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Derivation of an Explicit Function in Non-Singular Hamilton Symmetric Matrices of Rank 1 Via Linear Quadratics Inverse Eigenvalue Problem

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Abstract**-- This paper deals with derivation of an explicit function in a non-singular Hamilton symmetric matrices of Rank 1 via linear quadratics inverse eigenvalue problem (LQIEP in the neighborhood of the first type of Hamilton matrices through numerical illustration and examples.**

Keywords: **Explicit, functions, Hamilton, Symmetric, Non-singular, Neighborhood,**

I. Introduction

Various solvability and solubility of the inverse eigenvalue problem for Hamilton matrices together with numerical examples are systematically reviewed under certain singular and non-singular Hamilton matrices by Oduro et al (2012) and Oduro (2012a, b),Baah Gyamfi (2013) as well as Oladejo et al (2014) and Oladejo et.al (2015).This paper deals with derivation of an explicit function in a non-singular Hamilton symmetric matrices of Rank 1 via linear quadratics inverse eigenvalue problem (LQIEP in the neighborhood of the first type of Hamilton matrices through numerical illustration and examples.

The linear-quadratic optimal control (LQOC)

Given that $A \in \Re^{n\times n}, B \in \Re^{n\times m}, Q \in \Re^{n\times n} : Q = Q^T \geq 0$ and $R \in \Re^{m\times m} : R = R^T > 0$ we consider the linear-quadratic optimal control (LQOC) for the functional;

$$I_{xi}(u) = \int_{t_i}^{t_f} \frac{1}{2}\left[x(t)^T Qx(t) + u(t)^T Ru(t)\right]dt \qquad (1)$$

Subject to differential equation

$$\dot{x}(t) = Ax(t) + Bu(t), t \in \left[t_i, t_f\right], x(t_i) = x_i \qquad (2)$$

Then the Hamiltonian function is given by:

$$H(p,x,u,t) = \frac{1}{2}\left[x^T Qx + u^T Ru\right] + p^T\left[Ax + Bu\right] \quad (3)$$

Given any optimal input $u_\bullet$ and the corresponding state $x_\bullet$ we solve equation (3) which is LQOCP arising from the Pontryagin minimum principle in equation (3)

Thus: $\frac{\partial H}{\partial u}(p_\bullet(t), x_\bullet(t), u_\bullet(t), t) = 0$

$$\Rightarrow u_\bullet(t)^T R + p_\bullet(t)^T B = 0 \qquad (4)$$

Then $u_\bullet(t) = -R^{-1}B^T p_\bullet(t) \qquad (5)$

and the adjoint equation is given as:

$$\left[\frac{\partial H}{\partial x}\left(p_\bullet(t), k_\bullet(t), u_\bullet(t), t\right)\right]^T$$
$$= -\dot{p}_\bullet(t), t \in \left[t_i, t_f\right], p_\bullet(t_f) = 0 \qquad (6)$$
$$\Rightarrow \left(x_\bullet(t)^T Q + p_\bullet(t)^T A\right)^T$$
$$= -\dot{p}_\bullet(t), t \in \left[t_i, t_f\right], p_\bullet(t_f) = 0 \qquad (7)$$

We then have;

$$\dot{p}_\bullet(t) = A^T p_\bullet(t) - Qx_\bullet(t), t \in \left[t_i, t_f\right],$$
$$p_\bullet(t_f) = 0 \qquad (8)$$

Consequently;

$$\frac{d}{dt}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix}\begin{bmatrix} x_\bullet(t) \\ p_\bullet(t) \end{bmatrix}, t \in \left[t_i, t_f\right]$$
$$x_\bullet(t_i) = x_i, p_\bullet(t_f) = 0 \qquad (9)$$

Equation (9) is a linear, time variant differential equation in $(x_\bullet, p_\bullet)$

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IEP for $2n\times 2n$ *Hamiltonian Matrix of Rank 1 in respect of LQOCP*

