The focus of the book is on meeting the Mathematical needs of students in Senior High Schools who will be taking the West Africa Senior School Certificate Examination (WASSCE) and students preparing for the the Private Candidates Examination. For the reason that the student-teacher ratio is uncomfortably high in our SHS, individual attention to students in the classroom is generally not practicable. Hence, the need for text books written for SHS to be necessarily detailed as this book to enable students follow it independently without supervision. This book is also written to serve as an introductory text for undergraduates and other tertiary students.

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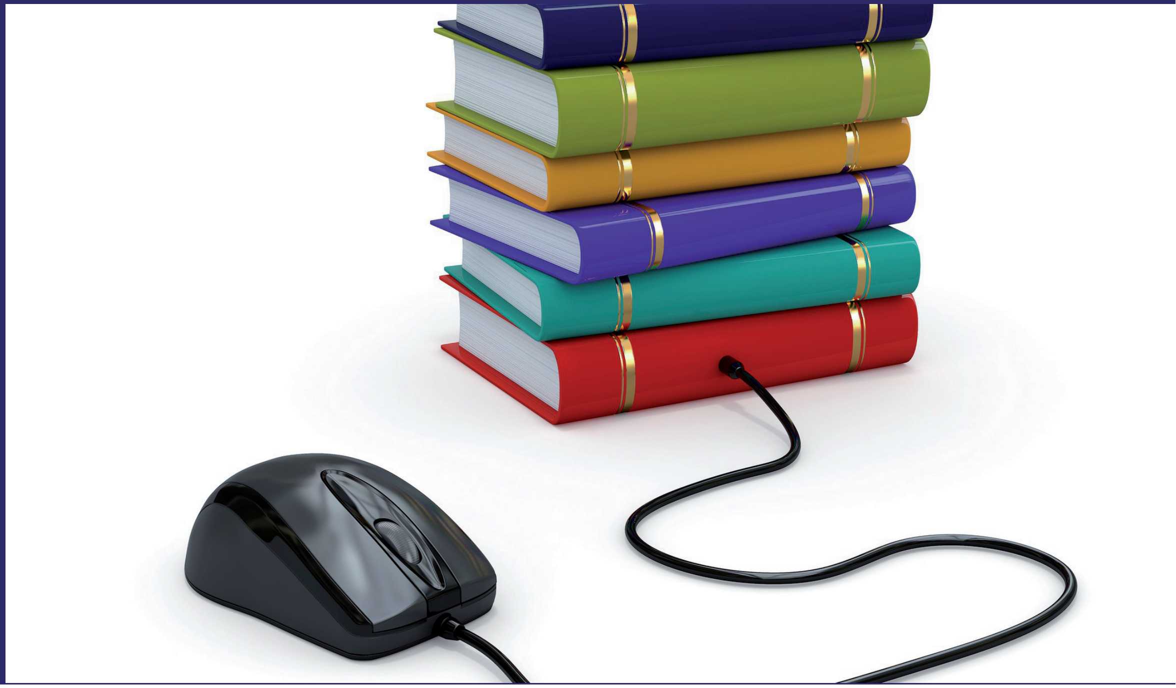
Alhassan, Alhassan, Oladejo



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CHAPTER 1
RATES II
(INCOME TAX, CUSTOM DUTY, ELECTRICITY AND WATER TARIFFS)

1.1 **Introduction**

Schools, hospitals and clinics, water supplies, electricity, civil service, police, army and so on all cost money to run. This money is paid either by the government or by the district assemblies and the communities themselves. One way governments raise money is through ***taxation***. Some of these taxes are income tax, custom duty, exercise duty, purchase tax, sales tax and license fees.
An important source of government revenue is ***income tax*** which is usually the ***tax*** deducted from an employee's monthly salary before the salary is paid to him or her.
Similarly, the tax imposed on goods entering or leaving the country is termed ***custom duty***.

NB

Custom duty is calculated in two forms:
One is from the ***value of the goods*** which for imports includes the cost, the insurance, the freight charges and the other from the ***cost of getting the freight*** on board the ship which is usually the duty for exports.

1.2 **General Deductions**

1. Tax free income (pay) = sum of all allowances
2. Taxable income (pay) = salary – tax free pay (allowance)
3. Total deductions = tax + social security contributions
4. Tax = rate of tax x corresponding amount to tax
5. Take home pay = net income
   = annual salary – total deductions
6. Net monthly pay = $\frac{net\ annual\ income}{12}$

Example 1.1

A woman takes GH¢300 per annum. She is allowed a tax free pay of GH¢100. If she pays 120Gp as tax on her taxable income, how much is she left with?

Solution

Taxable income (pay) = salary – tax free pay = GH¢300 - GH¢100 = GH¢200
Tax = 120Gp x GH¢200 = GH¢1.20 x GH¢200 = GH¢240
Amount left = salary – tax = GH¢300 - GH¢240 = GH¢60

Example 1.2

In evaluating the annual salary of someone, the following were allowed free of tax:
Personal allowance --------- GH¢350
Children allowance ---------GH¢300
Marriage allowance --------GH¢250
Find the taxable pay of the following:

(a) Prof. Elvis, who earns GH¢1500 a year, not married and has no child
(b) Madam Akos, who receives GH¢2500 a year, is married with two children

Solution

(a) Prof. Elvis will enjoy only personal allowance of GH¢350 since he is not married and has no child
Hence, Prof. Elvis' taxable pay = salary – allowance
= GH¢1500 - GH¢350
= GH¢1,150
(b) Madam Akos also enjoys a personal allowance of GH¢350, children allowance of GH¢300 and marriage allowance of GH¢250 x 2 = GH¢500 (for her two children)
Hence, her total allowance = 350 + 300 + 500 = GH¢1,150

Taxable pay = salary – allowance = 2500 – 1,150
= GH¢1,350

Example 1.3

In a certain country, the annual income tax payable by an individual is as follows:

AMOUNT	RATE OF TAX
First ¢140,700	Free
Next ¢100,000	5%
Next ¢150,000	15%
Next ¢200,000	25%
Next ¢300,000	35%
Next ¢350,000	45%
Next ¢400,000	55%

Mr. Hamosa's annual salary is ¢1,390,700. Calculate (a) his taxable income (b) his annual income tax (c) the percentage of his income that went into tax, correct to two significant figures.

Solution

(a) Annual salary = ¢1,390,700
Allowance = 140,700
Hence, Mr. Hamosa's taxable income = annual salary – Allowance
= ¢1,390,700 – ¢140,700
= ¢1,250,000
(b) For the first ¢100,000, rate of tax = 5%
Implies, tax to be paid = $\frac{5}{100} \times 100{,}000 = 5{,}000$
Amount left to be taxed = ¢1,250,000 - ¢100,000 = ¢1,150,000
Rate of tax for the next ¢150,000 = 15%
Hence, tax to be paid = $\frac{15}{100} \times 150{,}000 = 22{,}500$
Amount left to be taxed = ¢1,150,000 - ¢150,000 = ¢1,000,000
Rate of tax for the next ¢200,000 = 25%

Hence, tax to be paid = $\frac{25}{100} \times 200{,}000 = 50{,}000$

Amount left to be taxed = ¢1,000,000 - ¢200,000 = ¢800,000

Rate of tax for the next ¢300,000 = 35%

Tax to be paid = $\frac{35}{100} \times 300{,}000 = 105{,}000$

Amount left to be taxed = ¢800,000 – ¢300,000 =¢500,000

Rate of tax for the next ¢350,000 = 45%

Tax to be paid = $\frac{45}{100} \times 350{,}000 = 157{,}500$

Amount left to be taxed = ¢500,000 - ¢350,000 = ¢150,000

Rate of tax for the next ¢400,000 = 55%

Hence, tax to be paid = $\frac{55}{100} \times 150{,}000 = 82{,}500$

Hence, annual income tax

= ¢5,000 + ¢22,500 + ¢50,000 + ¢105,000 + ¢157,500 + ¢82,500

= ¢442,500

(c) Total income tax = ¢442,500

Annual income = ¢1,390,700

Hence, percentage of his income that went into tax

$= \frac{442{,}500}{1{,}390{,}700} \times 100 = 30.38\% = 30\%$

Example 1.4

In a certain country, the tax payable by an individual in 1997 was assessed at the following rates:

For the first $200.00 --------- Nil

For the next $300.00 --------10%

For the next $500.00 -------15%

For the next $800.00 -------20%

The remaining amount -----30%

(a) Calculate the income tax payable by Vonda whose annual income was $2,800.00

(b) If Miss Nyenge paid a monthly tax of $29.00, calculate her annual income

Solution

(a) Mr. Vonda's annual income = $2,800.00
His allowance = $200.00
Hence, his taxable income = $2,800 - $200 = $2,600.00
(b) For the first $300, rate of tax = 10%
Hence, tax paid = $\frac{10}{100} \times 300 = 30$
Amount left to be taxed = $2,600 - $300 = $2,300.00
Rate of tax for the next $500 = 15%
Hence, tax paid = $\frac{15}{100} \times 500 = 75$
Amount left to be taxed = $2,300 - $500 = $1,800.00
Rate of tax for the next $800 = 20%
Tax paid = $\frac{20}{100} \times 800 = 160$
Rate of tax for the remaining amount = 30%
Remaining amount = $1800 – $800 = $1000.00
Tax paid = $\frac{30}{100} \times 1000 = 300$
Income tax payable = $30 + $75 + $160 + $300 = $565.00
Monthly tax paid by Miss Nyenge = $29.00
Hence, total annual tax = $29 x 12 = $348.00
For the first $200 tax is free
Next $300; tax = $30
Next $500; tax = $75
Next $800; tax = $160
Remaining amount taxed = $348 - $265 = $83
Rate of tax for the remaining amount = 30%
Let the remaining amount = $x
Implies, 30% of $x = $83.00
Hence, $x = \frac{100 \times 83}{30} = 276.67$
Therefore, Miss Nyenge's annual income
= $200 + $300 + $500 + $800 + $276.67 = $2076.67

Example 1.5

Mr. Ansu's salary was ¢3,450,000.00 per annum. He contributed 5% of his salary per annum to a Social Security Fund on which he did not pay any tax. In addition, he was allowed ¢240,000.00 per annum free of tax. After these deductions, Mr. Ansu paid tax 17.5% on the first 70% of his taxable income. He also paid tax at the rate of 45% on the remaining 30% of his taxable income. Calculate:

(a) Mr. Ansu's annual contribution to the Social Security Fund;

(b) The annual amount on which he paid tax; (c) His annual income tax;

(d) The percentage of his income that went into tax, correct to three significant figures.

Solution

(a) Mr. Ansu's annual salary = ¢3,450,000.00

Percentage of his annual salary contributed to the SSF = 5%

Hence, his annual cont. to SSF = $\frac{5}{100} \times 3{,}450{,}000 = 172{,}500$

(b) His total annual tax free allowance = SSF + tax free allowance

= ¢172,500+¢240,000

= ¢412,500

Annual amount on which he paid tax

= his annual salary – his total annual tax free allowance

= ¢3,450,000 - ¢412,500 = ¢3,037,500

(c) Tax on the first 70% of his taxable income = 17.5%

Implies, 70% of taxable income

= $\frac{70}{100} \times 3{,}037{,}500 = 2{,}126{,}250 = 372{,}093.75$

The remaining 30% of his taxable income = ¢3,037,500 - ¢2,126,250 = ¢911,250

45% of the remaining 30% = 45% of ¢911,250

= $\frac{45}{100} \times 911{,}250 = 410{,}062.50$

His annual income tax = ¢372,093.75 + 410,062.50 = ¢782,156.25

(d) The percentage of his income that went into tax is:

$$\frac{total\ \ tax}{total\ \ income} \times 100\% = \frac{782156.25}{3450000} \times 100 = 22.7\%$$

Example 1.6

The monthly electricity charges in a country are calculated as follows:
First 50 units -----------¢4,000.00
Next 100 units ---------¢120.00 per unit
Next 150 units --------¢150.00 per unit
Next 300 units --------¢220.00 per unit
Remaining units -----¢350.00 per unit
(a) How much did Mr. Owusu pay for using 720 units in a month?
(b) A man paid ¢73,260.00 for electricity consumed in a month. How many units of electricity did he consume?

Solution

(a) Total units consumed = 720 units
For the first 50 unit, the charge = ¢4,000.00
Units left to be charged = 720 – 50 = 670 units
Next 100 units, the charge = ¢120 per unit
Implies, charge for the 100 units =¢12,000.00
Units left to be charged = 670 – 100 = 570units
Next 150 units, the charge = ¢150 per unit
Implies, charge for the 150 units = 150 x 150 = ¢22,500
Units left to be charged = 570 – 150 = 420 units
Next 300 units, the charge = ¢220 per unit
Implies, charge for the 300 units = 300 x 220 = ¢66,000
Units left to be charge = 420 – 300 = 120 units
For the remaining units the charge= ¢350 per unit
Implies charge for the 120 units left = ¢350 x 120 = ¢42,000
Therefore, the amount paid for using 720 units
= ¢4,000 + ¢12,000 + ¢22,500 + ¢66,000 + ¢42,000 = ¢146,500
(b) Total amount paid = ¢73,260.00
For the first 50 units, the amount paid = ¢4,000
Amount left = ¢73,260 – ¢4,000 = ¢69,260.00

For the next 100 units, the amount paid = ¢12,000
Amount left = ¢69,260 – ¢12,000 = ¢57,260.00
For the next 150 units, the amount paid = ¢22,500
Amount left = ¢57,260 - ¢22,500 = ¢34,760
For the next 300 units, the amount paid = ¢220 per units
If ¢220 equals 1unit
Then ¢34,760 will equal $\frac{34760}{220} \times 1 = 158 units$
Hence, total units consumed = 50 + 100 + 150 + 158 = 458units

Example 1.7

In a household, the meter reading for water at the end of October 1999 was 7848 thousand litres. The metre reading at the end of November, 1999 was 7908 thousand litres. The house was charged for consumption at the following rates:
The first 10 thousand litres at ¢500.00 per thousand litres
The next 30 thousand litres at ¢1,300.00 per thousand litres
The next 40 thousand litres at 1820 per thousand litres
Calculate (a) The consumption at the end of November
(b) The total charge for the consumption.

