J. Stat. Mech.* P11011: 7-26.

Diogo Aguiar Gomes (2007), Hamilton-Jacobi methods for Vakonomic, *Nonlinear differ. equ. appl.* 14 (2007): 233-257

Fabio Camilli, Lars Grüne, and Fabian Wirth (2010), Control Lyapunov Functions and Zubov's Method, *SIAM J. Control Optim.*, 47(1): 301-326.

Ioannis Karatzas, John P. Lehoczky and Steven E. Shreve I., (2010), Optimal Portfolio and Consumption Decisions for a 'Small Investor' on a Finite Horizon, *SIAM J. Control Optim*., 25: 1557-1586.

Arnab Basu and Vivek S. Borkar (2010), Stochastic Control with Imperfect Models, *SIAM J. Control and Optimization*, 47(3): 1274-1300

Hua Deng, Miroslav Krstic, and Ruth J. Williams (2001), Stabilization of Stochastic Nonlinear Systems Driven by Noise of Unknown Covariance, *IEEE Transactions On Automatic Control*, 46(8): 1237-1253.

Daniel Anderson, (2008), A mixed relaxed singular maximum principle for linear SDEs with random coefficients, *arXiv: 0812.0136v2*: 1-5.

Kurt Helmes and Richard H. Stockbridge (2010), Thinning and harvesting in stochastic forest models, *Journal of Economic Dynamics and Control*, 35(1): 1-21.

Jussi Keppo (1998), *On arbitrage, optimal portfolio and equilibrium under frictions and incomplete markets*, Otaniemi, ISBN 9512239469, ECONIS - Online Catalogue of the ZBW

J. J. Westman and F. B. Hanson, (1999), O*n Arbitrage, Optimal Portfolio and Equilibrium under Frictions and Incomplete Markets*, Otaniemi, p1-12

S. S. Sritharan (2000), Optimal stopping-time problem for stochastic Navier–Stokes equations and infinite-dimensional variational inequalities, *Journal of Nonlinear Analysis: Theory, Methods & Applications*, 64(5): 1018-1024.

L.A. Socha (2000), Application of true linearization in stochastic quasi-optimal control problems, *Journal of Structural Control*, 7(2): 219-230.

David Siska (2007), Optimal stopping-time problem for stochastic Navier–Stokes equations and infinite-dimensional variational inequalities, *Journal of Economic Dynamics and Control*, 31(12): 3860-3880.

Z.G. Ying (2008), Stochastic Optimal Control of Structural Systems, *The Open Automation and Control Systems Journal*, 1(1): 44-49

Stroock and Varadhan (1979), *Multidimensional Diffusion Processes*, Springer, ISBN 3540903534, 9783540903536, 338 pages

Kumar, S. and Sethi, S.P. (2009), Dynamic Pricing and Advertising for Web Content Providers, *European Journal of Operations Research*, 197: 924-944.

E. Fernández-Cara and E. Zuazua, (2007), Null and approximate controllability for weakly blowing up semilinear heat equations, *Ann. Inst. Henri Poincaré, Analyse Non Linéaire*, 17: 583-616.

Wim Wiegerinck Bert Kappen (2007), Optimal Control in Large stochastic Multi – agent System, Adaptive Agents and Multi-Agent Systems III. *Adaptation and Multi ...*, 3: 15-24

Artur Nowoświat (2002), Optimal Control of a Duffing Oscillator under Parametric and External Excitations, *Journal of Theoretical And Applied Mechanics*, 2: 40.

Fleming, W. H. and Richel R. W. (1975). *Deterministic and stochastic optimal control*, Springer verlag, New York, Heidelberg Berlin.

Suhardio J., Spencer B.F. Jr., Sain M.K., (1992), Nonlinear optimal control of a Duffing system, *Int. J. Non-Linear Mechanics*, 27: 157-172

Nowoświat A., (1999), *Zastosowanie metody szeregów w analizie stochastycz - nych układów dynamicznych*, Praca magisterska nie publikowana, Gliwice

Mohamed Ben Alaya and Benjamin Jourdain, (2006), *Probabilistic approximation of a nonlinear parabolic equation occurring in rheology*, http://cermics.enpc.fr/reports/CERMICS-2006/CERMICS-2006-314.pdf.

Chen, N., and S. G. Kou (2009): Credit Spreads, Optimal Capital Structure, and Implied Volatility with Endogenous Default and Jump Risk, *Math. Finance*, 19: 343–378.

Ingersoll, J. (1977a), A Contingent – Claims Valuation of Convertible Securities, *Journal of Financial Economics*, 10(4): 289-322.

Ingersoll, J. E. (1977b), An Examination of Corporate Call Policies on Convertible Securities, *J. Finance*, 32: 463-478.

Mikkelson, W.H., (1981), Convertible calls and security returns, *Journal of Financial Economics*, 9: 237-264.

Cowan, A. R., N. Nayar, and A. K. Singh (1993): Calls of Out-of-the-Money Convertible Bonds, *Financial Management*, 22: 106-116.

Asquith, P. (1995),: Convertible Bonds Are Not Called Late, *J. Finance*, 50: 1275-1289.

L. Brenig (1980), Space–time memory functions and solution of nonlinear evolution equations, *J. Math. Phys.* 23: 2048 (1982); http://dx.doi.org/10.1063/1.525262 (5 pages)

Areski Cousin (2008), *Comparison results for credit risk portfolios*, Université Claude Bernard Lyon 1, ISFA, Groupe de travail - Chaire Risque de Crédit - Evry 17 Janvier 2008 Joint work with Jean-Paul LAUREN

Stephane Crepey (2003) Calibration of the local volatility in a generalized Black-Scholes model using Tikhonov regularization. *SIAM Journal on Mathematical Analysis*, 34(5): 1183-1206.

S. Cr´epey, (2012a), *A BSDE approach to counterparty risk under funding constraints*. Working Paper, 2011. URL grozny.maths.univ-evry.fr/pages_perso/crepey.

S. Cr´epey, (2012b), Bilateral counterparty risk under funding constraints Part I: Pricing. *Mathematical Finance*, doi: 10.1111/mafi.12004

S. Cr´epey (2012c), Bilateral counterparty risk under funding constraints Part II: CVA. Forthcoming in *Mathematical Finance*. doi: 10.1111/mafi.12005

Min Dai (2007), Intensity-based framework and penalty formulation of optimal stopping problems, *Journal of Economic Dynamics and Control*, 31(12): 3860-3880.

Zhongxing Ye (2006), Pricing CDS with Jump-Diffusion Risk in the Intensity-Based Model, Nonlinear Mathematics for Uncertainty and its Applications, *Advances in Intelligent and Soft Computing*, 100: 221-229.

Michael Kohlmann, Dewen Xiong, Zhongxing Ye (2007), Change of filtrations and mean-variance hedging stochastics, *IJPSP*, 79(6): 539-562.

Jarrow, R.A., and Yu, F., (2001), Counterparty risk and the pricing of defaultable securities. *Journal of Finance*, 56(5): 1765-1799.

Bai, Y.F., Hu, X.H., and Ye, Z.X. (2007), A model for dependent default with hyperbolic attenuation effect and valuation of credit default swap. *Applied Mathematics and Mechanics*, 28(12): 1643-1649.

Park, H.S. (2008), The survival probability of mortality intensity with jump-diffusion. *Journal of the Korean Statistical Society*, 37: 355-363.

J. Gough, V.P. Belavkin, and O.G. Smolyanov (2005), *J. Opt B: Quantum Semiclass, Opt*. 7: S237-S244.