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Numerical Evaluation of Nonlinear Hamiltonian Symmetric Matrix of Rank 1 of an Inverse Eigenvalue Problem Via Newton-Raphson Method

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Abstract**-- In this paper we consider numerical evaluation of nonlinear Hamiltonian symmetric matrix of Rank 1 in an inverse eigenvalue problem via Newton-Raphson method. The approach employed Newton-Raphson's method for solving the inverse eigenvalue problem in a class of Hamiltonian matrices in the neighborhood of a related nonsingular matrix of rank 1. A few numerical examples are presented to illustrate the result.**

Keywords**-- Nonlinear, eigenvalues, Hamiltonian, symmetric, nonsingular matrices**

I. Introduction

Recently solvability of the IEP for a class of singular Hermitian matrices has been obtained by Oduro et al. (2012). (2014) and an inverse eigenvalue problem for linear- quadratic optimal control by Oladejo et al (2014) together with derivation of an explicit functions via Linear-quadratics inverse eigenvalue problem by Oladejo and Anang (2016) .Based on these results we assess the numerical evaluation of nonlinear Hamiltonian symmetric matrix of of Rank 1 in an inverse eigenvalue problem via Newton-Raphson Method

Linear System and the Riccati Equation

We let $A \in \Re^{n\times n}, B \in \Re^{n\times m}, Q \in \Re^{n\times n} : Q = Q^T \geq 0$ and $R \in \Re^{m\times m} : R = R^T > 0$ Where Q is a symmetric positive semi definite matrix and R is a symmetric positive definite matrix

We consider the linear quadratic optimal control for the functional:

$$Ix_i(u) = \int_{t_0}^{t_1} \frac{1}{2}\left[x(t)^T Qx(t) + u(t)^T Ru(t)\right]dt \quad (1)$$

Subject to differential equation

$$\dot{X}(t) = Ax(t) + Bu(t), t \in [t_0, t_1], x(t_i) = x_i \quad (2)$$

Constructing the Hamiltonian equation from equation (1) and (2) yields:

$$H(p,x,u,t) = \frac{1}{2}\left[x^T Qx + u^T Ru\right] + p^T\left[Ax + Bu\right] \quad (3)$$

Given any optimal input $u_\bullet$ and the corresponding state $x_\bullet$

Then;

$$\frac{\partial H}{\partial u}(p_\bullet(t), x_\bullet(t), u_\bullet(t), t) = 0$$

$$\Rightarrow u_\bullet(t)^T R + p_\bullet(t)^T B = 0 \quad (4)$$

Thus, $u_\bullet(t) = -R^{-1}B^T p_\bullet(t)$ (5)

and the adjoint equation is given as:

$$\left[\frac{\partial H}{\partial x}(p_\bullet(t), k_\bullet(t), u_\bullet(t), t)\right]^T$$

$$\left(x_\bullet(t)^T Q + p_\bullet(t)^T A\right)^T = -\dot{p}_\bullet(t), t \in [t_0, t_1], p_\bullet(t_1) = 0 \quad (6)$$

Which yields;

$$\dot{p}_\bullet(t) = A^T p_\bullet(t) - Qx_\bullet(t), t \in [t_0, t_1], p_\bullet(t_1) = 0 \quad (7)$$

Consequently;

$$\frac{d}{dt}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix}, \quad (8)$$

$$t \in [t_0, t_1], x_\bullet(t_0) = x_i, p_\bullet(t_1) = 0$$

Equation (8) is then a linear, time variant differential equation in $(x_\bullet, p_\bullet)$

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From the Hamilton's equations (8) above which indicates inverse eigenvalue problem in Hamiltonian matrix of Rank 1 in respect of linear - quadratic optimal control problem.i.e

$$\frac{d}{dt}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T} \end{bmatrix}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t) \end{bmatrix},$$

$$t \in [t_0, t_1], x_{\bullet}(t_0) = x_i, p_{\bullet}(t_1) = 0$$

We consider the case where the Hamiltonian matrix

$$H = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T} \end{bmatrix}$$ Is a $2n \times 2x$ matrices so that A, Q, $BR^{-1}B^{T}$ are all is $2n \times 2n$ sub-matrices of H.