From equation (9) and the theory of the Linear quadratic optimal control we have the following (Hamilton's Equations):

$$\frac{d}{dt}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t)\end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T}\end{bmatrix}\begin{bmatrix} x_{\bullet}(t) \\ p_{\bullet}(t)\end{bmatrix}, t\in\left[t_i, t_f\right],$$

$$x_{\bullet}(t_i) = x_i,\ p_{\bullet}(t_f) = 0 \qquad (10)$$

We then consider the case where the Hamiltonian matrix

$H = \begin{bmatrix} A & -BR^{-1}B^{T} \\ -Q & -A^{T}\end{bmatrix}$ Is $2n\times 2n$ so that *A, Q, BR*$^{-1}B^{T}$ are all is 2×2 sub-matrices of *H.*

Inverse Eigenvalue Problem for a Non-Singular 4×4 *symmetric matrix Newton's Method*

We construct a Characteristic (Polynomial) function of the diagonal elements from the matrix $2n\times 2n$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44}\end{bmatrix} =$$

$$\begin{bmatrix} a_{11}-\lambda & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22}-\lambda & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33}-\lambda & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44}-\lambda\end{bmatrix}$$

$$X^{(0)} = \begin{bmatrix} a_{11}^{(0)} \\ a_{22}^{(0)} \\ a_{33}^{(0)} \\ a_{44}^{(0)}\end{bmatrix}$$

In other words, consider the function with independent variables defined on 4 selected elements of matrix A, precisely, the diagonal elements:

$$f(a_{11}, a_{22}, a_{33}, a_{44}) = \lambda^2 - (trA)\lambda + \det A$$

$$\left(\begin{array}{l} \lambda^4 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44}+ \\ a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44} \\ -a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44} \\ -a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{array}\right)$$

Thus, given 4 distinct eigenvalues $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ we have the following four (4) functions with 4 independent variables being the diagonal element of A.

Thus:

$$f_1(a_{11}, a_{22}, a_{33}, a_{44}) =$$

$$\left(\begin{array}{l} \lambda^4{}_1 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_1 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_1 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44}+ \\ a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44} \\ -a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_1 + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44} \\ -a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{array}\right)$$

$$f_2(a_{11}, a_{22}, a_{33}, a_{44}) =$$

$$\left(\begin{array}{l} \lambda^4{}_2 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_2 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_2 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44}+ \\ a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44} \\ -a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_2 + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44} \\ -a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{array}\right)$$

$$f_3(a_{11}, a_{22}, a_{33}, a_{44}) =$$

$$\left(\begin{array}{l} \lambda^4{}_3 - (a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_3 + (a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_3 - (a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44}+ \\ a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44} \\ -a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_3 + a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44} \\ -a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{array}\right)$$

$$f_4(a_{11},a_{22},a_{33},a_{44}) =$$

$$\left(\begin{matrix} \lambda^4{}_4-(a_{11}+a_{22}+a_{33}+a_{44})\lambda^3{}_4+(a_{11}a_{22}+a_{11}a_{33}+a_{11}a_{44}+a_{22}a_{33} \\ +a_{22}a_{44}+a_{33}a_{44}+a_{34}a_{43}+a_{12}a_{21})\lambda^2{}_4-(a_{11}a_{22}a_{33}+a_{11}a_{21}a_{44}+ \\ a_{11}a_{33}a_{44}+a_{11}a_{33}a_{43}+a_{22}a_{33}a_{44}+a_{11}a_{34}a_{43}-a_{12}a_{21}a_{33}-a_{12}a_{21}a_{44} \\ -a_{13}a_{21}a_{32}+a_{14}a_{21}a_{42})\lambda_4+a_{11}a_{22}a_{33}a_{44}-a_{11}a_{22}a_{34}a_{43}-a_{12}a_{21}a_{33}a_{44} \\ -a_{12}a_{21}a_{34}a_{43}+a_{13}a_{21}a_{32}a_{44}-a_{13}a_{21}a_{34}a_{42}-a_{14}a_{21}a_{43}a_{32}+a_{14}a_{21}a_{42}a_{33} \end{matrix}\right)$$