Solution

(a) The consumption at the end of November = 7,908,000 litres
The first 10,000 litres = $\frac{10000}{1000} = 10 \Rightarrow 10 \times 500 = 5000$
The next 30,000 litres = $\frac{30000}{1000} = 30 \Rightarrow 30 \times 1300 = 39000$
The next 40,000 litres = $\frac{40000}{1000} = 40 \Rightarrow 40 \times 1820 = 72800$
So 10,000 + 30,000 + 40,000 = 80,000 litres
Implies, 7908,000 – 80,000 = ¢7,828,000
Therefore $\frac{7828000}{1000} = 7828 \Rightarrow 7828 \times 1820 = 14{,}246{,}960$
At the end of November the charge is
= ¢14,246,860 + ¢72,800 + ¢39,000 + 5,000
= ¢14,363,760.00

(b) October meter reading was now 7,848,000litres

Implies, 7,848,000 – 80,000 = ¢7,768,000 $\frac{7768000}{1000} = 7768$

Hence,

$7768 \times 1820 = 14137760$

At the end of October, the charge was = 14137760 + 72800 + 39000 + 5000 = 14254560

Hence, November + October = 14363760 + 14254560

= ¢28618320.00

EXERCISE

QUE. A

Dr. Elliot has 6 children with his wife Love and has a total income of GH¢8,500 in 2009. He was allowed the following free of tax.

Personal --------- GH¢1,200

Wife -------------GH¢300

Each child ----GH¢250 for a maximum of 4

Dependent relatives ---GH¢400

Insurance -----GH¢250

The rest was taxed as follows:

The first GH¢2000 at 10%

Next GH¢2000 at 15%

Next GH¢2000 at 20%

Next GH¢2000 at 25%

Calculate;(a) His tax free pay (b) His income tax (c) His monthly income

(d) His net monthly pay

QUE. B

In Ghana, the annual income tax is calculated using the following rates:

Taxable pay	**Rate of tax**
First GH¢1000	5Gp
Next GH¢1000	10Gp
Next GH¢3000	25Gp
Next GH¢4000	60Gp

(a) If a person's taxable pay is GH¢2,450, calculate his income tax
(b) Also find a person's taxable pay if he pays a tax of GH¢500

QUE. C

In Ghana, the annual income tax payable by the individual in a certain year was assessed at the following rates:
For the first GH¢300 ---------- Nil
For the next GH¢240 ---------5Gp
For the next GH¢480 --------7.5Gp
For the next GH¢480 --------10Gp
For the next GH¢960 -------12.5Gp
For the next GH¢1140 ------15Gp
(a) Calculate the income tax payable by Dr. Emelia who earned GH¢3000 per annum
(b) What percentage of Dr. Erwin's annual income was payable as income tax
(c) If in the previous year the rate of tax assessment was the same as above and
Dr. Erwin paid GH¢216 as tax, what was his income for that year?

QUE. D

Dr. Elliot's annual salary is ¢423000.00. he pays 5% of this salary, on which he pays no tax, into a Social Security fund. Using the 1987 tax schedule, calculate:

(a) his chargeable income (tax pay p. a)
(b) his annual income tax
(c) his deductions (i.e. deductions due to Social Security func and income tax)
(d) his annual salary after total deductions

QUE. E

Mr. Gregory pays 5% of his salary into a Social Security fund. He pays no tax on this contribution. If his annual salary in 1987 was ¢250000.00, calculate

(a) his Social Security contribution
(b) his taxable pay
(c) his income tax
(d) his deductions
(e) his salary after total deductions (i.e. his **net** salary)

QUE. F

In June, 1986, the tax schedule was as follows.

Chargeable income p. a.	Rate of Tax
First ¢35000 of chargeable income	5%
Next ¢40000 of chargeable income	15%
Next ¢35000 of chargeable income	25%
Next ¢35000 of chargeable income	35%
Next ¢35000 of chargeable income	45%
Exceeding ¢180000 of chargeable income	55%

Use the 1986 schedule to calculate the income tax paid by Mr. Allotey, Mr. Duodo and Mr. Mensah, whose chargeable incomes were, respectively

(a) ¢120000.00p. a. (b) ¢145000.00 p. a. (c) ¢195000.00 p. a.

QUE. G

Suppose the following allowances were granted free of income tax from the employees annual salary in June 1986:
Employee's personal allowance ¢30000.00
Employee's wife's allowance ¢24000.00
Allowance for each child under 18 ¢12000.00

Find

(a) the annual taxable income

(b) the net income of the following employees

(i) Mr. Banini whose income was ¢195000.0 p. a. and who was married but had no children

(ii) Mr. Lokko who also earned ¢195000.00 p. a. was married and had two children under the age of 18

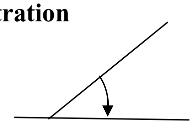
(iii) Mr. Alhassan whose salary was ¢195000 p. a. and who had four children, two of whom were above the age of 18 but had no wife.

CHAPTER 2
PLANE GEOMETRY
(ANGLES AND CIRCLES)

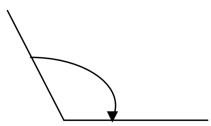
2.1 **Angle Properties Of Lines**

Generally, an ***angle*** is formed when two given lines meet at a common point. Angles can be described as being**:** ***acute, obtuse, reflex, complementary or supplementary.***

- ***Acute angles*** are angles found between 0^o and 90^o
  Illustration

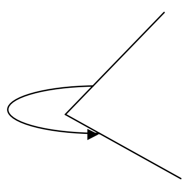


- ***Obtuse angles*** are angles between 90^o and 180^o
  Illustration



- ***Reflex angles*** are angles between 180^o and 360^o

  Illustration

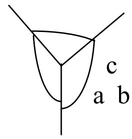


- ***Complementary angles*** are angles which add up to 90^o

- ***Supplementary angles*** are angles which add up to 180^o

- Angles that ***meet*** at any given point add up to 360^o

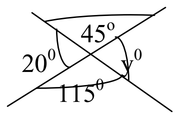
Illustration



i.e. $a + b + c = 360^o$

Example 2.1

Find the value of y in the diagram below



Since all angles meet at a common point, their sum will give 360^o

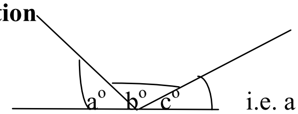
Implies, $y + 20 + 45 + 115 = 360$

$y + 180 = 360$

$y = 360 - 180 = 180^o$

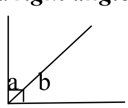
- Angles on a ***straight line*** add up to 180^o

Illustration



i.e. $a + b + c = 180^o$

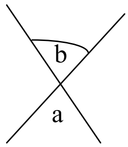
- Angles in a ***right angle*** add up to 90^o



i.e. $a + b = 90^o$

- When two straight lines cut each other at a point resulting to four angles, ***vertical opposite angles*** are formed. Vertical opposite angles are equal

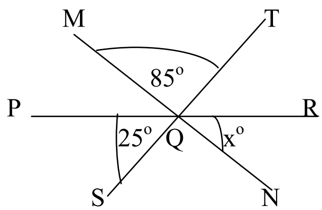
Illustration



i.e. a = b

'a' and 'b' are said to be vertical opposite angles.

Example 2.2

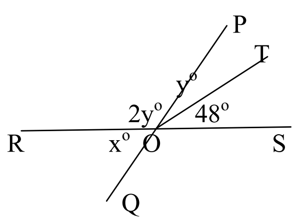


PQR, SQT and MQN are straight lines. PQR = 25^o and MQT = 85^o
Find the value of x.

Solution

Sum of angles on line ST (straight line) is 180^o
Implies, <SQP + <PQM + <MQT = 180^o
25 + <PQM + 85 = 180 Implies <PQM = 70^o
Since <PQM and x are vertical opposite angles, <PQM = x = 70^o

Example 2.3



In the diagram, PQ and RS intersect at O. angle TOS = 48^o
Calculate the value of x.

Solution

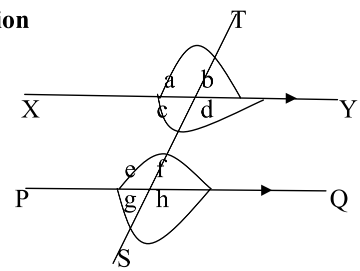
Consider the straight line PQ (sum of angle on a straight line is 180)

$x + 2y = 180$ ------- (1)

Again, from line RS,
$y + 2y = 180$ implies, $y = 44^o$
Hence, putting value of y into equation (1) above gives $x = 92^o$

- The line that cuts two ***parallel*** lines is called the ***transversal***.

Illustration



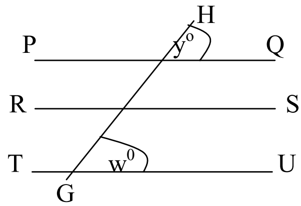
ST is a transversal that cuts the two parallel lines XY and PQ.

(a) Angles 'a and e', 'd and h', 'c and g', 'b and f' are all described as ***corresponding angles***. Corresponding angles are equal.
Hence, a = e, d = h, c = g, b = f. Thus, the ***top*** angles on the same sides of the two parallel lines ***corresponds*** and the ***down*** angles on the same sides of the two parallel lines corresponds

(b) Angles 'e and d', 'c and f' are described as ***alternate angles***. Alternate angles are equal. Hence, e = d and c = f. Thus, the opposite interior angles on the two parallel lines ***alternate*** each other.

(c) Angles ‘c and e’, ‘d and f’ are described as ***co-interior angles***.

Example 2.4

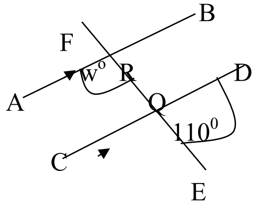


PQ, RS, TU and GH are straight lines. PQ is parallel to RS and TU. What kind of angles are v and w?

Solution

They are corresponding angles

Example 2.5



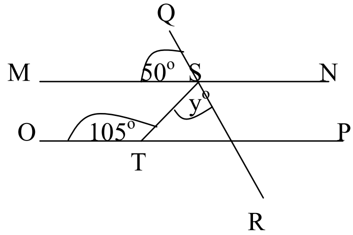
In the diagram AB and CD are parallel lines. What is the value of w?

Solution

<BRQ and <DHE are corresponding angles. Hence, <BRQ = <DHE = 110^o

Again, from straight line AB, <BRQ + w = 180 implies, w = 70^o

Example 2.6



In the diagram, MN and OP are parallel lines. Find the value of y.

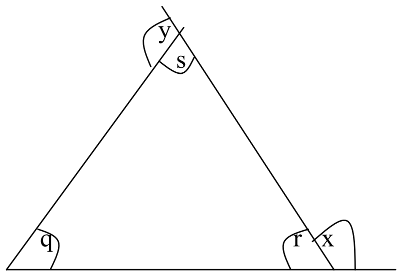
Solution

Sum of angles on a straight line MN equals 180^o.
Implies, <QSN = 180 – 50 = 130^o
<STO = <NST = 105^o (Alternate angles)
<NST = y + <NSR = 105
Sum of angles on line QR equals 180
Implies, <NSR = 180 – 130 = 50^o
Hence, y = 55^o

2.2 **Angle Properties Of A triangle**

The angles inside a triangle are described as ***interior angles*** and angles outside it are its ***exterior angles***.
The ***sum of all interior angles*** of any given triangle is **180^o**.
An exterior angle equals the sum of two opposite interior angles.

Illustration

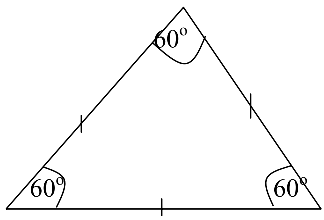


In the triangle above, q, r and s are its interior angles whiles x and y are the exterior angles.
Thus, $q + r + s = 180^o$ (sum of angles of a triangle)
And $x = q + s$, $y = q + r$ (exterior equals sum of opposite angles)

NB

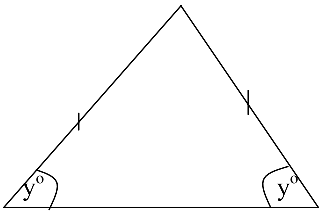
- The ***equilateral triangle*** has all three sides, three interior angles equal and three lines of symmetry

  Illustration



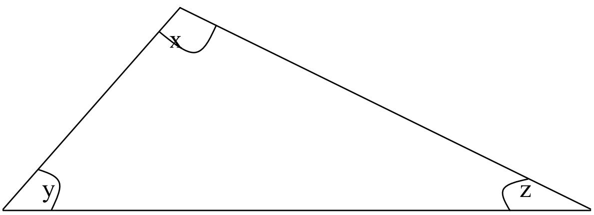
- The ***Isosceles triangle*** has two sides equal, two interior angles equal and one line of symmetry

Illustration

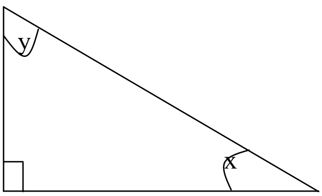


- The ***scalene triangle*** has no equal sides and angles and no line of symmetry

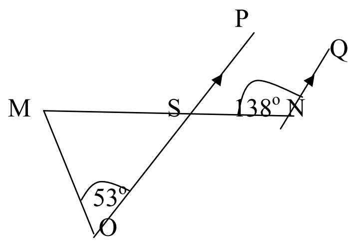
Illustration



- The ***right angled triangle*** has one of its angles being 90^o



Example 2.7

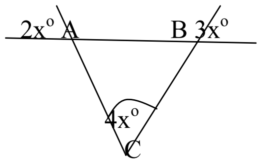


In the diagram, OP is parallel to NQ, MSN is a straight line, MOP = 53° and SNQ = 138°. Find OMS.

Solution

MN is the transversal of the parallel line OP and NQ.
SNQ = MSP = 138° (Corresponding angles)
Sum of angles in a triangle MSO = 180°
Implies, MSO + MOP + OMS = 180
Hence, OMS = 85°

Example 2.8

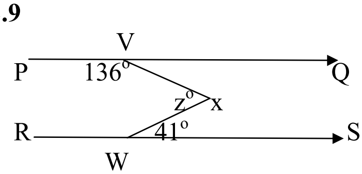


Calculate the value of x in the figure above.

Solution

<CAB = 2x and <ABC = 3x
(since vertical opposite angles are equal)
From triangle ABC, <ABC + <ACB + <BAC = 180
Implies, 2x + 3x + 4x = 180
Hence, x=20°

Example 2.9

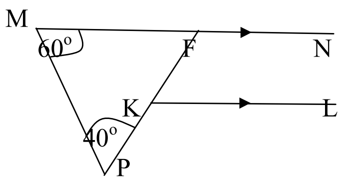


In the diagram, PQ is parallel to RS. Find the value of z.