By appropriate row dependence relations, a 4×4 singular Hamilton matrix representing H above can be constructed as follows:

$$R_{i+1} = k_i R_1$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = a_{11}\begin{bmatrix} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & k_1\bar{k}_2 & k_1\bar{k}_3 \\ k_2 & k_2\bar{k}_1 & |k_2|^2 & k_2\bar{k}_3 \\ k_3 & k_3\bar{k}_1 & k_3\bar{k}_2 & |k_3|^2 \end{bmatrix}$$

Using the given nonzero eigenvalue, we solve the inverse eigenvalue problem (IEP) for the singular matrix of rank:

Thus:

$$tr(A) = \lambda = a_{11}(1 + |k_1|^2 + |k_2|^2 + |k_3|^2)$$

So that

$$H = \frac{\lambda}{1 + tr(A)}\begin{bmatrix} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & k_1\bar{k}_2 & k_1\bar{k}_3 \\ k_2 & k_2\bar{k}_1 & |k_2|^2 & k_2\bar{k}_3 \\ k_3 & k_3\bar{k}_1 & k_3\bar{k}_2 & |k_3|^2 \end{bmatrix}$$

H is a Hamiltonian matrix of the Linear- quadratic optimal control problem; we may partition it as follows:

$$\left[\begin{array}{cc|cc} 1 & \bar{k}_1 & \bar{k}_2 & \bar{k}_3 \\ k_1 & |k_1|^2 & k_1\bar{k}_2 & k_1\bar{k}_3 \\ \hline k_2 & k_2\bar{k}_1 & |k_2|^2 & k_2\bar{k}_3 \\ k_3 & k_3\bar{k}_1 & k_3\bar{k}_2 & |k_3|^2 \end{array}\right] \Rightarrow \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T} \end{bmatrix}$$

Where

R and Q are Hamilton symmetric matrices and $A = -A^{T}$

Thus:

$$\bar{k}_1 = k_3\bar{k}_2 \Rightarrow k_1 = -k_3\bar{k}_2; \quad k_1 = -(\bar{k}_2 k_1)k_2$$

$$\bar{k}_2 = \sqrt{-1} \Rightarrow k_2 = -i.; \qquad k_3 = ik_1$$

Substituting k_1, k_2, k_3 into the H above give

$$H = a_{11}\begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix}$$

And $tr(A) = \lambda = 2a_{11}(1 + |k_1|^2)$

Thus the solution of the IEP is given by

$$H = \frac{\lambda}{2(tr(A))}\begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix}$$

Given that $a_{11} = 1, k_1 = 2 \Rightarrow \lambda = 10$

Hence;

$$H = \begin{bmatrix} 1 & \bar{k}_1 & i & -i\bar{k}_1 \\ k_1 & |k_1|^2 & i\bar{k}_1 & -i|k|^2 \\ i & -i\bar{k}_1 & 1 & -\bar{k}_1 \\ ik_1 & i|k_1|^2 & -k_1 & |k_1|^2 \end{bmatrix} \Rightarrow H = \begin{bmatrix} 1 & -2 & i & 2i \\ 2 & 4 & 2i & -4i \\ i & -2i & 1 & 2 \\ 2i & 4i & -2 & 4 \end{bmatrix}$$

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Since there are repeating diagonal elements we solve the IEP by Newton's method for two distinct target eigenvalues λ_1, λ_2 which therefore give rise to two (2) functions with independent variables being the diagonal elements of matrix A which is a sub-matrix of H:

$$f_1(a_{11}, a_{22}) = \lambda_1^{\ 2} - 2(trA)\lambda_1 + \det H$$

$$f_2(a_{11}, a_{22}) = \lambda_2^{\ 2} - 2(trA)\lambda_2 + \det H$$

Thus;

$$f_1(a_{11}, a_{22}) = \lambda_1^{\ 2} - 2(a_{11} + a_{22})\lambda_1 + \det H$$

$$f_2(a_{11}, a_{22}) = \lambda_2^{\ 2} - 2(a_{11} + a_{22})\lambda_2 + \det H$$

Then the Jacobian of the above functions is given as;