Derivation of an Explicit Formula for the Jacobian in the 4×4 *matrices case*

$$\frac{\partial f_1}{\partial a_{11}} = -\lambda^3{}_1+\lambda^2{}_1(a_{22}+a_{33}+a_{44})-\lambda_1(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) \qquad \frac{\partial f_1}{\partial a_{22}} = -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{33}+a_{44})-\lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_1}{\partial a_{33}} = -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{44})-\lambda_1(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) \qquad \frac{\partial f_1}{\partial a_{44}} = -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{33})-\lambda_1(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_2}{\partial a_{11}} = -\lambda^3{}_2+\lambda^2{}_2(a_{22}+a_{33}+a_{44})-\lambda_2(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) \qquad \frac{\partial f_2}{\partial a_{22}} = -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{33}+a_{44})-\lambda_2(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_2}{\partial a_{33}} = -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{44})-\lambda_2(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) \qquad \frac{\partial f_2}{\partial a_{44}} = -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{33})-\lambda_2(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_3}{\partial a_{11}} = -\lambda^3{}_3+\lambda^2{}_3(a_{22}+a_{33}+a_{44})-\lambda_3(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) \qquad \frac{\partial f_3}{\partial a_{22}} = -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{33}+a_{44})-\lambda_3(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_3}{\partial a_{33}} = -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{44})-\lambda_3(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) \qquad \frac{\partial f_3}{\partial a_{44}} = -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{33})-\lambda_3(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33})$$

$$\frac{\partial f_4}{\partial a_{11}} = -\lambda^3{}_4+\lambda^2{}_4(a_{22}+a_{33}+a_{44})-\lambda_4(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) \qquad \frac{\partial f_4}{\partial a_{22}} = -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{33}+a_{44})-\lambda_4(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44})$$

$$\frac{\partial f_4}{\partial a_{44}} = -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{22}+a_{33})-\lambda_4(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \qquad \frac{\partial f_4}{\partial a_{33}} = -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{22}+a_{44})-\lambda_4(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44})$$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial a_{11}} & \frac{\partial f_1}{\partial a_{22}} & \frac{\partial f_1}{\partial a_{33}} & \frac{\partial f_1}{\partial a_{44}} \\ \frac{\partial f_2}{\partial a_{11}} & \frac{\partial f_2}{\partial a_{22}} & \frac{\partial f_2}{\partial a_{33}} & \frac{\partial f_2}{\partial a_{44}} \\ \frac{\partial f_3}{\partial a_{11}} & \frac{\partial f_3}{\partial a_{22}} & \frac{\partial f_3}{\partial a_{33}} & \frac{\partial f_3}{\partial a_{44}} \\ \frac{\partial f_4}{\partial a_{11}} & \frac{\partial f_4}{\partial a_{22}} & \frac{\partial f_4}{\partial a_{33}} & \frac{\partial f_4}{\partial a_{44}} \end{bmatrix} \Rightarrow \begin{bmatrix} -\lambda^3{}_1+\lambda^2{}_1(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_1+\lambda^2{}_1(a_{11}+a_{22}+a_{33})- \\ \lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_1(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \\ -\lambda^3{}_3+\lambda^2{}_3(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_2+\lambda^2{}_2(a_{11}+a_{22}+a_{33})- \\ \lambda_2(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_2(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_2(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_2(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \\ -\lambda^3{}_3+\lambda^2{}_3(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_3+\lambda^2{}_3(a_{11}+a_{22}+a_{33})- \\ \lambda_3(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_3(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_3(a_{11}a_{22}+a_{11}a_{44}+a_{22}a_{44}) & \lambda_3(a_{22}a_{33}+a_{11}a_{22}+a_{11}a_{33}) \\ -\lambda^3{}_4+\lambda^2{}_4(a_{22}+a_{33}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{33}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_2(a_{11}+a_{22}+a_{44})- & -\lambda^3{}_4+\lambda^2{}_4(a_{11}+a_{22}+a_{33})- \\ \lambda_4(a_{22}a_{33}+a_{33}a_{44}+a_{22}a_{44}) & \lambda_4(a_{11}a_{33}+a_{33}a_{44}+a_{11}a_{44}) & \lambda_4(a_{11}a_{22}+a_{22}a_{44}+a_{11}a_{44}) & \lambda_4(a_{11}a_{22}+a_{22}a_{33}+a_{11}a_{33}) \end{bmatrix}$$