Solution

<VUS = <PVU = 136° (Alternate angles)
Implies, <WUX = 180 – 136 = 44° and <UWX + <WXU + <WUX = 180°
Hence, <WXU = 95° (straight line angles) <WXU + z = 180 implies, z = 85°

Example 2.10

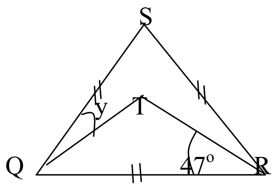


In the figure above, MN is parallel to KL. If angle NMP = 60° and angle MPK = 40°, calculate angle PKL.

Solution

<MPF + <PMF + <PFM = 180° (sum of interior angles of a triangle)
Implies, 40 + 60 + <PFM = 180 hence, <PFM = 80°
<PFM + <PFN = 180 implies, <PFN = 100°
<PFN + <FKL = 180
<FKL = 80°
Again, <FKL + <PKL =180 implies, <PKL = 100°

Example 2.11

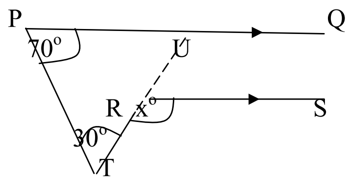


In the figure, QRS is an equilateral triangle and QRT is an isosceles triangle. If angle QRT = 47°, what is the value of y?

Solution

<QRT = <TQR = 47° (isosceles)
<TQR + y = 60° (for equilateral triangle) Implies, y = 13°

Example 2.12



In the diagram, PQ is parallel to RS, <QPT = 70°, <TRS = x° and <PTR = 30°. Find the value of x.

Solution

Sum of angles in a triangle PTU = 180°. Implies, <TUP = 80°.
<TUP = <URS = 80° (alternate angles)
From line TU, x + <URS = 180 hence, x = 100°

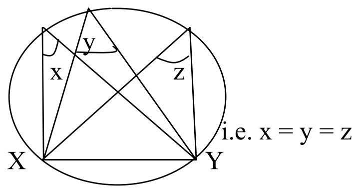
2.3 **Circles**

The circle is an ***oval*** shaped plane figure.
Refer to chapter 36 for notes on parts of the circle with solved examples.

2.3.1 **Angle Properties Of A Circle**

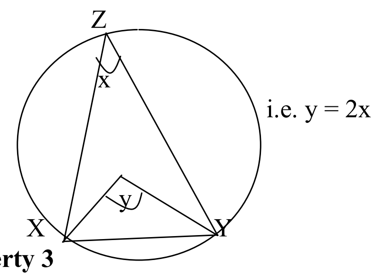
Property 1
The angles a chord or arc subtends at the circumference in the same segment of a circle are equal.

Illustration



Property 2

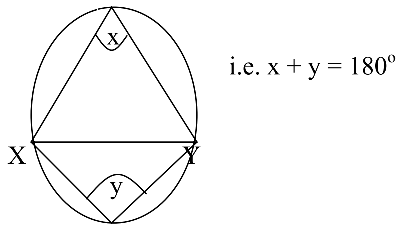
The angle a chord subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle.



Property 3

The sum of the angles a chord or an arc subtends at the circumference of opposite segments of a circle is equal to 180^o.

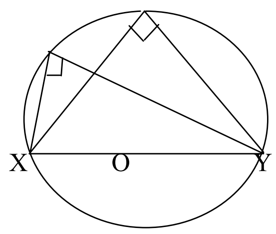
Illustration



Property 4

The angle that the diameter of a circle subtends at the circumference is 90^o

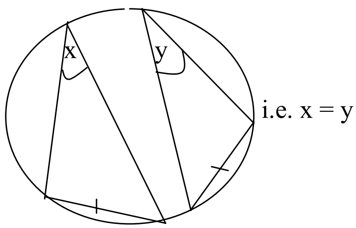
Illustration



Property 5

Equal chords or arcs subtends the same angles at the centre of a circle

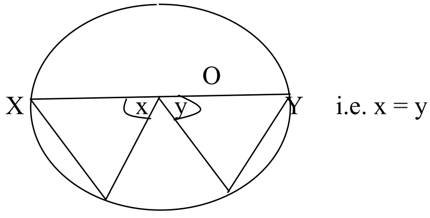
Illustration



Property 6

Equal chords or arcs subtend the same angles at the circumference of a circle

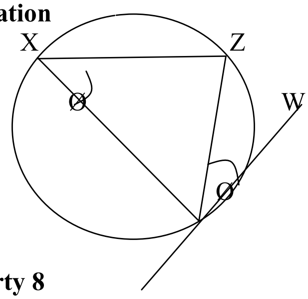
Illustration



Property 7

The angle between a chord and a tangent at the end of the chord equals the angle in the alternate segment.

Illustration

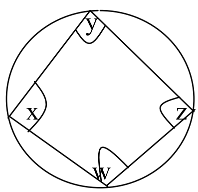


i.e. <ZYW = <YXZ

Property 8

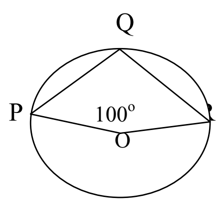
The opposite angles of a cyclic quadrilateral are supplementary. Thus, adding up to 180^o.

Illustration



$x + z = 180^o$ and $w + y = 180^o$

Example 2.13

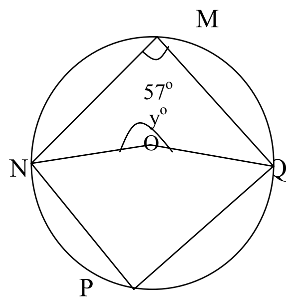


In the diagram P, Q and R are points on a circle with centre O. If angle POR = 100^o, find <POR.

Solution

The major arc PR subtends an angle of 360 – 100 = 260^o
at the centre and at the circumference, it subtends an angle
PQR = ½ x 260 = 130^o (Refer to Property 2)

Example 2.14

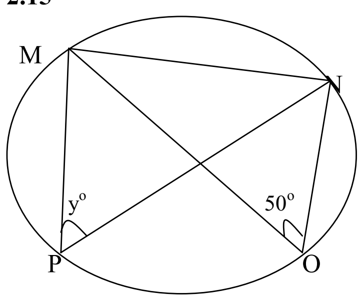


In the diagram O is the centre of MNPQ and angle NMQ = 57^o.
find the value of y.

Solution

<NOQ = 2 x <NMQ = 2 x 57 = 114^o
(Property 2 applied)
Hence, y + <NOQ = 360 implies, y = 246^o

Example 2.15

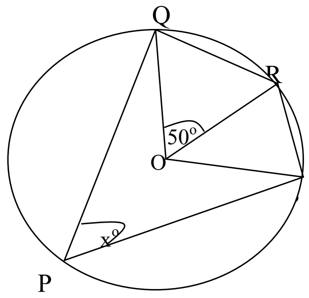


In the diagram, MN is a chord and <MON = 50°.
Find the value of the angle marked y.

Solution

<MON = <MNP = 50° (Refer to Property 1)

Example 2.16

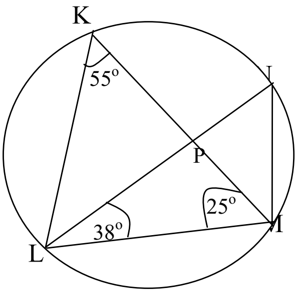


In the diagram, PQRS is a circle with centre O,
$|QR| = |RS|$ *and* $< RPQ = 50^o$. *If* $< QPS = x$, *find* x

Solution

Chord QR = Chord RS
Implies, QOR = ROS = 50° (Refer property 6)
<QOS = 100°
x = ½ <QOS = ½ x 100 = 50° (Refer to property 2)

Example 2.17

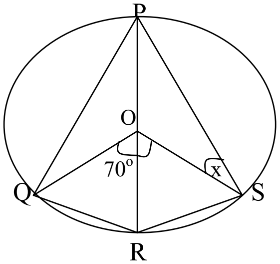


In the diagram, K, L, M, N are points on a circle. If <LKM = 55°, <NLM = 38° and <NMP = 25°, find <LPM

Solution

The chord LM subtends angles <LKM and <LNM at the circumference.
Hence, <LKM = <LNM = 55°
From <LNM + <NMP + <MPN = 180° implies, <MPN = 100°
From line LN, <MPN + <LPM = 180° implies, <LPM = 80°

Example 2.18



In the diagram PQRS is a circle with centre O.
$|QR| = |RS|$ *and* $< QOR = 70^o$.

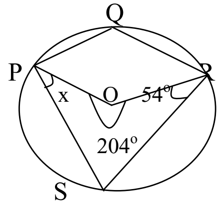
Find the value of x.

Solution

QR and RS are equal chords and subtends equal angles at the centre
Hence, <QOR = <ROS = 70^o
By property 2,
$< ROS = 2\times < RPS$

$\Rightarrow 70 = 2x \Rightarrow x = 35^o$

Example 2.19

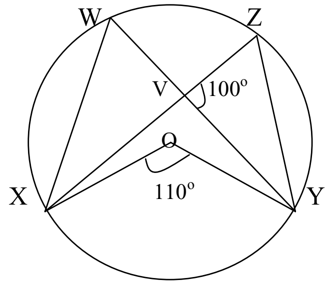


The diagram shows a circle PQRS with center O. the reflex angle at O is 204^o,
angle ORS =54^o and angle OPS = x. Find x.

Solution

Sum of angles at the centre equals 360^o
Implies, <POR + 204^o = 360, hence, <POR = 156^o
But <PSR = ½ x <POR = ½ x 156 = 78^o
Now, sum of angles in a quadrilateral PORS is 360^o
Implies, x + 204 + 54 + 78 = 360
Therefore, x = 24^o

Example 33.20

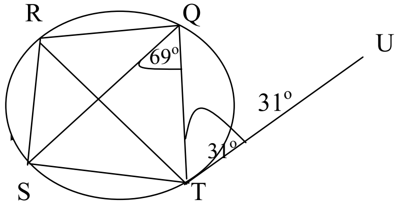


In the diagram, WXYZ is a circle with center O. XZ and WY intersect at V.
<XOY = 110° and <YVZ = 100°. Calculate: (i) <XZY (ii) <WXZ

Solution

<XOY = 110°
Implies, <XZY = ½ x 110 = 55° (Property 2)
Hence, <XWV = <XZY = 55° (Refer to property 1)
<WVX = <YVZ = 100° (Vertically opposite angles are equal)
Implies, <WXZ = 180 – (<XWV + <WVX) = 25°

Example 2.21



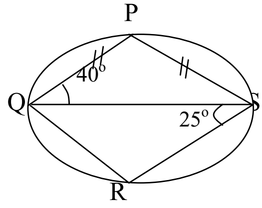
In the diagram TU touches the circle at T and RT is diameter. Angle UTQ = 31°
And angle TQS = 69°. Calculate (i) angle QRS (ii) angle QTS (iii) angle SQR

Solution

(i) <UTQ = <QRT = 31^o (Refer to property 7)
<SQT = <SRT = 69^o (Property 1)
Hence, angle QRS = 69 + 31 = 100^o
(ii) <QRT = <QST = 31^o (since are subtended by same chord QT)
Hence, <QTS = 180 – 100 = 80^o
(iii) <TQR = 90^o
But <TQR = <SQT + <SQR implies, <SQR = 21^o

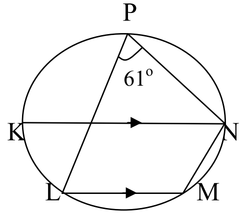
EXERCISE

QUE. A



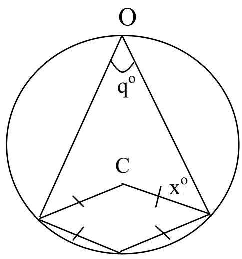
In the diagram, P, Q, R, S are points on a circle.
$|PQ| = |PS|, < PQS = 40^o, < QSR = 25^o$
Calculate the value of: (i) <QPS (ii) <QRS (iii) <RQS

QUE. B



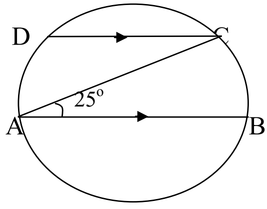
In the figure above, LM is a chord parallel to the diameter KN of the circle KLMNP.
If angle NPL = 61^o, calculate the angle MLN.

QUE. C



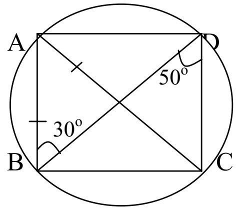
The diagram shows a circle PQRS with center C. quadrilateral CPSR is a rhombus, <QPC = <CRQ = x° and <PQR = q°. find (i) q (ii) x (iii) <QRS

QUE. D



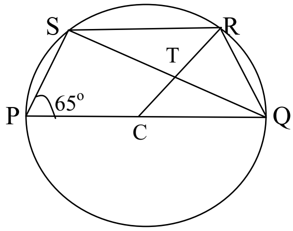
In the diagram AB is a diameter of the circle ABCD. DC is parallel to AB and
<BAC = 25° calculate (i) <ADC (ii) <CAD

QUE. E



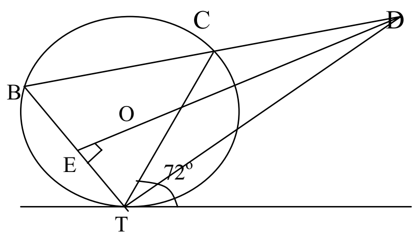
In the diagram above, A, B, C and D are points on a circle.
$|AB| = |AC|, > BDC = 50^o$ *and* $< ABD = 30^o$. *Calculate* $< CAD$

QUE. F



In the diagram, C is the centre of the circle, PQRS, STQ and PCQ are straight lines and RS is parallel to QP. Angle SPC = 65° (a) show that triangle CQT and RST are similar (b) find (i) <RSQ (ii) <CRQ

QUE. G

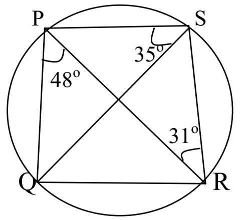


In the diagram A, B, C are points on a circle with centre O. AT is the tangent to the circle at A. the line DOE is perpendicular to AB, $|AB| = |AC|$ *and* $< TAC = 72^o$

(a) Calculate (i) <BCA (ii) <CAD (iii) <CDA

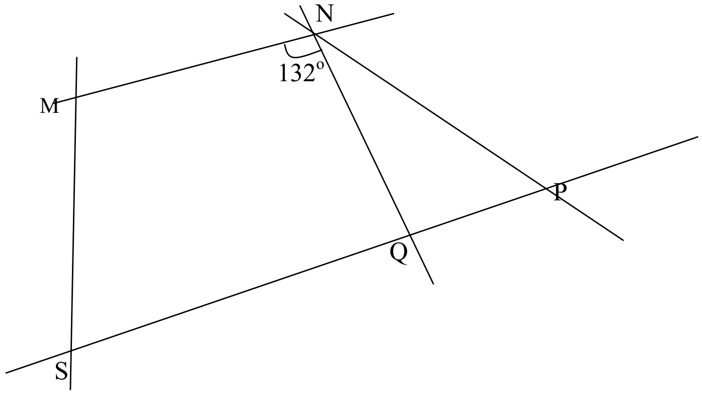
(b) Use your results in (a) to show that (i) AD bisects angle TAC (ii) $|CD| = |CA|$

QUE. H



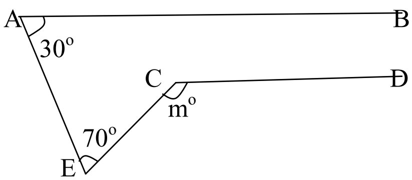
In the diagram, P, Q, R and S are points on the circle. Angle QPR = 48°,
angle PSQ = 35° and angle PRS = 31°. calculate (i) angle PQR (ii) angle QRS

QUE. I



In the diagram, MNPS is a quadrilateral. A line is drawn through N to cut SP at Q. angle MNQ = 132^{o}, angle SMN is twice angle MSQ and angle NPQ is twice angle QNP. If NP bisects the acute angle at N, find (i) angle SQN (ii) angle MSQ

QUE. J

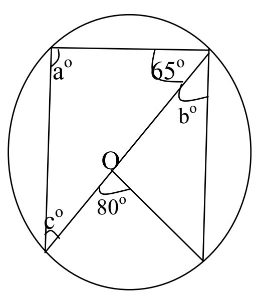


In the diagram, AB is parallel to CD. <BAE = 30^{o}, <AEC = 70^{o}, <ECD = m^{o}. Find m?