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial a_{11}} & \frac{\partial f_1}{\partial a_{22}} \\ \frac{\partial f_2}{\partial a_{11}} & \frac{\partial f_2}{\partial a_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda_1 + 2a_{22} & -\lambda_1 + 2a_{11} \\ -\lambda_2 + 2a_{22} & -\lambda_2 + 2a_{11} \end{bmatrix}$$

While the general Newton's method is given by the following iteration;

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(0)})\underline{f}(X^{(n)})$$

$$: X^{(0)} = \begin{bmatrix} a_{11}^{\ (0)} \\ a_{22}^{\ (0)} \end{bmatrix}$$

While the Determinant $Det = 2(\lambda_1 - \lambda_2)(a_{22} - a_{11})$
The inverse of Jacobian matrix is gives;

$$J^{-1} = \frac{1}{2(\lambda_1 - \lambda_2)(a_{22} - a_{11})} \begin{bmatrix} 2a_{22} - \lambda_1 & 2a_{11} - \lambda_1 \\ 2a_{22} - \lambda_2 & 2a_{11} - \lambda_2 \end{bmatrix}$$

II. Numerical Illustration

We consider to solve the missile intercept problem as movement of an object O_1 in the xy plane described by the parameterized equations: $X_1(t) = t$; $Y_1(t) = 1 - e^{-t}$. and the second object O_2 moves according to the equations: $X_2(t) = 1 - \cos(\alpha)\, t$, $Y_2(t) = \sin(\alpha)\, t - 0.1t^2$.

Converting the parametric equations to a system of nonlinear equations, by setting x and y coordinates equal to each other. i.e

$$1 - \cos(\alpha)\, t = t \quad \sin(\alpha)\, t - 0.1t^2 = 1 - e^{-t}.$$

Rearranging the equations by representing the system in functional form as follows:

$$f_1(t, \alpha) = 1 - \cos(\alpha)\, t - t = 0$$

$$f_2(t, \alpha) = \sin(\alpha)\, t - 0.1t^2 - 1 + e^{-t} = 0.$$

Defining an initial guess for the required solution as: $\mathbf{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Better approximation of the solution of the system is then obtained by evaluating $\mathbf{x}_1$;

$$\mathbf{x}_1 = \mathbf{x}_0 - \mathbf{J}^{-1}(\mathbf{x}_0)\mathbf{f}(\mathbf{x}_0).$$

The calculated function value $\mathbf{f}(\mathbf{x})$ at $\mathbf{x}_0$ is as follows;

$$\mathbf{f}(\mathbf{x}_0) = \begin{bmatrix} 1 - \cos(\alpha)\, t - t \\ \sin(\alpha)\, t - 0.1t^2 - 1 + e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - \cos(1) - 1 \\ \sin(1) - 0.1(1)^2 - 1 + e^{-1} \end{bmatrix} = \begin{bmatrix} -0.540302305 \\ -0.095107509 \end{bmatrix}.$$

We proceed to calculate the Jacobian at $\mathbf{x}_0$;

$$J(x_0) = \begin{bmatrix} -\cos(\alpha) - 1 & \sin(\alpha)t \\ \sin(\alpha) - 0.2t - e^{-t} & \cos(\alpha)t \end{bmatrix} = \begin{bmatrix} -\cos(1) - 1 & \sin(1)(1) \\ \sin(\alpha) - 0.2(1) - e^{-1} & \cos(1)(1) \end{bmatrix}$$

$$= \begin{bmatrix} -1.540302306 & 0.841470984 \\ 0.273591543 & 0.540302305 \end{bmatrix}.$$