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Thus

Computing the Determinant we get

$$trA = (-\lambda_1 + a_{22})(-\lambda_2 + a_{11}) - (-\lambda_1 + a_{11})(-\lambda_2 + a_{22})$$
$$\Rightarrow Det. = (\lambda_1 - \lambda_2)(a_{22} - a_{11})$$

And the inverse of the Jacobian matrix is given as:

$$J^{-1} = \frac{1}{(\lambda_1 - \lambda_2)(a_{22} - a_{11})}\begin{bmatrix} a_{22} - \lambda_1 & a_{11} - \lambda_1 \\ a_{22} - \lambda_2 & a_{11} - \lambda_2 \end{bmatrix}$$

Note that the existence of the inverse of the Jacobian matrix requires that both the target eigenvalues and the diagonal elements of the starting matrix be distinct.

While the *(n+1) th* iteration of the Newton's method is given by the following recursive relation

$$X^{(n+1)} = X^{(n)} - J^{-1}(X^{(n)})\underline{f}(X^{(n)})$$

Under these conditions, the first step of Newton's method is given

$$\begin{bmatrix} a_{11}{}^{(1)} \\ a_{22}{}^{(1)} \end{bmatrix} = \begin{bmatrix} a_{11}{}^{(0)} \\ a_{22}{}^{(0)} \end{bmatrix} - \frac{1}{Det}\begin{bmatrix} a_{22}{}^{(0)} - \lambda_1 & a_{11}{}^{(0)} - \lambda_1 \\ a_{22}{}^{(0)} - \lambda_2 & a_{11}{}^{(0)} - \lambda_2 \end{bmatrix}\begin{bmatrix} a_{11}{}^{(0)} \\ a_{22}{}^{(0)} \end{bmatrix}$$

Numerical Examples

Given that the IEP to be solved is to determine completely the nonsingular symmetric coefficient of $2n \times 2n$ matrix of the system

$$\dot{x}_1 = a_{11}x_1 + 2x_2 + 3x_3 + 4x_4$$
$$\dot{x}_2 = 2x_1 + 4a_{22}x_2 - 6x_3 + 8x_4$$
$$\dot{x}_3 = 3x_1 - 6x_2 + 9a_{33}x_3 + 12x_4$$
$$\dot{x}_4 = 4x_1 + 8x_2 + 12x_3 + 16a_{44}x_4$$

$$\text{A} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & -6 & 8 \\ 3 & -6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$$

For Non Singular 2×2 *symmetric matrices case*

$$\dot{x}_1 = a_{11}x_1 + 2x_2$$
$$\dot{x}_2 = 2x_1 + 4a_{22}x_2$$

Given the eigenvalues $\lambda_1 = -1, \lambda_2 = 3$. i.e. we let the target solution of the form

$$x = c_1 ue^{-t} + c_2 ve^{3t}$$

Assuming the initializing matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$; $X^{(0)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Moreover, $A^{(0)}$ is singular

with $k = 2, a_{11} = 1, \lambda = trA^{(0)} = 5$

Computing the values of the functions at the initial point:

$$f_1(a_{11}, a_{22}) = \lambda_1{}^2 - (a_{11} + a_{22})\lambda_1 + (a_{11}a_{22} - a_{12}{}^2)$$
$$f_2(a_{11}, a_{22}) = \lambda_2{}^2 - (a_{11} + a_{22})\lambda_2 + (a_{11}a_{22} - a_{12}{}^2)$$
$$\underline{f}(X^{(0)}) = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