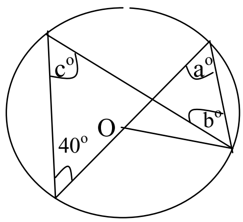
QUE. K

In the diagrams below, O is the centre of each circle. Find the values of a, b, c.

1.



2.

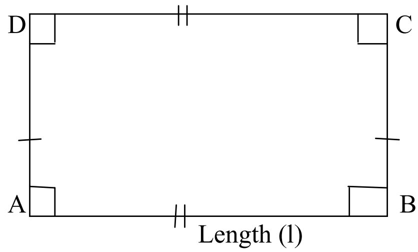


CHAPTER 3
MENSURATION
(PLANE FIGURES, CIRCLES, AREAS & VOLUMES)

3.1 **The Rectangle**

The rectangle is a plane figure with four sides all of which are at 90^o to each other and with equal parallel opposite sides.

Illustration



3.1.1 **Perimeter Of The Rectangle**

This is the total distance round the figure obtained by summing the lengths of all the sides and measured in ***units of lengths***.
Perimeter, P = l + w + l + w = 2l + 2w (Where l is ***length*** and w is ***width***)

3.1.2 **Area Of The Rectangle**

This is the amount of surface between the boundaries and measured in ***square units***.
Area, A = l x w = lw

Example 3.1

If a rectangle has width 5cm and length 12cm, find the perimeter and area of the rectangle.

Solution

From question, l = 12cm, w = 5cm
Implies, perimeter = 2l + 2w = 2(12) + 2(5) = 24 + 15 = 39cm
Area = l x w = 12 x 5 = 60cm^2

Example 3.2

Find the width of a rectangle whose perimeter is 24cm and length 6cm.

Solution

From question, perimeter = 24cm and length = 6cm
From p = 2l + 2w
Implies, 24 = 2(6) + 2w
24 = 12 + 2w
2w = 24 – 12 = 12
Implies, w = 6cm

Example 3.3

If the area of a rectangle is 32cm^2 and its width is 12cm, find its length.

Solution

Area = 32cm^2 and width = 12cm
From A = l x w
32 = 12l
Implies, length, l = 2.67cm

Example 3.4

The annual rent of a rectangular plot of land is ¢1,024,000.00 at a rate of ¢320.00 per square meter. If the length is 80m, find the width of the plot.

Solution

Annual rent = ¢1,024,000.00, Rate = ¢320.00 per square meter, length = 80m

If ¢320.00 is equivalent to 1m^2

Then, ¢1,024,000.00 will be equivalent to $\frac{1024000 \times 1}{320} = 3200m^2$

Therefore, the area of the rectangle is 3200m^2

But area = l x w

Implies, 3200 = 80w

Hence, width, w = 40m

Example 3.5

A rectangle is (3x + 2)cm long and 5cm wide. If a square has the same perimeter as that of the rectangle, what is the length of the side of the square?

Solution

Length, l = (3x + 2), width, w = 5cm

Perimeter = 2(3x + 2) + 2(5) = 6x + 4 + 10 = (6x + 14)cm

From question, perimeter of square = perimeter of rectangle = (6x + 14)cm

But Perimeter of square = 4 x length

Implies, 6x + 14 = 4l

Hence, length = $\frac{6x + 14}{4} cm$

Example 3.6

A rectangle has a perimeter of 54cm. if the ratio of the length of the rectangle to the width is 5:4, what is the length of the rectangle?

Solution

Perimeter = 54cm

Ratio = length: width = 5:4

Total ratio = 9
Perimeter = 2l + 2w = 2(l + w)
Implies, 54 = 2(l + w)

$$\Rightarrow \frac{54}{2} = l + w = 27cm$$

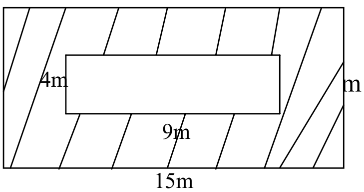
$$\Rightarrow 9 \equiv 27$$

$$then,\ 5 \equiv \frac{5 \times 27}{9} = 15cm$$

(where total ratio 9 corresponds to sum of length and width 27cm, and 5 ratio of length)
Hence, length of the rectangle is 15cm.

Example 3.7

Find the area of the shaded portion in the diagram above.

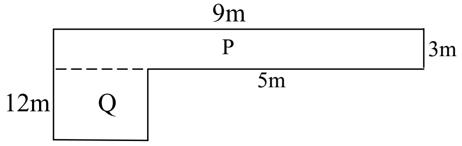


Solution

First, area of bigger rectangle = 15 x 7 = 105cm^2
Area of smaller rectangle = 9 x 4 = 36cm^2
Area of shaded portion
= area of bigger rectangle - area of smaller rectangle
= 105 – 36 = 69cm^2

Example 3.8

Find the area of the figure below



Solution

Area of P = l x w = 9 x 3 = 27m^2
Area of Q = 9 x 4 = 36m^2
Area of the whole figure = area of P + area of Q = 27 + 36 = 63cm^2

Example 3.9

The perimeter of a rectangle is 45cm and has a length of 7cm. Find its area.

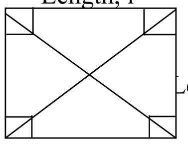
Solution

Perimeter = 45cm, length = 7cm
From p = 2l + 2w
Implies, w = 45 – 2(7) = 31cm
Hence, area = l x w = 7 x 31 = 217cm^2

3.2 **The Square**

The square is a four sided plane figure with equal sides and diagonals, each side at 90^o to each other, diagonals bisects each other at 90^o and has four lines of symmetry.

Illustration



3.2.1 **Perimeter Of A Square**

Perimeter = 4 x length of side = 4 x l = 4l

3.2.2 **Area Of A Square**

Area = $(\text{length})^2 = l^2$

Example 3.10

Find the perimeter and area of the square of side 8m.

Solution

Perimeter = 4 x l = 4 x 8 = 32m
Area = $L^2 = 8^2 = 64m^2$

Example 3.11

The area of a square figure is $64m^2$. Find its length and perimeter.

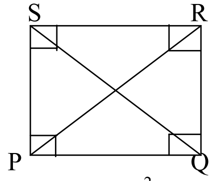
Solution

Area = $64m^2$
From area = L^2
Implies, 64 = L^2, L = $\sqrt{64} = 8m$
Perimeter = 4L = 4 x 8 = 32m

Example 3.12

The area of a square is 36m^2. Calculate the size of its diagonals.

Solution

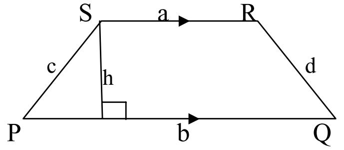


From area = L^2
Implies, Length, L = 6m
Considering triangle PQS,
By Pythagoras theorem,
$|QS|^2 = |PS|^2 + |PQ|^2 = 6^2 + 6^2 = 72 \Rightarrow |QS| = \sqrt{72} = 8.485m$
Therefore, each diagonal is 8.485m long

3.3 **The Trapezium**

The trapezium is a four sided plane figure with two unequal opposite parallel sides

Illustration



3.3.1 **Perimeter Of A Trapezium**

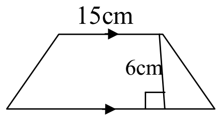
Perimeter = a + b + c + d

3.3.2 **Area Of A Trapezium**

Area = $\frac{1}{2}\times(sum\ of\ \ parallel\ \ sides)\times height = \frac{1}{2}\times(a+b)\times h$

Example 3.13

Find the area of the trapezium below:

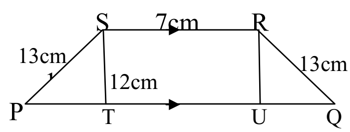


Solution

Area = $\frac{1}{2}\times(sum\ of\ \ parallel\ \ sides)\times height = \frac{1}{2}\times(a+b)\times h$

$= \frac{1}{2}\times(a+b)\times h = \frac{1}{2}\times(15+24)\times 6 = \frac{1}{2}\times 39\times 6 = 117cm^2$

Example 3.14



In the diagram PQRS is a trapezium. SR is parallel to PQ.

$|PS| = |QR| = 13cm\ \ and\ \ |SR| = 7cm.\ Find\ \ |PQ|$

Solution

$|PQ| = |PT| + |TU| + |UQ|\ \ \ But\ |TU| = |RS| = 7cm$

$Since\ |PS| = |QR|\ \Rightarrow |PT| = |UQ|$

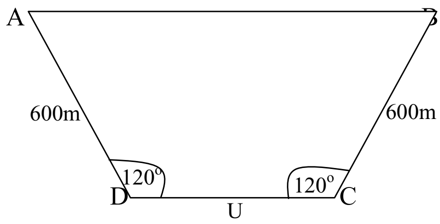
From triangle PST,

$|PS|^2 = |PT|^2 + |TS|^2 \Rightarrow 13^2 = |PT|^2 + 12^2 \Rightarrow |PT| = 5cm$

$\therefore |PQ| = 5+7+5 = 17cm$

Example 3.15

The diagram shows a field ABCD in the form of a trapezium.
If $|AD| = |BC| = 600m$,
$< ADC = < BCD = 120^o$ *and* $|DC| = 450m$

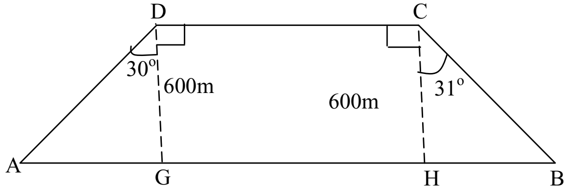


NOT DRAWN TO SCALE

(a) Find the perimeter of the field
(b) Calculate, correct to three significant figures, the area of the field.

Solution

Redrawing, the trapezium becomes;



$From\ \Delta ADG,\ \sin 30^o = \frac{|AG|}{|AD|} = \frac{|AG|}{600}$

$\Rightarrow 600\sin 30^o = |AG|$

$\Rightarrow |AG| = 300m$

$But\ |AG| = |BH| = 300m\ and\ |DC| = |GH| = 450m$

$Again,\ |AB| = |AG| + |GH| + |BH| = 300 + 450 + 300 = 1050m$

$(a)\ Perimeter = |AB| + |BC| + |CD| + |AD| = 1050 + 600 + 450 + 600 = 2700m$

$(b)\ First\ apply\ Pythagoras\ theorem\ to\ \Delta ADG$

$\Rightarrow |AD|^2 = |AG|^2 + |DG|^2 \Rightarrow 600^2 = 300^2 + |DG|^2$

$\Rightarrow 360000 = 90000 + |DG|^2 \Rightarrow 450000 = |DG|^2 \Rightarrow |DG| = \sqrt{450000} = 300\sqrt{5}$

$\Rightarrow |DG| = 519.6m = Height\ of\ field$

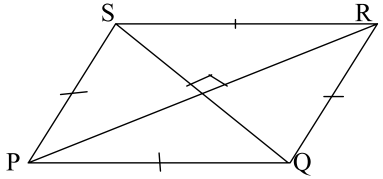
$\therefore\ Area = \frac{1}{2} \times height(Sum\ of\ parallel\ sides) = \frac{1}{2} \times 519.6(450 + 1050)$

$\therefore\ Area = 389700m^2 = 390{,}000m^2\ to\ 3\ significant\ figures$

3.4 **The Rhombus**

The rhombus is a type of parallelogram with equal sides. Its diagonals bisect each other at right angles and the opposite angles are equal

Illustration



3.4.1 **Perimeter Of A Rhombus**

Perimeter = Sum of lengths of all four sides = 4 x length of one side

3.4.2 **Area Of A Rhombus**

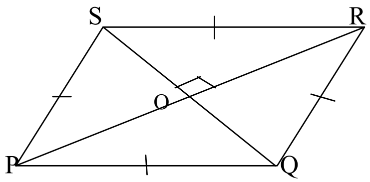
Area = length of base x height

$Or\ \ Area = \frac{1}{2} \times \text{Pr}\, oduct\ \ of\ \ diagonals$

Example 3.15

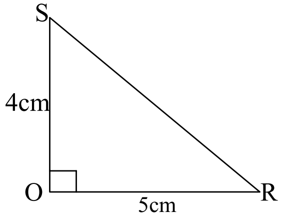
The diagonals of a rhombus are 8cm and 10cm long. Calculate correct to 1 decimal place, the length of a side of the rhombus.