We then calculate the inverse of the Jacobian at $\mathbf{x}_0$;

$$\mathbf{J}^{-1}(\mathbf{x}_0) = \begin{bmatrix} -0.508544594 & 0.79201128 \\ 0.257510469 & 1.449766926 \end{bmatrix}.$$

Evaluating $\mathbf{x}_1$: $\mathbf{x}_1 = \mathbf{x}_0 - \mathbf{J}^{-1}(\mathbf{x}_0)\mathbf{f}(\mathbf{x}_0)$

$$\mathbf{x}_1 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} -0.508544594 & 0.79201128 \\ 0.257510469 & 1.449766926 \end{bmatrix} \begin{bmatrix} -0.540302305 \\ -0.095107509 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} - \begin{bmatrix} 0.199441596 \\ -0.27701722 \end{bmatrix} = \begin{bmatrix} 0.800558404 \\ 1.27701722 \end{bmatrix}.$$

Evaluating $\mathbf{x}_2$:

$$\mathbf{x}_2 = \mathbf{x}_1 - \mathbf{J}^{-1}(\mathbf{x}_1)\mathbf{f}(\mathbf{x}_1).$$

$$f(x_1) = \begin{bmatrix} 1 - (0.800558404)\cos(1.27701722) - 0.800558404 \\ (0.800558404)\sin(1.27701722) - 0.1(0.800558404)^2 - 1 + e^{-(0.800558404)} \end{bmatrix}$$

$$= \begin{bmatrix} -0.032377286 \\ 0.151248344 \end{bmatrix}.$$

$J(x_1) =$

$$\begin{bmatrix} -\cos(1.27701722) - 1 & (0.800558404)\sin(1.27701722) \\ \sin(1.27701722) - 0.2(0.800558404) - e^{-0.800558404} & (0.800558404)\cos(1.27701722) \end{bmatrix}$$

$$= \begin{bmatrix} -1.28957148 & 0.766259593 \\ 0.347966583 & 0.231818882 \end{bmatrix}.$$

$$\mathbf{J}^{-1}(\mathbf{x}_1) = \begin{bmatrix} -0.409878326 & 1.354821476 \\ 0.615238757 & 2.280087782 \end{bmatrix}.$$

$$\therefore x_2 = \begin{bmatrix} 0.800558404 \\ 1.27701722 \end{bmatrix} - \begin{bmatrix} -0.409878326 & 1.354821476 \\ 0.615238757 & 2.280087782 \end{bmatrix}\begin{bmatrix} -0.032377286 \\ 0.151248344 \end{bmatrix}$$

$$= \begin{bmatrix} 0.800558404 \\ 1.27701722 \end{bmatrix} - \begin{bmatrix} 0.218185252 \\ 0.32493974 \end{bmatrix} = \begin{bmatrix} 0.582373152 \\ 0.95207748 \end{bmatrix}$$

Evaluating $\mathbf{x}_3$: $\quad \mathbf{x}_3 = \mathbf{x}_2 - \mathbf{J}^{-1}(\mathbf{x}_2)\mathbf{f}(\mathbf{x}_2).$

$$f(x_2) = \begin{bmatrix} 1 - (0.582373152)\cos(0.95207748) - 0.582373152 \\ (0.582373152)\sin(0.95207748) - 0.1(0.582373152)^2 - 1 + e^{-(0.582373152)} \end{bmatrix}$$

$$= \begin{bmatrix} 0.079855089 \\ -9.305420505 \times 10^{-4} \end{bmatrix}.$$

$J(x_2) =$

$$\begin{bmatrix} -\cos(0.95207748) - 1 & (0.582373152)\sin(0.95207748) \\ \sin(0.95207748) - 0.2(0.582373152) - e^{-0.582373152} & (0.582373152)\cos(0.95207748) \end{bmatrix}$$

$$= \begin{bmatrix} -1.579991981 & 0.474414088 \\ 0.139576335 & 0.337771758 \end{bmatrix}.$$

$$\mathbf{J}^{-1}(\mathbf{x}_2) = \begin{bmatrix} -0.563052732 & 0.790830323 \\ 0.232668466 & 2.633786813 \end{bmatrix}.$$