The inverse of the Jacobian matrix yields:

$$J^{-1} = \frac{1}{Det}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix} = -\frac{1}{12}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix}$$

Substituting into the Newton's equation, $X^{(n+1)} = X^{(n)} - J^{-1}(X^{(n)})\underline{f}(X^{(n)})$ we get:

$$X^{(0)} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \frac{1}{12}\begin{bmatrix} -2 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 6 \\ -6 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \Rightarrow X^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Thus: $A(X^1) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

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Non Singular $2n \times 2n$ Symmetric Matrices case

Suppose the IEP to be solved is to determine completely the nonsingular 4×4 symmetric coefficient matrix of the system

$$\dot{x}_1 = a_{11}x_1 + 2x_2 + 3x_3 + 4x_4$$
$$\dot{x}_2 = 2x_1 + 4a_{22}x_2 + 6x_3 + 8x_4$$
$$\dot{x}_3 = 3x_1 + 6x_2 + 9a_{33}x_3 + 12x_4$$
$$\dot{x}_4 = 4x_1 + 8x_2 + 12x_3 + 16a_{44}x_4$$

Given the eigenvalues $\lambda_1 = 1, \lambda_2 = 1, \lambda_3 = 3, \lambda_4 = 5$ i.e. We let the target solution of the form

$$x = c_1 ue^{t} + c_2 ve^{2t} + c_3 we^{5t}$$

Given the initializing matrix A= $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{bmatrix}$;

$$X^{(0)} = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \end{bmatrix}$$

Here, $A^{(0)}$ is singular while $k_1 = 2, k_2 = 3, k_3 = 4, a_{11} = 1$ and $\lambda = trA^{(0)} = 30$

We compute the values of the functions at the initial point:

Thus; $\underset{-}{f}\left(X^{(0)}\right) = \begin{bmatrix} -28 \\ -28 \\ 10 \\ 112 \end{bmatrix}$ and

$$J = \begin{bmatrix} -11 & -8 & -12 & -6 \\ 35 & -8 & -12 & -6 \\ -3 & -6 & -12 & -6 \\ 5 & -20 & -20 & -30 \end{bmatrix}$$

Estimating determinant and substituting into the Newton's equation yielded

$$X^{(1)} =$$
$$\begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \end{bmatrix} - \frac{1}{23040} \begin{bmatrix} -11 & -35 & -3 & 5 \\ -8 & -8 & -6 & -20 \\ -12 & -12 & -12 & -20 \\ -6 & -6 & -6 & 30 \end{bmatrix} \begin{bmatrix} -28 \\ -28 \\ 10 \\ 112 \end{bmatrix}$$

$$X^{(1)} = \begin{bmatrix} 1 \\ 4 \\ 9 \\ 16 \end{bmatrix} - \begin{bmatrix} 0.0672 \\ -0.0803 \\ -0.0733 \\ 0.1578 \end{bmatrix} = \begin{bmatrix} 1.0672 \\ 4.0803 \\ 9.0733 \\ 15.8422 \end{bmatrix}$$

Hence,

$$A(X^{(1)}) = \begin{bmatrix} 1.067 & 2 & 3 & 4 \\ 2 & 4.080 & 4 & 8 \\ 3 & 6 & 9.073 & 12 \\ 4 & 8 & 12 & 15.842 \end{bmatrix}$$

II. CONCLUSION

Various theoretical results have been systematically reviewed and discussed in respect of the inverse eigenvalue problem (IEP). Based on these results we developed the derivation of an explicit function in non-singular Hamilton symmetric matrices of Rank 1 via linear quadratics inverse eigenvalue problem (LQIEP in the neighborhood of the first type of Hamilton matrices through numerical illustration and examples.

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