Solution



$From\ \ question,\ |PR| = 10cm,\ |QS| = 8cm \Rightarrow |PO| = \tfrac{1}{2} \times |OR| = \tfrac{1}{2} \times 10 = 5cm,\ |OS| = 4cm$

Removing triangle ORS;



By Pythagoras theorem,

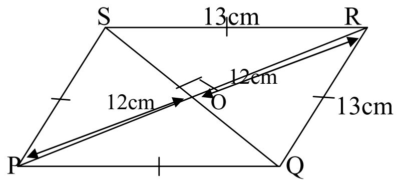
$|SR|^2 = |OR|^2 + |OS|^2 = 5^2 + 4^2 = 25 + 16 = 41 \Rightarrow |SR| = \sqrt{41} = 6.4cm$

Therefore, the length of side of the rhombus is 6.4cm

Example 3.16

A rhombus has sides of length 13cm and one of its diagonals is 24cm long. Find the area of the rhombus.

Solution



Length of side = 13cm, length of diagonal = 24cm

Applying Pythagoras theorem to triangle ORS,

$|SR|^2 = |OR|^2 + |OS|^2 \Rightarrow 13^2 = 12^2 + |OS|^2 \Rightarrow |OS| = 5cm$

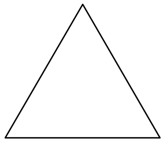
Hence, length of diagonal SQ = 2(5) = 10cm

$Area\ of\ rhombus = \frac{1}{2} \times 24 \times 10 = 120cm^2$

3.5 **The Triangle**

The triangle is a three sided plane figure.

Illustration



3.5.1 **Perimeter Of A Triangle**

Perimeter = Sum of lengths of all three sides

3.5.2 **Area Of A Triangle**

$Area = \frac{1}{2} \times base \times height = \frac{1}{2}bh$

Example 3.17

If the base and height of a triangle are respectively 6cm and 8cm, find the area of the triangle.

Solution

Base = 6cm, height = 8cm

$Area = \frac{1}{2} \times base \times height = \frac{1}{2} bh = \frac{1}{2} \times 6 \times 8 = 24cm^2$

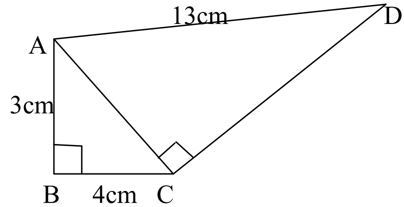
Example 36.18

The sides of a triangle are pcm, (p + 7)cm and (p + 8)cm.
Find the length of the hypotenuse, if its perimeter is 30cm.

Solution

Perimeter of triangle = sum of all three sides
Implies, 30 = p + p +7 + p + 8 = 3p +15
Then, 30 – 15 = 3p implies, p = 5cm
But hypotenuse = longest side = (p + 8) = 5 + 8 = 13cm

Example 3.19



In the diagram, ABCD is a quadrilateral, <ABC = <ACD = 90^o,
$|AB| = 3cm, |BC| = 4cm\ \ and\ \ |AD| = 13cm.\ Find\ its\ area.$

Solution

Area of quadrilateral =area of triangle ABC + area of triangle ACD
For triangle ABC, h = 3cm, b = 4cm, implies, area of ABC
= $\frac{1}{2}bh = \frac{1}{2}\times 4\times 3 = 6cm^2$
Apply Pythagoras theorem to ABC, $\Rightarrow |AC| = 5cm$
Similarly, from triangle ACD, $|CD| = 12cm$
Therefore, area of ACD = $\frac{1}{2}bh = \frac{1}{2}\times 12\times 5 = 30cm^2$
Hence, area of quadrilateral = 6 + 30 = 36cm^2

3.6 **Parts Of The Circle**

The Circumference**:** This is the total distance round a circle also called ***perimeter*** of the circle.

The Arc**:** This is any portion of the circumference.

The Chord**:** This is a straight line that joins two points on the circumference.

The Diameter**:** This is a chord that passes through the centre of a circle dividing the circle into two equal parts.

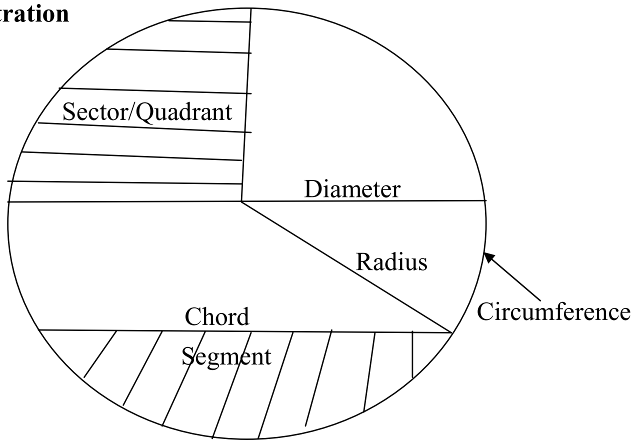
The Radius**:** This is the distance from the centre to any part of the circumference.

The Segment**:** A chord drawn divides the circle into two parts called the ***segment*** which are named the ***minor*** (smaller) and ***major*** (bigger) segments.

The Sector**:** The area bounded by two radii of a circle is called the sector.

The Quadrant*:* one quarter of a circle is called a quadrant obtained by dividing the circle into four equal parts.

Illustration



3.7 **Finding The Circumference Of A Circle**

The circumference, C of a circle is given by: $C = 2\pi r = \pi d$
Where r = radius and d = diameter = 2r

Example 3.20

Find the circumference of a circle whose radius is 9cm.
(take $\pi = \frac{22}{7}$)

Solution

Radius, r = 9cm
$C = 2\pi r = 2 \times \frac{22}{7} \times 9 = 56.57cm$

Example 3.21

If the diameter of a circle is 20cm, find its circumference.

Solution

Diameter, d = 20cm

$$C = \pi d = \frac{22}{7} \times 20 = 62.857cm$$

3.8 **Finding The Area Of A Circle**

The area, A of a circle is given by:

$$A = \pi r^2 = \tfrac{1}{4}\pi d^2$$

Example 3.22

Find the area of a circle with radius 9cm. (take $\pi = \frac{22}{7}$)

Solution

Radius = 9cm

$$Area = \pi r^2 = \frac{22}{7} \times 9^2 = 254.57cm^2$$

Example 3.23

If the diameter of a circle is 15cm, find its area

Solution

Diameter = 15cm

$$Area = \tfrac{1}{4}\pi d^2 = \tfrac{1}{4} \times \tfrac{22}{7} \times 15^2 = 176.7857cm^2$$

Example 3.24

The area of a circle is $154m^2$. Find its circumference.

Solution

Area = 154m^2

$But \ \ area = \pi r^2 \Rightarrow 154 = \frac{22}{7} r^2 \Rightarrow r = 7m$

Again, $C = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44m$

3.9 **Finding The Length Of Arc Of A Circle**

The length of arc of a circle is given by:

$$L = \frac{\theta}{360} \times 2\pi r = \frac{\theta}{360} \times Circumference$$

Example 3.25

Find the length of arc of a circle of radius 7cm and subtends an angle of 60^o at the centre.

Solution

Radius = 7cm, Ø = 60^o

$$L = \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2 \times \frac{22}{7} \times 7 = 7.33cm$$

Example 3.26

An angle of 30^o is subtended by an arc at the centre of a circle of circumference 40cm. find the length of the arc.

Solution

Circumference = 40cm, Ø = 30^o

$$L = \frac{\theta}{360} \times Circumference = \frac{30}{360} \times 40 = 3.33cm$$

Example 3.27

The length of an arc of a circle is 8.8cm. the radius of the circle is 3.5cm. find the angle that the arc subtends at the centre of the circle.

Solution

L = 8.8cm, radius = 3.5cm

$$L = \frac{\theta}{360} \times 2\pi r \Rightarrow 8.8 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 3.5 \Rightarrow \theta = 144^o$$

Example 3.28

A sector of a circle of a radius of 14cm subtends an angle of 54° at the centre. Find the length of the arc. $(Take\ \pi = \frac{22}{7})$

Solution

Radius = 14cm, Ø = 54°

$$L = \frac{\theta}{360} \times 2\pi r \Rightarrow L = \frac{54}{360} \times 2 \times \frac{22}{7} \times 14 = 13.2cm$$

3.10 **Finding The Area Of Sector**

Area of Sector $= \frac{\theta}{360} \times \pi r^2 = \frac{\theta}{360} \times area\ of\ Circle$

NB: Area of a quadrant $= \frac{1}{4}\pi r^2$

And area of semi-circle $= \frac{1}{2}\pi r^2$

Example 3.29

The minute hand of a clock moved from 12 to 4. If the length of the minute hand is 3.5cm, find the area covered by the minute hand.

Solution

If the minute hand moved from 12 to 12 covers an angle of 360°

Then moving from 12 to 4 covers $\frac{4}{2}\times 360 = 120^o$

But length of minute hand = radius of circle = 3.5cm

Hence, area of sector $= \frac{\theta}{360}\times 2\pi r \Rightarrow 8.8 = \frac{120}{360}\times\frac{22}{7}\times 3.5\times 3.5 = 12.8cm^2$

Perimeter of a sector is given by: P = 2r + length of minor arc

Area of minor segment = Area of whole sector – Area of triangle

$$= \frac{\theta}{360}\times\pi r^2 - \frac{1}{2}r^2\sin\theta$$

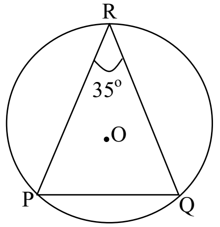
Similarly, **Area of major segment** $= \frac{(360-\theta)}{360}\times\pi r^2 + \frac{1}{2}r^2\sin\theta$

Example 3.30

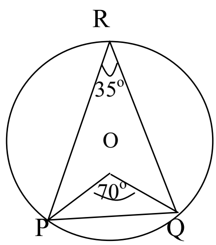
In the diagram P, Q and R are points on the circle with center O and diameter 14cm. angle PRQ = 35°. find correct to one decimal place

(a) The length of the minor arc PQ

(b) The chord PQ



Solution

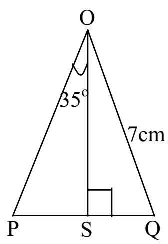


(a) <POQ = 2<PRQ = 2(35) = 70°

From r = d/2 implies, r = 14/2 = 7cm

Implies, length of arc $= L = \frac{\theta}{360} \times 2\pi r = \frac{70}{360} \times 2 \times 3.142 \times 7 = 8.6cm$

(b)



Considering triangle QSO,

$$Sin35^o = \frac{Opp}{Hyp} = \frac{|QS|}{7} \Rightarrow |QS| = 7\sin 35^o = 4.015$$

$$\Rightarrow |PQ| = 2|QS| = 2 \times 4.015 = 8.0cm\ to\ 1dp$$

Example 3.31

A chord PQ of a circle of radius 5cm subtends an angle of 70° at the centre, O. find correct to 3 significant figures,

(i) the length of the chord PQ

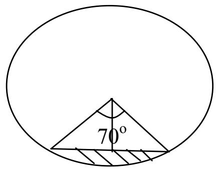
(ii) the length of the arc PQ

(iii) the area of the sector POQ

(iv) the area of the minor segment cut off by PQ

Solution

(i) Chord PQ = 5cm, Ø = 70°



Dividing the triangle PQO into two, we have

$$Sin35^o = \frac{Opp}{Hyp} = \frac{\frac{1}{2}|PQ|}{|PO|} = \frac{\frac{1}{2}|PQ|}{5} \Rightarrow \frac{1}{2}|PQ| = 2.868$$

$$\Rightarrow |PQ| = 2(2.868) = 5.74cm \ to \ 3 \ sig \ figs.$$

(ii) Length of arc PQ $= \frac{70}{360} \times 2 \times \frac{22}{7} \times 5 = 6.11cm \ to \ 3 \ Sig. \ figs$

(iii) Area of sector POQ $= \frac{70}{360} \times \frac{22}{7} \times 5^2 = 15.3cm^2 \ to \ 3 \ Sig. \ figs$

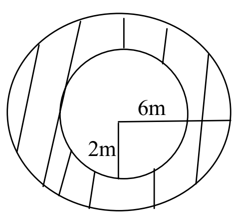
(iv) Area of minor segment of shaded region = Area of sector POQ – Area of ΔPOQ

$But \ area \ of \ \Delta POQ = \frac{1}{2}ab\sin\theta = \frac{1}{2} \times 5 \times 5 \sin 70 = 11.7cm^2$

Hence, area of minor segment = 15.3 – 11.7 = 3.6cm^2

Example 3.32

Find the area of the shaded region of the figure below:



Solution

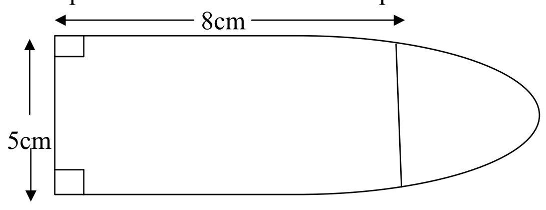
$Area\ of\ inner\ circle = \frac{22}{7} \times 2 \times 2 = 12.57m^2$

$Area\ of\ outer\ circle = \frac{22}{7} \times 6 \times 6 = 113.143m^2$

Area of shaded region = area of outer circle – area of inner circle
= 113.143 – 12.57
= 100.573m^2

Example 3.33

The diagram below is a compound which is made up of a rectangular portion, 5cm by 8cm with a semi-circle attached. Calculate the perimeter and area of the compound.