$$\therefore x_3 = \begin{bmatrix} 0.582373152 \\ 0.95207748 \end{bmatrix} - \begin{bmatrix} -0.563052732 & 0.790830323 \\ 0.232668466 & 2.633786813 \end{bmatrix}\begin{bmatrix} 0.079855089 \\ -9.305420505 \times 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 0.582373152 \\ 0.95207748 \end{bmatrix} - \begin{bmatrix} -0.045698526 \\ 0.016128911 \end{bmatrix} = \begin{bmatrix} 0.628071678 \\ 0.935948569 \end{bmatrix}.$$

Evaluating $\mathbf{x}_4$; $\quad \mathbf{x}_4 = \mathbf{x}_3 - \mathbf{J}^{-1}(\mathbf{x}_3)\mathbf{f}(\mathbf{x}_3).$

$$f(x_3) = \begin{bmatrix} 1 - (0.628071678)\cos(0.935948569) - 0.628071678 \\ (0.628071678)\sin(0.935948569) - 0.1(0.628071678)^2 - 1 + e^{-(0.628071678)} \end{bmatrix}$$

$$= \begin{bmatrix} -5.526903117 \times 10^{-4} \\ -1.281584331 \times 10^{-4} \end{bmatrix}.$$

$J(x_3) =$

$$\begin{bmatrix} -\cos(0.935948569) - 1 & (0.628071678)\sin(0.93594856 \\ \sin(0.935948569) - 0.2(0.628071678) - e^{-0.628071678} & (0.628071678)\cos(0.93594856 \end{bmatrix},$$

$$= \begin{bmatrix} -1.593054942 & 0.505699444 \\ 0.145927857 & 0.372481012 \end{bmatrix}.$$

$$\mathbf{J}^{-1}(\mathbf{x}_3) = \begin{bmatrix} -0.558293011 & 0.757967403 \\ 0.218723908 & 2.387749744 \end{bmatrix}.$$

$$\therefore x_4 = \begin{bmatrix} 0.628071678 \\ 0.935948569 \end{bmatrix} - \begin{bmatrix} -0.558293011 & 0.757967403 \\ 0.218723908 & 2.387749744 \end{bmatrix}\begin{bmatrix} -5.526903117 \times 10^{-4} \\ -1.281584331 \times 10^{-4} \end{bmatrix}$$

$$= \begin{bmatrix} 0.628071678 \\ 0.935948569 \end{bmatrix} - \begin{bmatrix} 2.114232238 \times 10^{-4} \\ -4.26896896851 \times 10^{-4} \end{bmatrix} = \begin{bmatrix} 0.627860254 \\ 0.936375465 \end{bmatrix}.$$

Evaluating $\mathbf{x}_4$; $\quad \mathbf{x}_5 = \mathbf{x}_4 - \mathbf{J}^{-1}(\mathbf{x}_4)\mathbf{f}(\mathbf{x}_4).$

$$f(x_4) = \begin{bmatrix} 1 - (0.627860254)\cos(0.936375465) - 0.627860254 \\ (0.627860254)\sin(0.936375465) - 0.1(0.627860254)^2 - 1 + e^{-(0.627860254)} \end{bmatrix}$$

$$= \begin{bmatrix} -3.7941886 \times 10^{-8} \\ -9.2568569 \times 10^{-8} \end{bmatrix}.$$

$J(x_4) =$

$$\begin{bmatrix} -\cos(0.936375465) - 1 & (0.627860254)\sin(0.93637546 \\ \sin(0.936375465) - 0.2(0.627860254) - e^{-0.627860254} & (0.627860254)\cos(0.93637546 \end{bmatrix}$$

$$= \begin{bmatrix} -1.592711167 & 0.505688125 \\ 0.14611041 & 0.372139783 \end{bmatrix}.$$

$$\mathbf{J}^{-1}(\mathbf{x}_4) = \begin{bmatrix} -0.558267605 & 0.758610907 \\ 0.21918836 & 2.389314687 \end{bmatrix}$$

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$$\therefore \mathbf{x}_5 = \begin{bmatrix} 0.627860254 \\ 0.936375465 \end{bmatrix} - \begin{bmatrix} -0.558267605 & 0.758610907 \\ 0.21918836 & 2.389314687 \end{bmatrix} \begin{bmatrix} -3.7941886 \times 10^{-8} \\ -9.2568569 \times 10^{-8} \end{bmatrix}$$

$$= \begin{bmatrix} 0.627860254 \\ 0.936375465 \end{bmatrix} - \begin{bmatrix} -4.904180032 \times 10^{-8} \\ -2.294918613 \times 10^{-7} \end{bmatrix} = \begin{bmatrix} 0.627860303 \\ 0.936375694 \end{bmatrix}$$

.

Evaluating $\mathbf{x}_6$: $\mathbf{x}_6 = \mathbf{x}_5 - \mathbf{J}^{-1}(\mathbf{x}_5)\mathbf{f}(\mathbf{x}_5)$.

$$f(x_5) = \begin{bmatrix} 1 - (0.627860303)\cos(0.936375694) - 0.627860303 \\ (0.627860303)\sin(0.936375694) - 0.1(0.627860303)^2 - 1 + e^{-(0.627860303)} \end{bmatrix}$$

$$= \begin{bmatrix} -1.82134 \times 10^{-10} \\ -1.89156 \times 10^{-10} \end{bmatrix}.$$

$$J(x_5) = \begin{bmatrix} -\cos(0.936375694) - 1 & (0.627860303)\sin(0.9363756 \\ \sin(0.936375694) - 0.2(0.627860303) - e^{-0.627860303} & (0.627860303)\cos(0.9363756 \end{bmatrix}$$

$$= \begin{bmatrix} -1.592710983 & 0.505688249 \\ 0.146110562 & 0.372139697 \end{bmatrix}.$$

$$\mathbf{J}^{-1}(\mathbf{x}_5) = \begin{bmatrix} -0.558267568 & 0.758611219 \\ 0.219188624 & 2.389314807 \end{bmatrix}$$

$$\therefore \mathbf{x}_6 = \begin{bmatrix} 0.627860303 \\ 0.936375694 \end{bmatrix} - \begin{bmatrix} -0.558267568 & 0.758611219 \\ 0.219188624 & 2.389314807 \end{bmatrix} \begin{bmatrix} -1.82134 \times 10^{-10} \\ -1.89156 \times 10^{-10} \end{bmatrix}$$

$$= \begin{bmatrix} 0.627860303 \\ 0.936375694 \end{bmatrix} - \begin{bmatrix} -4.181635845 \times 10^{-11} \\ -4.918749326 \times 10^{-10} \end{bmatrix} = \begin{bmatrix} 0.627860303 \\ 0.936375694 \end{bmatrix}$$

.

$$f(x_6) = \begin{bmatrix} 1 - (0.627860303)\cos(0.936375694) - 0.627860303 \\ (0.627860303)\sin(0.936375694) - 0.1(0.627860303)^2 - 1 + e^{-(0.627860303)} \end{bmatrix}$$

$$= \begin{bmatrix} -1.82134 \times 10^{-10} \\ -1.89156 \times 10^{-10} \end{bmatrix}.$$

Hence the solution to the missile intercept problem is approximated as $(t, \alpha) = (0.627860303, 0.936375694)$, correct to nine decimal places.