Solution

$Length\ of\ semi-circle = \frac{Circumference}{2} = \frac{1}{2} \times 2\pi r = \pi r = \frac{22}{7} \times 2.5 = 7.857cm$

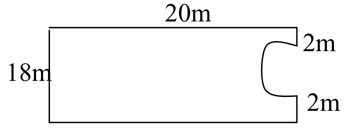
Perimeter = sum of lengths of straight sides + length of semi-circle
= 8 + 5 + 8 + 7.857 = 28.857cm

Next, area of rectangular portion = L x W = 8 x 5 = 40cm^2

$Area\ of\ semi-circle = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 5 = 7.857cm^2$

Area = area of rectangle + area of semi-circle
= 40 + 7.857 = 47.857cm^2

Example 3.34



The diagram above represents a rectangular compound 20m by

18m with a semi-circular portion cut off. Calculate
(a) the perimeter of the compound
(b) the area of the compound (Take $\pi = \frac{22}{7}$)

Solution

(a) Perimeter = 18 + 20 + 20 + 2 + 2 + length of semi-circle
= 62 + length of semi-circle

$But\ length\ of\ semi-circle = \frac{1}{2}\pi d = \frac{1}{2}\times\frac{22}{7}\times 14 = 22m$

Implies, perimeter = 62 + 22 = 84m^2

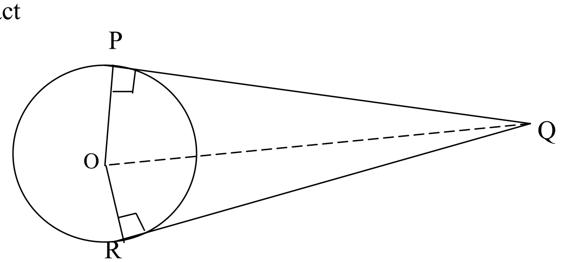
(b) Area of rectangle = L x W = 20 x 18 = 360m^2

$Area\ of\ semi-circle = \frac{1}{2}\pi r^2 = \frac{1}{2}\times\frac{22}{7}\times 7\times 7 = 77m^2$

Area of compound = area of rectangle – area of semi-circle = 360 – 77 = 283m^2

3.11 **Tangents**

A tangent to a circle is a straight line drawn from the centre to the point of contact



Properties Of Tangents

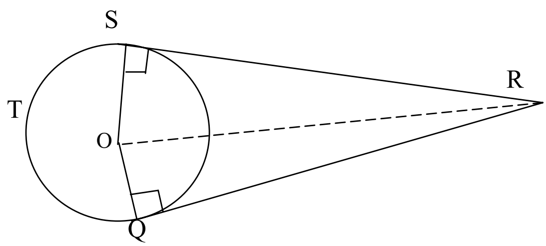
Using the figure above, we have the following properties of tangents.

1. $|QP| = |QR|$
2. $|OP| = |OR|$

3. <OPQ = <ORQ = 90^o
4. <OQP = <OQR
5. triangle OPQ and ORQ are right-angled

Example 3.35

The diagram shows a belt QRST round a shaft R (of negligible radius) and a pulley of radius 0.6m. O is the centre of the pulley, $|OR|1.5m$ and the straight portion QR and RS of the belt are tangents at Q and S to the pulley.



Calculate (i) angle QOS, correct to the nearest degree (ii) the total length of the belt (QRST) to the nearest meter $(take\ \pi = 3.142)$

Solution

(i) From angle properties of circles, <QOS = <QOR + <SOR and <QOR= <SOR
Implies, <QOS = 2<QOR
Consider triangle QOR (right angled)

$$Cos\,\theta = \frac{Adj}{Hyp} = \frac{|OQ|}{|OR|} = \frac{0.6}{1.5} = 0.4 \quad \Rightarrow \theta = 66.4^o$$

Implies, <QOS = 2 x 66.4 = 132.8^o = 133^o to nearest degree

$(ii)\ length\,of\ belt\ QRST = |QR| + |SR| + length\,of\ arc\ STQ$

$But\ |QR| = |SR|$

Applying Pythagoras theorem to triangle QOR,

$|OR|^2 = |OQ|^2 + |QR|^2 \ \Rightarrow |QR| = 1.375m$

Again, arc STQ subtends angle = 360 - <QOS = 360 – 133 = 227°

Length of arc = $\frac{227}{360} \times 2 \times 3.142 \times 0.6 = 2.377m$

Therefore, total length of belt QRST = 1.375 + 1.375 + 2.377 = 5.127 = 5m

3.12 **Surface Area And Volume Of Solid Figures**

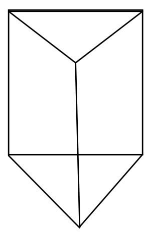
3.12.1 **The Prism**

This is a solid figure with uniform cross sections such as rectangular, triangular, circular etc.

Surface Area Of Prisms = Sum of area of all faces

Volume Of Prisms = product of area of cross section and length

Illustration



3.12.2 **The Cylinder**

This is a prism having circular cross section. Examples are the tin of Milo or Milk.

$Curved\ Surface\ are\ of\ cylinder = 2\pi rh$

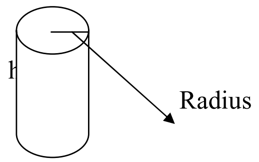
$Total\ Surface\ area\ of\ a\ closed\ cylinder = 2\pi r^2 + 2\pi rh$

$Total\ Surface\ area\ of\ a\ hollow\ cylinder\ (with\ one\ end\ opened) = \pi r^2 + 2\pi rh$

$Total\ Surface\ area\ of\ a\ hollow\ cylinder\ (with\ two\ ends\ opened) = 2\pi rh$

$Volume\ of\ a\ cylinder = \pi r^2 h$

Illustration



Example 3.36

A cylinder has diameter 14cm and height 11cm. calculate the curved surface area of the cylinder. $(Take\ \pi = \frac{22}{7})$

Solution

Diameter, d = 14cm, implies, radius, r = 7cm

Curved surface area = $2\pi rh = 2 \times \frac{22}{7} \times 7 \times 11 = 484cm^2$

Example 3.37

Find the volume of a cylindrical tin of radius 1.25cm and height 3.5cm. $(Take\ \pi = \frac{22}{7})$

Solution

Radius, r = 1.25cm, height, h = 3.5cm,

Volume = $\pi r^2 h = \frac{22}{7} \times 1.25 \times 1.25 \times 3.5 = 17.2cm^3$

Example 3.38

Water flows from a tap into an empty cylindrical jar at the rate of $32\pi cm^3$ per second. If the radius of the jar is 4cm, find the height of water in the jar after 2 seconds.

Solution

$Rate\ of\ flow\ of\ water\ per\ second = 32\pi\, cm^3$

$After\ two\ seconds\ the\ volume\ of\ water = 2 \times 32\pi\, cm^3$

$Radius\ of\ jar = 4cm,\ volume\ of\ cylindrical\ jar = \pi r^2 h$

$\Rightarrow \pi(4)^2 h = 64\pi \Rightarrow h = 4cm$

Example 3.39

A cylinder with one end open has a radius of 5cm and height 15cm. calculate the
(i) area of the base (ii) curved surface area (iii) total surface area (iv) volume of the cylinder

Solution

(i) $Area\ of\ base = \pi r^2 = \frac{22}{7} \times 5 \times 5 = 78.57cm^2$

(ii) $Curved\ Surface\ area = 2\pi rh = 2 \times \frac{22}{7} \times 5 \times 15 = 157.143cm^2$

(iii) $Total\ Surface\ area = \pi r^2 + 2\pi rh = 78.57 + 157.143 = 235.7cm^2$

(iv) $Volume = \pi r^2 h = \frac{22}{7} \times 25 \times 15 = 1178.57cm^3$

3.12.3 **The Sphere**

$Surface\ area\ of\ a\ Sphere = 4\pi r^2$

$Volume\ of\ a\ Sphere = \frac{4}{3}\pi r^3$

$Volume\ of\ a\ Hemisphere = \frac{2}{3}\pi r^3 (i.e.\ half\ volume\ of\ sphere)$

$Curved\ Surface\ area\ of\ a\ Hemisphere = 2\pi r^2$

Example 3.40

Calculate the surface area and volume of a sphere of radius 9cm.

Solution

$Surface\ area = 4\pi r^2 = 4 \times \frac{22}{7} \times 81 = 1018.29cm^2$

$Volume = \frac{4}{3}\pi r^3 = \frac{4}{3}\times\frac{22}{7}\times 9^3 = 3054.857cm^3$

3.12.4 **The Cone**

$Curved\ Surface\ area = \pi rl$

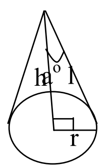
$Total\ Surface\ area(with\ covered\ base) = \pi r^2 + \pi rl$

$Total\ Surface\ area\ of\ a\ hollow\ cone(without\ base) = \pi rl$

$Volume\ of\ a\ cone = \frac{1}{3}\pi r^2 h$

NB: l = slant height, r = radius, h = height of cone, ά = semi-vertical angle

Illustration



Example 3.41

A solid cube of side 8cm was melted to form a solid circular cone. The base radius of the cone is 4cm. calculate correct to one decimal place, the height of the cone.

Solution

Volume of cube = LBH = 8 x 8 x 8 = 512cm^3

Volume of cone = Volume of cube (Since Cube melted into cone)

$\Rightarrow \frac{1}{3}\pi r^2 h = 512 \Rightarrow h = 30.55cm$

Therefore, height of cone = 30.6cm

Example 3.42

A solid brass cube of side 10cm is melted down. It is recast to form a solid cone of height 10cm and base radius r cm. Calculate (i) the radius of the cone

(ii) the curved surface area of the cone

Solution

(i) side of brass cube = 10cm

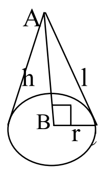
Volume of cube = $10^3 = 1000cm^3$

$Volume\ of\ cone = \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 10 = 10.47619r^2$

But volume of cube = volume of cone

$\Rightarrow 1000 = 10.47619r^2 \quad \Rightarrow r = 9.8cm$

(ii)



h = 10cm, r = 9.8cm, l = slant height

Applying Pythagoras theorem to triangle ABC,

$\Rightarrow l^2 = r^2 + h^2 = 9.8^2 + 10^2 = 196.04 \Rightarrow l = 14cm$

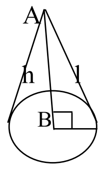
$Curved\ Surface\ area = \pi rl = \frac{22}{7} \times 9.8 \times 14 = 431.2cm^2$

Example 3.43

A sector of area $427cm^2$ is cut out from a thin circular metal sheet of radius 17cm. it is then folded with the straight edges coinciding to form a cone. Calculate correct to three significant figures (a) the angle of the sector (b) the length of arc of the sector

(c) the height of the right circular cone.

Solution



(a) Area of sector, A = 427cm^2, radius of circular metal, R = 7cm, let Ø = angle of sector

$$\Rightarrow A = \frac{\theta}{360} \times \pi R^2 \Rightarrow 427 = \frac{\theta}{360} \times 3.142 \times 17^2 \Rightarrow \theta = 169^o$$

$$(b)\ \ Length\ \ of\ \ arc = \frac{\theta}{360} \times 2\pi R = \frac{169}{360} \times 2 \times 3.142 \times 17 = 50.2cm$$

(c) Circumference of cone = length of arc = 50.2cm

$$\Rightarrow 2\pi r = 50.2 \Rightarrow r = 7.99cm$$

From figure of the cone above l = R = 17cm,
By Pythagoras theorem, $l^2 = h^2 + r^2$ implies, h = 15.01cm

NB:

1. in finding the base radius of a cone, we use the following formulae:

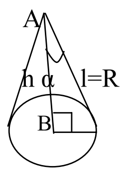
$$\frac{r}{R} = \frac{\theta}{360} \quad \Rightarrow r = \frac{\theta}{360} \times R$$

2. Again, we find the semi-vertical angle ά by using: $Sin\alpha = \frac{r}{R}$

Example 3.44

A sector of a circle of radius 7cm has an angle of 200^o. if it is bent to form a cone, find
(i) the radius of the base circle of the cone (ii) the height of the cone
(iii) the vertical angle (iv) the volume of the cone
(v) the curved surface area of the cone (take π = 3.142)

Solution



(i) $\frac{r}{R}=\frac{\theta}{360} \quad \Rightarrow r=\frac{\theta}{360}\times R=\frac{200}{360}\times 7=3.89cm$

(ii) From figure above, $L^2 = h^2 + r^2$ implies, h = 4cm

(iii) $Sin\alpha=\frac{r}{R}=\frac{3.89}{7}=0.5557 \quad \alpha=33.7599^o$

(iv) Volume, $V=\frac{1}{3}\pi r^2 h=190.180cm^3$

(v) $Curved\ Surface\ area=\pi rl=3.142\times 3.89\times 7=85.5567cm^2$

3.12.5 **Cuboids**

A cuboid is a prism having a rectangular cross-section.

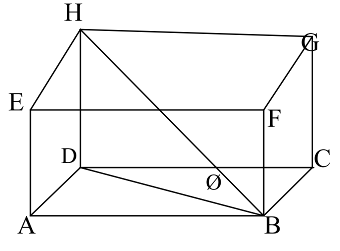
Total Surface Area Of A Cuboid = Sum of area of all the six faces = 2(ab + bc + ab)

Volume = length x breadth x height = base area x height = abc

NB

A cuboid has six faces, eight vertices and 12 edges

Illustration



NB

1. the principal diagonals on the cuboid above are: AG, FD, EC and HB

In finding one of the principal diagonals, say HB, we first apply Pythagoras theorem to triangle ADB to find side DB.

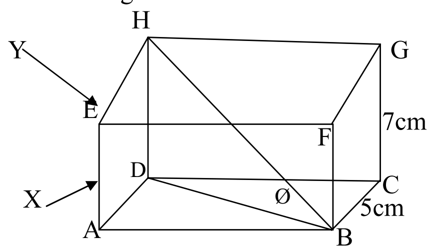
$Thus, |DB|^2 = |AB|^2 + |AD|^2$

$Now, \ from \ \Delta HDB, |HB|^2 = |HD|^2 + |DB|^2$

2. if the angle the diagonal makes with the base is Ø, then from triangle HDB, we $have \ Tan\theta = \frac{|HD|}{|DB|}$

Example 3.45

Consider the figure below

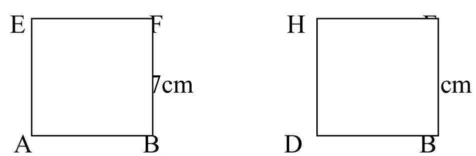


(a) draw the views as seen from X and Y

(b) find (i) the total surface area (ii) the volume (iii) the diagonal HB (iv) the angle the diagonal makes with the base

Solution

(a)



View as seen from X View as seen from Y

(b) (i)total surface area = 2[(4 x 7)+(4 x 5)+(5 x 7)]
= 2[(28)+(20)+(35)]
= 2[83] = 166cm
(ii) Volume = length x breadth x height = 4 x 5 x 7 = 140cm
(ii) From triangle ADB, $|DB|^2 = |AB|^2 + |AD|^2 \Rightarrow |DB| = 5cm$

$|HB|^2 = |HD|^2 + |DB|^2 \Rightarrow |HB| = 7.1cm$

(iv) $Tan\theta = \frac{|HD|}{|DB|} = \frac{5}{5} = 1 \Rightarrow \theta = 45^o$

3.12.6 **Pyramids**

The shape of the base of the pyramid determines the name of the pyramid. For instance, a pyramid with a rectangular base is called a ***rectangular pyramid*** and that with a square base is called a ***square pyramid***.
Total Surface Area Of A Pyramid = area of the net
= area of base + area of triangular faces
Volume = $\frac{1}{3} \times area\ of\ base \times height$

Example 3.46

A right pyramid has a square base of side 10cm. if its volume is 700cm^3, find its height.