Table below shows a summary of the results from solving missile intercept problem.

n	**Approximated** t_n **− values**	**Approximated** α_n **− values**	$f_1(\mathbf{x}_n)$	$f_2(\mathbf{x}_n)$
0	1	1	−0.540302305	−0.095107509
1	0.800558404	1.27701722	−0.032377286	0.151248344
2	0.582373152	0.95207748	0.079855089	$-9.305420505 \times 10^{-4}$
3	0.628071678	0.935948569	$-5.526903117 \times 10^{-4}$	$-1.281584331 \times 10^{-4}$
4	0.627860254	0.936375464	$-3.7941886 \times 10^{-8}$	$-9.2568569 \times 10^{-8}$
5	0.627860303	0.936375694	-1.82134×10^{-10}	-1.89156×10^{-10}
6	0.627860303	0.936375694	-1.82134×10^{-10}	-1.89156×10^{-10}

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From Table we observed that the Newton-Raphson's method facilitated a convergence to a solution of the system as the approximated t_n and α_n values converge simultaneously to the solution of the system while the function values of the system decreasing systematically and converging to zero.

When $(t_1, \alpha_1) = (0.800558404, 1.27701722)$, $f_1(\mathbf{x}_1) = -0.032377286$, $f_2(\mathbf{x}_1) = 0.151248344$ .when $f_1(\mathbf{x}_5) = -1.82134 \times 10^{-10}$,and when $f_2(\mathbf{x}_5) = -1.89156 \times 10^{-10}$ $(t_5, \alpha_5) = (0.627860303, 0.936375694)$.

This indicates that the values of t_5 and α_5 were approximate solution to the system of nonlinear equations since $f_1(\mathbf{x}_5) \cong 0$ and $f_2(\mathbf{x}_5) \cong 0$ implying that $\mathbf{f}(\mathbf{x}_5) \cong 0$. Meanwhile the value of (t_5, α_5) and (t_6, α_6) does not change as shown in the table, indicating that the required real roots of the equation are $(t, \alpha) = (0.627860303, 0.936375694)$, correct to nine decimal places.

III. Conclusion

In this paper we successfully assess and numerical evaluate nonlinear Hamiltonian symmetric matrix of Rank 1 in an inverse eigenvalue problem via Newton-Raphson method.

The approach employed Newton-Raphson's method for solving the inverse eigenvalue problem in a class of Hamiltonian matrices in the neighborhood of a related nonsingular matrix of rank 1. Numerical examples are presented to illustrate the result.

REFERENCES

[1] Bhattacharyya S P.(1991) Linear control theory; structure, robustness and optimization Journal of control system, robotics and automation. CRS Press Vol IX. San Antonio Texas.

[2] Boley D and Golub.G.H (1987).A survey of matrix inverse eigenvalue problem. Inverse Problems.Vol,3 pp 595–622,

[3] Cai, Y.F. Kuo, Y.C Lin W.W and Xu, S.F (2009) Solutions to a quadratic inverse eigenvalue problem, Linear Algebra and its Applications. Vol. 430 pp. 1590-1606.

[4] Datta B.N and Sarkissian, D.R (2004) Theory and computations of some Inverse Eigenvalue Problems for the quadratic pencil. Journal of Contemporary Mathematics,Vol.280 221-240.

[5] Miranda.J.O.A.O. Optimal Linear Quadratic Control. Control system, Robotics and Automation.Vol.VIII INESC.JD/IST,R.Alves,Redol 9.1000-029.Lisboa, Portugal

[6] Oduro F.T., A.Y. Aidoo, K.B. Gyamfi, J. and Ackora-Prah, (2012) Solvability of the Inverse Eigenvalue Problem for Dense Symmetric Matrices" Advances in Pure Mathematics, Vol. 3, pp 14- 19,

[7] Oladejo N.K, Oduro F.T and Amponsah S.K (2014) An Inverse eigenvalue problem for Linear- quadratic optimal control. International Journal of Mathematical Archive-5(4) 306-314

[8] Oladejo N.K and Anang R (2016) Derivation of an explicit function in non-singular Hamilton symmetric matrices of Rank 1 via linear quadratics inverse eigenvalue problem. International Journal of Emerging Technology and Advance Engineering.Vol.6 Issue 7 pg 257-263

[9] Raymond A. B and Michael R. Z (2000). Pre-calculus: Functions and Graphs,

[10] Peter Linz and L. C. Wang (2003) Exploring Numerical Methods, an Introduction to Scientific Computing using MATLA