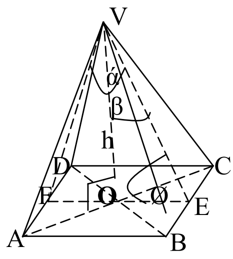
Solution

Volume of pyramid = 700cm^3, side of the square base = 10cm
Area of square base = 10 x 10 = 100cm^2
Volume of pyramid = $\frac{1}{3} \times area\ of\ base \times height = \frac{1}{3} \times 100h$

$\Rightarrow 100 = \frac{1}{3} \times 100h \Rightarrow h = 21cm$

Rectangular Or Square Pyramids

Illustration



NB

1. From the pyramid above, VO is the height, h

To find the height, h, we remove triangle VOA and apply Pythagoras theorem to it. Thus, $|VA|^2 = |VO|^2 + |AO|^2$

Similarly, to find the diagonal, AC of the base, we apply Pythagoras theorem to triangle ABC. Thus, $|AC|^2 = |AB|^2 + |BC|^2$ (since base is rectangular or square, each vertex angle is 90^o)

2. From the pyramid above, the angle Ø is the angle the face VBC makes with the base ABCD. In finding Ø, we remove triangle VOE and use $\tan\theta = \frac{Opp}{Adj} = \frac{|VO|}{|OE|}$ *where* $|OE| = \frac{1}{2}|AB|$

3. From the pyramid above, ά is the angle between the faces VBC and VAD.

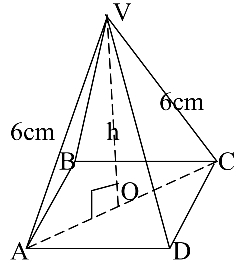
i.e. <FVE.

We divide triangle FVE into two equal part with angle OVE being β. We then find angle β first using and multiply the result by 2 to get ά. i.e. tan β = OE/OV

Example 3.47

A right pyramid has a square base of side 5cm. each sloping edge is 6cm long. Find correct to 2 decimal places, the height of the pyramid.

Solution



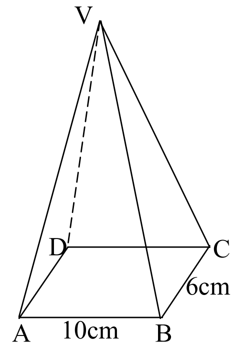
Height of pyramid = h cm

$First,\ from\ \Delta ACD, |AC|^2 = |AD|^2 + |DC|^2 = 5^2 + 5^2 = 50 \Rightarrow |AC| = 7.07cm$

$But\ |AO| = \frac{1}{2}|AC| = 3.54cm$

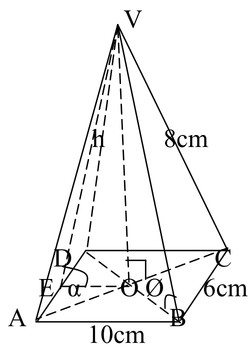
$From\ \Delta AOV, |AV|^2 = |AO|^2 + |OV|^2 \Rightarrow |OV| = h = \sqrt{6^2 - 3.54^2} = 4.85cm$

Example 3.47



The diagram shows a right pyramid VABCD. $|AD| = 6cm, |AB| = 10cm$
And each of the slant sides is 8cm long. Calculate, correct to two decimal places:
(i) the height of the pyramid (ii) the angle that VB makes with ABCD (iii) the height of triangle VAD makes with ABCD.

Solution



(i) From triangle ABC,
$|AC|^2 = |AB|^2 + |BC|^2 = 10^2 + 6^2 = 136 \Rightarrow |AC| = 11.66cm$
Therefore, OC = $\frac{1}{2}|AC| = \frac{1}{2} \times 11.66 = 5.83cm$
Considering triangle VOC, $|VC|^2 = |VO|^2 + |OC|^2 \Rightarrow |VO| = 5.478cm$
Hence, the height of the pyramid is 5.48cm
(ii) Ø = angle VB makes with ABCD, from triangle VOB,
$$Sin\theta = \frac{|VO|}{|VB|} = \frac{5.48}{8} = 0.685 \Rightarrow \theta = 43.24^o$$
(iii) remove triangle VAB and divide it into two forming a right angle at M on line AD.
Then considering triangle VAM,
$|AV|^2 = |AM|^2 + |MV|^2 \Rightarrow |MV| = 7.42cm\ (Since\ |AM| = \frac{1}{2}|AD| = 3cm)$
Hence, height of triangle VAD is 7.42cm

(iv) let ά = angle VAD makes with ABCD

Implies, from triangle VOE, $Tan\alpha = \frac{|VO|}{|EO|} = \frac{5.48}{5} = 1.096 \Rightarrow \alpha = 47.62^o$

Example 3.48

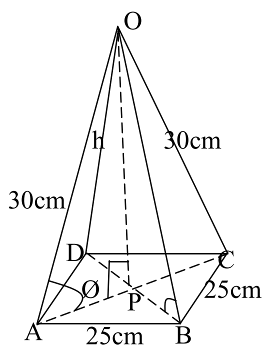
OABCD is a right square pyramid with vertex O such that $|AB| = |OB| = |OC| = |OD| = 30cm \ and |AB| = 25cm.$

Calculate (a) the height of the pyramid (b) the volume of the pyramid

(c) the angle between OA and AC (d) the total surface area of the pyramid

(excluding the base)

Solution



(a) From triangle ABC,

$|AC|^2 = |AB|^2 + |BC|^2 = 25^2 + 25^2 = 1250 \Rightarrow |AC| = 35.355cm$

$|AP| = \frac{1}{2}|AC| = 17.6775cm$

But $h^2 = |OA|^2 - |AP|^2 \Rightarrow h = 24.24cm$

(b) Volume of pyramid =

$\frac{1}{3} \times area \ of \ base \times height = \frac{1}{3} \times 25 \times 25 \times 24.24 = 5050cm^3$

(c) from triangle OAP, $Cos\theta = \frac{|AP|}{|OA|} = \frac{17.68}{30} = 0.5893 \Rightarrow \theta = 54^o$

(d) total surface area(excluding base) = area of four triangular faces
= 4(Area of one face)

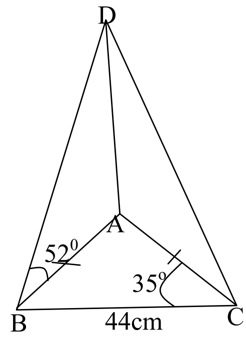
Considering triangle AOQ. i.e. half of triangle AOB, we find the height, h using: $h^2 = |AO|^2 - |AQ|^2 = 30^2 - 12.5^2 = 743.75 \Rightarrow h = 27.27cm$

Hence, area of face AOB = $\frac{1}{2}bh = \frac{1}{2}\times 25\times 27.27 = 340.875cm^2$

Total surface area = 4 x 340.875 = 1363.5cm^2

Example 3.49

In the diagram ABC is a right-angled triangle on a horizontal ground. AD is a vertical tower. BAC = 90o, ACB = 35o, ABD = 52^o and side BC = 44cm. Find the height of the tower.

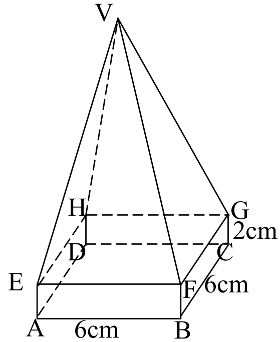


from triangle ABC, $Sin35 = \frac{|AB|}{44} \Rightarrow |AB| = 25.24m$

Again, from triangle ABD, $Tan52 = \frac{|AD|}{|AB|} \Rightarrow |AD| = 32.31m$

Example 3.50

The model below shows a pyramid EFGHV on a cuboid ABCDEFGH. $|AB| = 6cm, |BC| = 6cm$



The volume of the model is 132cm^3. Find correct to the nearest whole number (a) the height of the pyramid (b) the length of the slant edge, VG (c) the slant height of the pyramid (d) the angle between the face VFG and the base EFGH.

Solution

(a) $Area\ of\ Square\ base = 6 \times 6 = 36cm^2$

$Volume\ of\ pyramid = \frac{1}{3} \times 36h = 12h\,cm^3$

Volume of cuboid = L x B x H = 6 x 6 x 2 = 72cm^3

But volume of model = volume of pyramid + volume of cuboid

Implies, volume of pyramid = volume of model – volume of cuboid

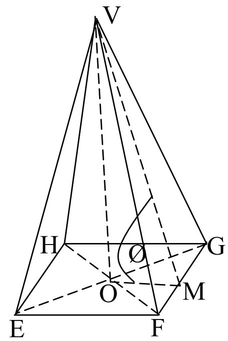
= 132 – 72 = 60cm^3

$Volume\ of\ pyramid = \frac{1}{3}Ah = 12hcm^3$

$But\ volume\ of\ pyramid = 60cm^3$

Implies, 60 = 12h, Hence, height of pyramid, h = 5cm

(b)



$From\ \Delta EFG, |EG|^2 = |EF|^2 + |FG|^2 = 72 \Rightarrow |EG| = 8.485cm$

Implies, OG = 4.242cm

$From\ \Delta VOG, |VG|^2 = |VO|^2 + |OG|^2 = 43 \Rightarrow |VG| = 6.56cm$

Hence, the length of the sloping edge, VG of the pyramid is 7cm to nearest whole number

(c) $From\ \Delta VOM, |VM|^2 = |VO|^2 + |OM|^2 = 34 \Rightarrow |VM| = 5.8cm$

Hence, slant height is 6cm to nearest whole number

(d) $Again,\ from\ VOM, \tan\theta = \frac{|VO|}{|OM|} = \frac{5}{3} = 1.667 \Rightarrow \theta = 59.04^o$

EXERCISE

QUE. A

A flower bed is in the form of a rectangle with semi-circular ends. The straight sides are 25m long and the flower bed is 14m wide. (Take $\pi = \frac{22}{7}$)

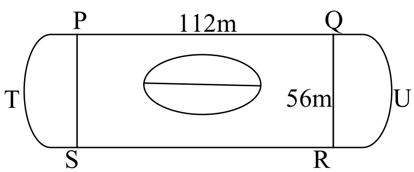
(i) find the area of the flower bed

(ii) if the cost of black soil is ¢955.75 per square meter, find the cost of covering the flower bed with black soil.

QUE. B

The diagram below represents a field with a circular pond of diameter 14m. PTS and QUR are semi-circles. PQRS is a rectangle with PQ = 112m and QR = 56m. find (a) the distance round the field (b) the area of the field excluding the pond.

$(Take\ \pi = \frac{22}{7})$



QUE. C

A right circular cone has a base radius 5cm and height 12cm.
calculate (i) its volume
(ii) its total surface area

QUE. D

A right pyramid has square base of side 10cm. if its volume is $500cm^3$, find its height.

QUE. E

The dimensions of a water tank in the form of a cuboid are 60cm by 20cm by 15cm.
Find the capacity of the tank in litres.

QUE. F

Two squares P and Q have sides $2\frac{1}{4}$ cm and $2\frac{1}{4}$ respectively. Express the area of P as a fraction of the area of Q.

QUE. G

The diameter of a five hundred cedis coin is 2.3cm and its thickness is 0.3cm. find correct to 3 significant figures, the volume of metal used. (take $\pi = 3.142$)

QUE. H

A circle has a radius 7.5cm. A sector with an angle of 240^{o} is cut out from the circle.
(a) Find the length of the arc of the sector (b) If the sector is folded to form a cone, find correct to one decimal place (i) the height of the cone (ii) the volume of the cone

QUE. I

ABCV is a right pyramid with base ABC which is an equilateral triangle of side 18cm. each slopping edge of the pyramid is 13cm. G is the centre of symmetry of triangle ABC. If AG = 10.4cm, calculate (a) the angle between AV and the base ABC
(b) correct to one decimal place the height VG of the pyramid
(c) correct to the nearest whole number, the volume of the pyramid
(d) the angle between the face BCV and the base ABC

QUE. J

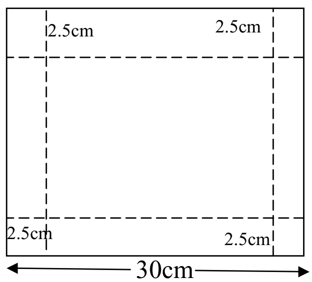
A hollow right circular cone stands with its base on a horizontal table. It is 100cm high with a base of 20cm. it is filled with water through the vertex up to a depth of 25cm. calculate (i) the radius (ii) correct to the nearest whole number, the volume of the water inside the cone

QUE. K

A pyramid ABCD, whose base BCD is an equilateral triangle of side 8cm, has its stand edge AB, AC and AD of lengths 10cm. the foot of the pyramid from A to the base BCD is M. Calculate (a) $(a)\,|BM|$ $(b)\,|AM|$ (c) the angle between the face ABC and the base (d) the volume of the pyramid

QUE. L

A piece of cardboard, 30cm squared has small squares of sides 2.5cm cut from each corner as shown in the diagram.



The sides are then folded along the dotted lines to form an open box of height 2.5cm. calculate (i) the total surface area of the open box (ii) the volume of the box formed.

QUE. M

A swimming pool with length 30m and width 12cm is 4m deep at one end and 1m deep at the other end. Find the volume of water that will fill it completely.

QUE. N

The volume of a cone of height 10.5cm is 396cm^3. Find the radius of the cone.

QUE. O

The base of a prism, whose height is 12cm is a right angled triangle with dimensions 17cm, 15cm and 8cm. calculate the total surface area of the prism.

QUE. P

Find the circumference of the circle with the following diameters.
$1.\ 28cm \quad 2.\ 35m \quad 3.\ 7m \quad 4.\ 21cm \quad 5.\ 63mm \quad (Take\ \pi = \frac{22}{7})$

QUE. Q

Find the circumference of the circles with the following radii.
$1.\ 3.5m \quad 2.\ 5cm \quad 3.\ 14cm \quad 4.\ 6m \quad 5.\ 42mm \quad (Take\ \pi = \frac{22}{7})$

QUE. R

Find the diameters of the circles with the following circumferences.
$1.\ 220mm \quad 2.\ 44m \quad 3.\ 66cm \quad 4.\ 33m \quad 5.\ 121cm \quad (Take\ \pi = \frac{22}{7})$

QUE. S

A 400metre running track has two parallel straight portions. Each end is in the form of a semicircle. If each section is 112m long, find the diameter of the semicircles.
$(Take\ \pi = \frac{22}{7})$

QUE. T

The radius of a circle is 10cm. the angle subtended by an arc of the circle at the centre is 63^o. find

(a) The length of the arc
(b) The length of the chord of the sector
(c) The perimeter of the minor segment

QUE. U

The angle of a sector of a circle is 70^{o}. the radius of the circle is 6cm. find

(a) The length of the arc of the sector
(b) The area of the sector

QUE. V

Find the total surface area of the cone with radius rcm and height hcm for each of the following pairs of values of r and h. $(Take\ \pi = \frac{22}{7})$

$(a)\ r = 6cm,\ h = 8cm \quad (b)\ r = 7cm,\ h = 24cm \quad (c)\ r = 4cm,\ h = 3cm$

$(d)\ r = 10cm,\ h = 24cm$

QUE. W

Find the total surface area of the cylindrical solid of radius rcm and height hcm for each of the following pairs of values of r and h. $(Take\ \pi = \frac{22}{7})$

$(a)\ r = 5cm,\ h = 7cm \quad (b)\ r = 10cm,\ h = 20cm \quad (c)\ r = 15cm,\ h = 25cm$

$(d)\ r = 2m,\ h = 2m$

QUE. X

The radius of a cone is 7cm and the height is 24cm. find

(a) The slant height
(b) The curved surface area
(c) The total surface area
(d) The volume

QUE. Y

The radius of a cylinder is 2.1cm and the height is 4.9cm. find

(a) The curved surface area
(b) The volume

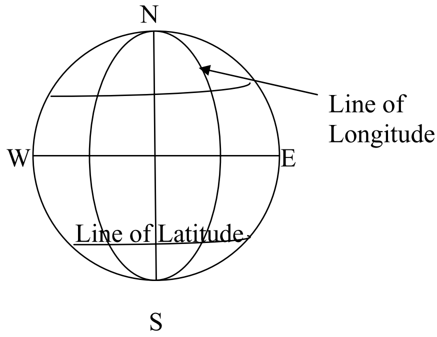
QUE. Z

A pyramid has a square base of side 6m. if the height is also 6m, find the volume of the pyramid.

CHAPTER 4
GLOBAL MATHEMATICS
(THE EARTH AS A GLOBE)

The two types of ***imaginary lines*** used in finding distances and positions on the earth surface are the ***lines of latitude*** and the ***lines of longitude***.
Lines of latitude are usually drawn from the ***west*** to the ***east*** whiles the lines of longitude are drawn from the ***north*** to the ***south***.

Illustration



Recall

Length Of Arc = $\frac{\theta}{360} \times 2\pi R$

4.1 **Distances On Great Circles**

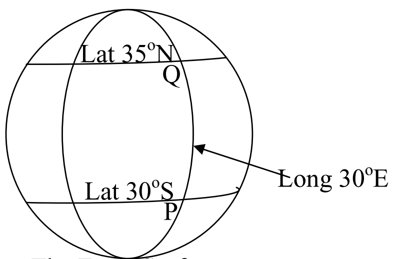
Great Circles are lines that divide the earth into two equal parts. Examples of great circles are the ***equator*** and all ***lines of longitude***. The radius of the earth is taken to be the radius of every great circle, R = 6400km.

Example 4.1

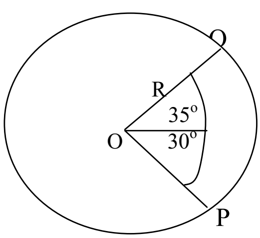
Find the shortest distance between the two stations P(Lat 30°S, Long 30°E) and Q(lat 35°N, Long 30°E) along the line of longitude.

Solution

P(Lat 30°S, Long 30°E) and Q(lat 35°N, Long 30°E)



The Earth Surface



Long 30°E

From question, the common line is the longitude. Hence, it is a great circle with R = 6400km and Ø = 30 + 35 = 65° (Since uncommon line is in different directions, i.e. N and S).

Hence, shortest distance = length of arc

$$= \frac{\theta}{360} \times 2\pi R = \frac{65}{360} \times 2 \times 3.142 \times 6400 = 7{,}261.5km$$

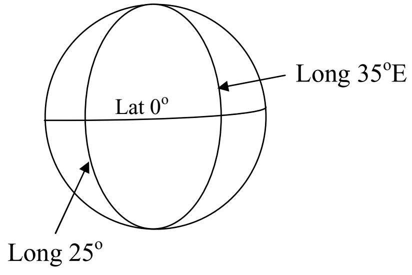
NB

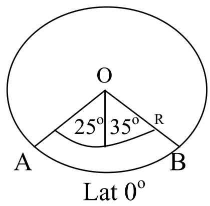
1. The imaginary line ***common to both*** points is used to determine whether it will be a great or small circle. Thus, when the common line is on longitude, it will have a great circle with radius, R = 6400km and when the common line is on latitude, it will have a small circle with $r = R\cos\alpha$. Where α is the common latitude of the points.
2. The other line with no common angles is used to determine the angle Ø in the formulae for length of arc above. Thus, if this other line in the two points are in the same direction (say N and N, S and S, E and E or W and W), we find their difference for the value of Ø and if they are in different directions (say N and S or E and W), we find their sum to obtain the value of Ø.
3. Lastly, we use the formulae for length of arc to calculate the shortest distance between two given points.

Example 4.2

Find the shortest distance between the two points A(lat 0^oN, long 25^oW) and B(lat 0^oN, long 35^oE) along the line of latitude and through the earth.

Solution



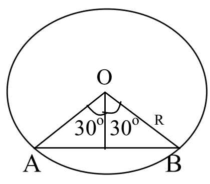


Shortest distance along line of lat

$= \frac{\theta}{360} \times 2\pi R = \frac{60}{360} \times 2 \times 3.142 \times 6400 = 6{,}702.9km$

NB

Shortest distance through the earth = length of chord joining A and B



Considering half of triangle AOB,

Sin 30° =

$\frac{\frac{1}{2}AB}{R} \Rightarrow \frac{1}{2}AB = R\sin 30^o = 3200km \;\; Hence,\; AB = 2 \times 3200 = 6400km$

Therefore, the shortest distance through the earth is 6400km.

4.2 **Distances On Small Circles**

All the lines of latitude except the equator are referred to as ***Small Circles***. These lines do not divide the earth into two equal parts and has a radius, r given by

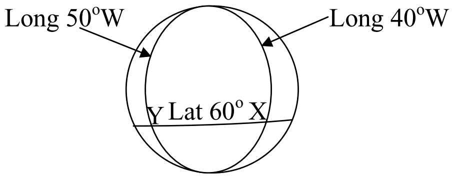
r = Rcosά where ά is the common latitude of the two points.

Example 4.3

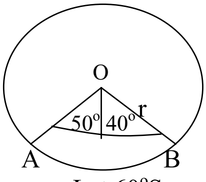
Find the shortest distance along the line of latitude between the two towns
X(lat 60°S, long 40°E) and Y(lat 60°S, long 50°W).
(take R = 6400km and $\pi = 3.142$)

Solution

X(lat 60°S, long 40°E) and Y(lat 60°S, long 50°W)



The earth surface



Lat 60°S

Here, we have a small circle with r = Rcosά where ά = 60°
and Ø = 50 + 40 = 90°

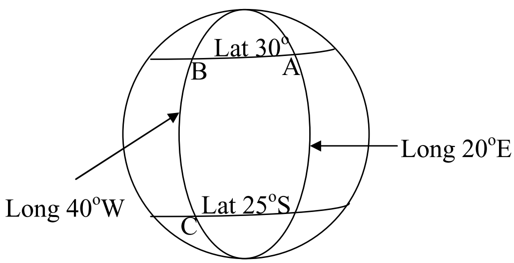
$$\Rightarrow \frac{\theta}{360} \times 2\pi r = \frac{90}{360} \times 2 \times 3.142 \times (6400 \cos 60^o) = 5027.2km$$

Example 4.4

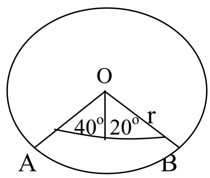
Three towns A(lat 30°N, long 20°E), B(lat 30°N, long 40°W) and C(lat 25°S, long 40°W) are situated on the earth surface. Find (i) the distance between A and B along the line of lat (ii) the distance between B and C along the long (iii) the time it will take a cyclist to travel the distance AB and BC if he moves at an average speed of 930km/h.

Solution

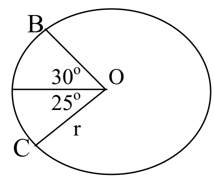
A(lat 30°N, long 20°E), B(lat 30°N, long 40°W) and C(lat 25°S, long 40°W)



The Earth Surface



Lat 30°N



Long 40°W

(i) Distance between A and B

$$= \frac{\theta}{360} \times 2\pi r = \frac{60}{360} \times 2 \times 3.142 \times (6400 \cos 30^{o}) = 5{,}804.9 km$$

(Since it's a small circle)

(ii) Distance between B and C along the longitude

$$\frac{\theta}{360} \times 2\pi R = \frac{55}{360} \times 2 \times 3.142 \times 6400 = 6{,}144.36 km$$

(Since it's a great circle)

(iii)

$$Time = \frac{total\ distance}{average\ speed} = \frac{|AB| + |BC|}{930} = \frac{5804.9 + 6144.36}{930} = \frac{11{,}949.26}{930} = 12.8487$$

EXERCISE

QUE. A

Find the length of the chord that joins A(60^{o}N, 18^{o}W) and B(30^{o}S, 18^{o}W).

QUE. B

Find the distance between the following points:

(a) A(45^{o}N, 60^{o}W) and B(45^{o}N, 35^{o}E)
(b) P(70^{o}N, 30^{o}W) and Q(40^{o}N, 30^{o}W)

QUE. C

The distance between two points P(70^{o}S, 15^{o}W) and Q(70^{o}S, 48^{o}E) on the surface of the earth along their line of latitude is 2090km. Find the radius of this latitude.

QUE. D

An aircraft from P(65^{o}N, 40^{o}W) to Q(65^{o}N, 30^{o}W) at the speed of 500km/hr. calculate the time it takes to complete the journey.

QUE. E

Find the radius of the circle of latitude 270^o. Hence find the distance between the points I(270^{o}S, 14^{o}E) and J(270^{o}S, 72^{o}E).

QUE. F

Find the distance between A(25°S, 21°W) and B(25°S, 72°W) along its circle of latitude and along the great circle passing through A and B.

QUE. G

An aircraft flies from town A(60°N, 10°E) to town B(60°N, 170°W) along the line of latitude. It then flies back to town A along the line of longitude through the north pole. Calculate correct to the ***nearest*** km, the difference in the distance of the two routes. (take radius of the earth as 6400km and $\pi = 3.142$)

QUE. H

Find the angle between the latitudes of the following places:

1. A(50°N, 30°E), B(20°N, 30°E)
2. C(60°N, 25°W), D(15°S, 25°W)
3. E(20°S, 15°W), F(75°S, 15°W)
4. G(10°N, 20°E), H(35°N, 20°E)
5. I(45°N, 35°E), J(85°N, 35°E)
6. K(80°N, 40°W), L(35°N, 40°W)
7. M(15°N, 31.5°W), N(15°N, 1.7°W)
8. O(25°S, 25°W), P(25°S, 57.9°E)
9. Q(13°N, 10.7°E), R(13°N, 21.3°E)
10. S(14°S, 85°W), T(14°S, 14°W)

ANSWERS TO EXERCISES

CHAPTER 1

A. GH¢3150 (ii) GH¢770 (iii) GH¢64 (iv) GH¢644.17
B. GH¢262.50 (b) GH¢3,400
C. GH¢297 (b) 9.9% (c) GH¢2,460
D. a. ¢401850.00 b. ¢139017.50 c. ¢160167.50
d. ¢262832.50
E. a. ¢12500.00 b. ¢237500.00 (of which ¢10000.00 is tax free, giving ¢227500.00)
F. a. ¢20000.00 b. ¢28750.00 c. ¢52750.00
G. a. (i) ¢141000.00 (ii) ¢117000.00 (iii) ¢141000.00
b. (i) ¢27350.00 (ii) ¢18950.00 (iii) 27350.00

CHAPTER 2

A. (i) 100^o (ii) 80^o (iii) 75^o
B. 29^o
C. (i) 60^o (ii) 30^o (iii) 90^o
D. (i) 115^o (ii) 40^o
E. 35^o
F. (b) (i) 25^o (ii) 65^o
G. (a) (i) 72^o (ii) 360^o (iii) 36^o
H. (i) 97^o (ii) 66^o
I. (i) 72^o (ii) 52^o
J. 100^o
K. 1. $a = 90^o, b = 40^o, c = 25^o$ 2. $a = 60^o, b = 40^o, c = 60^o$

CHAPTER 3

A. (i) 504m^2 (ii) ¢884898
B. (a) 400m (b) 8582m^2
C. (i) 314.2cm^3 (ii) 204.23cm^2
D. 15cm
E. 18litres
F. $\frac{9}{100}$
G. 1.25cm^3
H. (a) 31.43cm (b) (i) 5.6cm (ii) 146.7cm^3
I. (a) 36.9^o (b) 7.8 (c) 365cm^3 (d) 56^o
J. (i) 15cm (ii) 24226cm^3
K. (a) 4.62cm (b) 8.9cm (c) 75.4^o (d) 81.9cm^3
L. (i) 875cm^2 (ii) 1,562.5cm^3
M. 900m^3
N. 6cm
O. 600cm^2
P. 1. 88cm 2. 110m 3. 22m 4. 66cm 5. 198mm
Q. 1. 22m 2. 31.43cm 3. 88cm 4. 37.71m 5. 264mm
R. 1. 70mm 2. 14m 3. 21cm 4. 10.5m 5. 38.5cm
S. 56m
T. a. 11cm b. 10.45cm c. 31cm d. 21.45cm
U. a. 7.33cm b. 22cm^2
V. a. 301.7cm^2 b. 704cm^2 c. 113.1cm^2 d. 1131cm^2
W. a. 377.1cm^2 b. 1886cm^2 c. 3771cm^2 d. 50.28m^2
X. a. 25cm b. 550cm^2
Y. a. 64.68cm^2 b. 67.91cm^3
Z. 72m^2

CHAPTER 4

A. 2019km
B. (i) 8^{o}W (ii) 560km **C.** 3352km

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